Applying the Probability Method for Multi-Objective Optimization of the X12M Steel Grinding Process

Nguyen Quang Manh
Institute of Technology, Vietnam Defence Industry, Hanoi, Vietnam
nguyenquangmanhvdi@gmail.com & manhmin@gmail.com

***Abstract*—Optimizing mechanical machining processes plays a crucial role and significantly impacts the efficiency of the manufacturing process. This study applies the probability method to optimize the grinding process of X12M material. An experimental process was implemented consisting of 9 experiments designed according to the Taguchi matrix. In each experiment, the values of the cutting parameters, including workpiece velocity, feed rate, and depth of cut, were varied. Four parameters were measured in each experiment: surface roughness, the cutting force component in the X direction, the cutting force component in the Y direction, and the cutting force component in the Z direction. Two methods, MEREC and SPC, were used to calculate the weights for the responses, while the Probability method was used to rank the experiments. The results indicated that the experiment corresponding to the workpiece velocity, feed rate, and depth of cut values of 8 (m/min), 4 (mm/stroke), and 0.02 (mm), respectively, was the best among the performed experiments. These represent the optimal values of the cutting parameters. At these optimal cutting parameter values, the surface roughness and the cutting force components in the X, Y, and Z directions were 0.22 (μm), 40.48 (N), 39.52 (N), and 90.88 (N), respectively.**

Keywords—Grinding, X12M steel, MCDM, Probability, weight method

1. INTRODUCTION

Grinding is the most commonly used method for finishing mechanical part surfaces that require high precision and low surface roughness [1-3]. Numerous factors influence the grinding process, such as cutting parameters, grinding wheel types, coolant types, workpiece characteristics, the grinding machine, and various other elements [4-6]. Among these, cutting parameters significantly impact the efficiency of the grinding process and have been investigated in many studies [7-9]. Determining the optimal values of cutting parameters in grinding to simultaneously ensure multiple objectives has also been carried out in various published works, where different optimization methods have been employed. Some examples include using the TOPSIS method for multi-objective optimization of the SKD11 steel grinding process using CBN wheels [10]. In [11], the CURLI method was applied for the multi-objective optimization of the X12M steel grinding process. The TOPSIS and AHP methods were integrated to optimize the gear grinding process [12]. Furthermore, the MOORA and COPRAS methods were combined for the multi-objective optimization of the SKD11 steel grinding process [13]. The application of the MARCOS method for multi-objective optimization of the SCM400 steel grinding process was performed in [14], etc.

Probability is a multi-objective optimization method with the advantage of not using "additive" calculations, thereby ensuring high accuracy [15]. This method has been used for multi-objective optimization in several recently published studies, such as optimizing material selection [15], optimizing machine tool selection [16], ranking logistics industry benchmarks of several universities [17], and designing the flight plate of a scraper conveyor [18]. In this study, the Probability method will be applied for the multi-objective optimization of the X12M steel grinding process. Section 2 presents the steps for applying the MEREC and SPC methods to calculate weights for criteria, as well as the steps for solving the optimization problem using the Probability method. These methods are applied to solve the multi-objective optimization problem of the X12M steel grinding process in Section 3 of this article. The conclusions drawn wrap up this research.

2. METHODS USED

To apply the Probability method for multi-objective optimization, it is essential to calculate the weights for the criteria. In this study, two methods, MEREC and SPC, were used to calculate these weights.

To calculate the weights for the criteria using the MEREC method, the following steps are applied sequentially [19].

Step 1: Construct the decision matrix with *m* rows and *n* columns, where *m* is the number of alternatives to be ranked (number of experiments) and *n* is the number of criteria to evaluate each alternative (number of criteria to evaluate the grinding process). Let x_{ij} be the value of criterion j for alternative i, with j = 1÷ n, i = 1 ÷ m.

Step 2: Calculate the normalized values according to formulas (1) and (2).

