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Comparison of Two Levels of Generalization of a Circuit Design Problem

Alexander Zemliak
Department of Physics and Mathematics
Autonomous University of Puebla
Puebla, Mexico
azemliak@yahoo.com

***Abstract*—The design process of the analog circuit is formulated based on the optimal control theory. A special control vector is defined to redistribute the computational costs between the circuit analysis and parametric optimization. This redistribution permits the minimization of a computer time. The problem of the minimal-time circuit design can be formulated in this case as a classical problem of the optimal control for some functional minimization. The fundamental difference between this new approach and the previously developed generalized methodology is a higher level of generalization. In this case, the structural basis of various design strategies turns out to be more complete, and this circumstance makes it possible to obtain a greater amount of machine time gain. Numerical results demonstrate the effectiveness and prospects of the proposed approach.**

Keywords—Circuit optimization; optimal algorithm; control theory; generalized methodology

I. INTRODUCTION

Reducing the computational time of designing large systems is one source of overall improvement in design quality. This problem is of great importance because it has many applications, for example, in the design of VLSI electronic circuits. Any traditional system design strategy includes two main parts: the mathematical model of the physical system that can be defined by the algebraic equations or differential-integral equations and optimization procedure that achieves the optimum point of the design objective function. In limits of this conception it is possible to change optimization strategy and use the different models and different methods of analysis but in each step of the circuit optimization process there are a fixed number of the equations of the mathematical model and a fixed number of the independent parameters of the optimization procedure.

There are some powerful methods that reduce the necessary time for the circuit analysis. Because a matrix of the large-scale circuit is a very sparse, the special sparse matrix techniques are used successfully for this purpose [1]-[2]. Other approach to reduce the amount of computational required for both linear and nonlinear equations is based on the decomposition techniques. The partitioning of a circuit matrix into bordered-block diagonal form can be done by branches tearing as in [3], or by nodes tearing as in [4] and jointly with direct solution algorithms gives the solution of the problem. The extension of the direct solution methods can be obtained by hierarchical decomposition and macro model representation [5]. Other approach for achieving decomposition at the nonlinear level consists on a special iteration techniques and has been realized in [6] for the iterated timing analysis and circuit simulation. Optimization technique that is used for the circuit optimization and design, exert a very strong influence on the total necessary computer time too. The numerical methods are developed both for the unconstrained and for the constrained optimization [7] and will be improved later on. The practical aspects of these methods were developed for the electronic circuits design with the different optimization criterions [8]-[9]. The fundamental problems of the development, structure elaboration, and adaptation of the automation design systems have been examine in some papers [10]-[13].

The above described system design ideas can be named as the traditional approach or the traditional strategy because the analysis method is based on the Kirchhoff laws.

The other formulation of the circuit optimization problem was developed in heuristic level some decades ago [14]. This idea was based on the Kirchhoff laws ignoring for all the circuit or for the circuit part. The special cost function is minimized instead of the circuit equation solving. This idea was developed in practical aspect for the microwave circuit optimization [15] and for the synthesis of high-performance analog circuits [16] in extremely case, when the total system model was eliminated. The last papers authors affirm that the design time was reduced significantly. This last idea can be named as the modified traditional design strategy.

Nevertheless all these ideas can be generalized