For the larger-the-better criteria.

$$n_{ij} = \frac{min x_{ij}}{x_{ij}} \quad (1)$$

For the smaller-the-better criteria.

$$n_{ij} = \frac{x_{ij}}{max\, x_{ij}} \quad (2)$$

Step 3: Calculate the overall performance of the alternatives according to formula (3).

$$S_i = Ln\left[1 + \left(\frac{1}{n}\sum_j^n \left|ln(n_{ij})\right|\right)\right] \quad (3)$$

Step 4: Calculate the performance of the alternatives by removing each criterion according to formula (4).

$$S'_{ij} = Ln\left[1 + \left(\frac{1}{n}\sum_{k,k\neq j}^n \left|ln(n_{ij})\right|\right)\right] \quad (4)$$

Step 5: Calculate the absolute values of the deviations according to formula (5).

$$E_j = \sum_i^m \left|S'_{ij} - S_i\right| \quad (5)$$

Step 6: Calculate the weights for the criteria according to formula (6).

$$w_j = \frac{E_j}{\sum_k^n E_k} \quad (6)$$

To calculate the weights for the criteria using the SPC method, the following steps are applied sequentially [20].

Step 1: Similar to Step 1 of the MEREC method.

Step 2: Calculate the SPC value for each criterion according to formula (7).

$$SPC_j = \frac{\max(x_{ij}) + \min(x_{ij})}{2};\ i = 1, 2, \ldots, m;\ \forall j \in [1 \div n] \quad (7)$$

Step 3: Create the absolute distance matrix according to formula (8).

$$D = \left|d_{ij}\right|_{mxn} = \begin{bmatrix} |x_{11} - SPC_1| & |x_{12} - SPC_2| & \cdots & |x_{1n} - SPC_n| \\ |x_{21} - SPC_1| & |x_{22} - SPC_2| & \cdots & |x_{2n} - SPC_n| \\ \cdots & \cdots & \cdots & \cdots \\ |x_{m1} - SPC_1| & |x_{m2} - SPC_2| & \cdots & |x_{mn} - SPC_2| \end{bmatrix} \quad (8)$$

Step 4: Create the matrix of symmetric modules according to formula (9).

$$R = [r_{ij}]_{m\times n} = \begin{bmatrix} \left|\frac{\sum_{i=1}^m d_{i1}}{m\times x_{11}}\right| & \left|\frac{\sum_{i=1}^m d_{i2}}{m\times x_{12}}\right| & \cdots & \left|\frac{\sum_{i=1}^m d_{in}}{m\times x_{1n}}\right| \\ \left|\frac{\sum_{i=1}^m d_{i1}}{m\times x_{21}}\right| & \left|\frac{\sum_{i=1}^m d_{i2}}{m\times x_{22}}\right| & \cdots & \left|\frac{\sum_{i=1}^m d_{in}}{m\times x_{2n}}\right| \\ \cdots & \cdots & \cdots & \cdots \\ \left|\frac{\sum_{i=1}^m d_{i1}}{m\times x_{m1}}\right| & \left|\frac{\sum_{i=1}^m d_{i2}}{m\times x_{m2}}\right| & \cdots & \left|\frac{\sum_{i=1}^m d_{in}}{m\times x_{mn}}\right| \end{bmatrix} \quad (9)$$

Step 5: Calculate the symmetric module of the criteria according to formula (10).

$$Q = [q_{1j}]_{1\times n} = \left[\frac{\sum_{i=1}^m r_{i1}}{m} \quad \frac{\sum_{i=1}^m r_{i2}}{m} \quad \ldots \quad \frac{\sum_{i=1}^m r_{in}}{m}\right] \quad (10)$$

Step 6: Calculate the weights of the criteria according to formula (11).

$$W = [w_{1j}]_{1\times n} = \left[\frac{q_1}{\sum_{j=1}^n q_j} \quad \frac{q_2}{\sum_{j=1}^n q_j} \quad \ldots \quad \frac{q_j}{\sum_{j=1}^n q_j}\right] \quad (11)$$

The steps to solve the optimization problem using the Probability method include [15]:

Step 1: Similar to Step 1 of the MEREC method.

Step 2: Calculate the probability of achieving a favorable result.

For benefit criteria, the probability of achieving a favorable result in the decision-making process increases linearly and is calculated according to formula (12).

$$P_{ij} \infty X_{ij}, \quad P_{ij} = \alpha_j X_{ij,} i = 1, 2, \ldots, m, \quad j = 1,2,..n \quad (12)$$

In which α_j is the normalization coefficient of the j^{th} benefit criterion and is calculated according to (13).

$$\alpha_j = \frac{1}{\sum_{i=1}^{m} x_{ij}} \quad (13)$$

For cost criteria, the probability of achieving a favorable result in the decision-making process is also a linear function and is calculated according to formula (14).

$$P_{ij} \infty \left(x_{jmax} + x_{jmin} - x_{ij}\right), \quad P_{ij} = \beta_j\left(x_{jmax} + x_{jmin} - x_{ij}\right) \\ i = 1, 2, \dots, m, \quad j = 1, 2, \dots n \quad (14)$$

In which β_j is the normalization coefficient of the j^{th} cost criterion and is calculated according to formula (15).

$$\beta_j = \frac{1}{m\left(x_{jmax} + x_{jmin} - \frac{\sum_{i=1}^{m} x_{ij}}{m}\right)} \quad (15)$$

Step 3: Calculate the overall favorable probability of each alternative according to formula (16).

$$P_i = \prod_{j=1}^{n} \left(P_{ij}\right)^{w_j} \quad (16)$$

Step 4: Rank the alternatives according to the principle that the best alternative is the one with the largest overall probability.

3. RESULTS AND DISCUSSION

Table 1 summarizes the data from an experimental grinding process of X12M steel. The data table consists of 9 experiments designed according to the Taguchi method. This is a common type of experimental design used in mechanical manufacturing research [21]. In each experiment, the values of three cutting parameters were varied: workpiece velocity (v), feed rate (f), and depth of cut (t). Also in each experiment, parameters including surface roughness (Ra), cutting force component in the X direction (Fx), cutting force component in the Y direction (Fy), and cutting force component in the Z direction (Fz) were measured.

Table 1. Experimental results of X12M steel grinding

Exp.	Input parameters			Responses			
	v(m/min)	f(mm/stroke)	t(mm)	Ra (µm)	Fx (N)	Fy (N)	Fz (N)
#1	4	4	0.01	0.37	47.74	29.38	86.72
#2	4	8	0.02	0.28	75.9	53.3	77.76
#3	4	12	0.03	0.34	86.68	42.64	83.84
#4	8	4	0.02	0.22	40.48	39.52	90.88
#5	8	8	0.03	0.23	49.5	53.56	97.28
#6	8	12	0.01	0.19	65.12	51.48	99.84
#7	10	4	0.03	0.43	69.74	59.02	72.96
#8	10	8	0.01	0.37	71.94	74.36	97.92
#9	10	12	0.02	0.33	61.82	47.84	100.8

According to the data in Table 1, it is observed that the smallest surface roughness occurs in experiment #6, the smallest cutting force component in the X direction in experiment #4, the smallest cutting force component in the Y direction in experiment #1, and the smallest cutting force component in the Z direction in experiment #7. Thus, clearly, there is no single experiment where all four components reach their minimum values. This leads to the necessity of solving a multi-objective optimization problem to find the experiment where all four parameters are considered "minimal."

The decision matrix consists of the columns containing the values of the parameters Ra, Fx, Fy, and Fz in Table 1.

Applying formulas (1) to (6), the weights for the criteria Ra, Fx, Fy, and Fz calculated by the MEREC method were 0.2954, 0.2676, 0.3376, and 0.0994, respectively. Applying formulas (7) to (11), the weights for the criteria Ra, Fx, Fy, and Fz calculated by the SPC method were 0.3270, 0.2725, 0.2646, and 0.1359, respectively. Figure 1 shows the chart illustrating the weights of the criteria calculated by the MEREC and SPC methods.

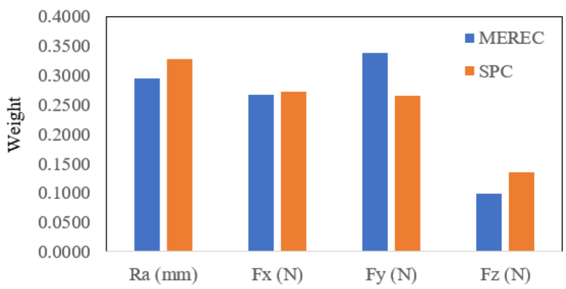

Figure 1. Weights of the criteria

Observing Figure 1, it is noted that although calculated by two different methods, the weights of each criterion Ra, Fx, Fy, and Fz have values that are relatively close to each other. In general, the three parameters Ra, Fx, and Fy have comparable weights, while Fz is the criterion with the smallest weight.

The next task is to use the weight values of the criteria calculated by the MEREC and SPC methods to rank the experiments using the Probability method. Applying formulas (12) to (16), the overall favorable probability values (Pi) for each experiment were calculated as summarized in Table 2. This table also summarizes the ranking of the experiments in the two cases where the weights of the criteria were calculated by the two different methods.

Table 2. Overall favorable probability (Pi) and ranking of experiments

TT	MEREC		SPC	
	Pi	Xếp hạng	Pi	Xếp hạng
#1	0.1234	2	0.1200	3
#2	0.1065	6	0.1077	6
#3	0.1001	7	0.0988	7
#4	0.1375	1	0.1369	1
#5	0.1210	3	0.1221	2
#6	0.1185	4	0.1193	4
#7	0.0892	8	0.0895	8
#8	0.0807	9	0.0834	9
#9	0.1092	5	0.1081	5

According to the data in Table 2, the ranking of the experiments shows a high level of consistency when the weights of the criteria are calculated by the MEREC and SPC methods. 7 out of 9 experiments have consistent rankings when the weights of the criteria were calculated by the two different methods. There is only a swap between rank 2 and rank 3 between experiment #1 and experiment #5. Specifically, when using the MEREC method to calculate the weights for the criteria, experiment #1 ranks 2nd and experiment #5 ranks 3rd; meanwhile, if using the SPC method to calculate the weights, experiment #1 ranks 3rd and experiment #5 ranks 2nd. Notably, experiment #4 is consistently identified as the best experiment among the 9 experiments performed. Accordingly, the optimal values for workpiece velocity, feed rate, and depth of cut are 8 (m/min), 4 (mm/stroke), and 0.02 (mm), respectively. At that point, the parameters Ra, Fx, Fy, and Fz have values of 0.22 (μm), 40.48 (N), 39.52 (N), and 90.88 (N), respectively.

4. CONCLUSIONS

For the first time, the Probability method has been applied in this study for the multi-objective optimization of the X12M steel grinding process, in which the MEREC and SPC methods were used to calculate the weights for the criteria. Several conclusions are drawn as follows:

The cutting force component in the Z direction has a significantly smaller weight compared to the other components, including surface roughness, the cutting force component in the X direction, and the cutting force component in the Y direction.

The weights of each parameter, surface roughness, cutting force component in the X direction, cutting force component in the Y direction, and cutting force component in the Z direction—are quite close in value when calculated by the two different methods, MEREC and SPC. This results in a high degree of stability in the ranking of the experiments when the weights of the criteria are determined by different methods.

The optimal values for workpiece velocity, feed rate, and depth of cut are 8 (m/min), 4 (mm/stroke), and 0.02 (mm), respectively. At these values, the parameters Ra, Fx, Fy, and Fz reach values of 0.22 (μm), 40.48 (N), 39.52 (N), and 90.88 (N), respectively.

REFERENCES

[1] R. K. Panthang, V. Naduvinamani. (2017). Optimization of Surface Roughness in Cylindrical Grinding Process, International Journal of Applied Engineering Research, 12(8), 7350-7354.

[2] S. Malkin, C. Guo. (2008). Grinding Technology: Theory and Applications of Machining with Abrasives, Industrial Press.

[3] N. -T. Nguyen, D. D.Trung. (2020). A study on the surface grinding process of the SUJ2 steel using CBN slotted grinding wheel, AIMS Materials Science, 7(6), 871–886.

[4] A. V. Agapovichev, A. V. Sotov, R. R. Kyarimov, V. P. Alexeev, V. G. Smelov, V. S. Sufiiarov, D. V. Masaylo. (2018). The investigation of microstructure and mechanical properties of tool steel produced by selective laser melting technology, Materials Science and Engineering 441 (012003), 1-10.

[5] L. X. Hung, V. N. Pi, L. A. Tung, H. X. Tu, G. Jun, B. T. Long. (2018). Determination of Optimal Exchanged Grinding Wheel Diameter when Internally Grinding Alloy Tool Steel 9CrSi, IOP Materials Science and Engineering 417(012026), 1-9.

[6] [J. M. Menendez-Aguado, Grinding and Concentration Technology of Critical Metals, MPDI, 2022.

[7] V. T. N. Uyen, N. H. Son. (2021). Improving accuracy of surface roughness model while turning 9XC steel using a Titanium Nitride-coated cutting tool with Johnson and Box-Cox transformation, AIMS Materials Science, 8(1), 1–17.

[8] D. D. Trung. (2021). The combination of TAGUCHI – ENTROPY – WASPAS - PIV methods for multi-criteria decision making when external cylindrical grinding of 65G steel, *Journal of Machine Engineering, 21(4), 2021, 90–105.*

[9] D. D. Trung. (2021). Influence of Cutting Parameters on Surface Roughness in Grinding of 65G Steel, Tribology in Industry, 43(1), 167-176.

[10] B. T. Danh, V. V. Khoa, N. H. Linh, H. X. Tu, N. V. Tung, B. T. Hien. (2023). Multi-criteria decision making in CBN grinding tool steel using

TOPSIS method, Mechanics & Mechanical engineering, 94-99.

[11] D. D. Trung, N. N. Ba, D. H. Tien. (2022). Application of the CURLI method for multi-critical decision of grinding process, Journal of Applied Engineering Science, 20(3), 634-643.

[12] K. H. *Nguyen, D. V. Pham, Q. V. Tran. (2023). A multi-criteria decision-making in relieving grinding process of surface of gear milling tooth based on the archimedean spiral using taguchi-ahp-topsis method. EUREKA: Physics and Engineering, 4, 87–103.*

[13] N. -T. Nguyen, Do Duc Trung. (2021). Combination of TAGUCHI method, MOORA and COPRAS techniques in multi-objective optimization of surface grinding process, ournal of Applied Engineering Science, 19(2), 390 – 398.

[14] D. D. Trung. (2022). Multi-criteria decision making under the MARCOS method and the weighting methods: applied to milling, grinding and turning processes, Manufacturing review, 9(3), 1-13.

[15] M. Zheng, H. Teng, Y. Wang. (2021). A simple approach for multi-criteria decision-making on basis of probability theory, Engineering Structures and Technologies, 13(1), 26-30.

[16] D. D. Trung, M. T. Diep, D. V. Duc, N. C. Bao, N. H. Son. (2024). Application of probability theory in machine selection, Applied Engineering Letters, 9(4), 203-214.

[17] V. T. N. Uyen, N. Ersoy, D. D. Trung, N. T. P. Giang. (2024). Applying MCDM to evaluate benchmark scores in the logistics sector for the period 2021–2023: Application to universities in Vietnam, Journal of Infrastructure, Policy and Development, 8(11), 8942.

[18] M. Zheng, J. Yu. (2024). Probabilistic approach for robust design with orthogonal experimental methodology in case of target the best, J. Umm Al-Qura Univ. Eng.Archit. 15, 55–59.

[19] D. D. Trung, H. X. Thinh. (2021). A multi-criteria decision-making in turning process using the MAIRCA, EAMR, MARCOS and TOPSIS methods: A comparative study, Advances in Production Engineering & Management, 16(4), 443-456.

[20] Z. Gligoric, M. Gligoric, I. Miljanovic, S. Lutovac, A. Milutinovic. (2023). Assessing Criteria Weights by the Symmetry Point of Criterion (Novel SPC Method)–Application in the Efciency Evaluation of the Mineral Deposit Multi-Criteria Partitioning Algorithm, Computer modeling in Engineering & Sciences, 136(1), 955-979.

[21] D. D. Trung. (2021). Application of TOPSIS and PIV methods for multi-criteria decision making in hard turning process, *Journal of Machine Engineering, 21(4), 57–71.*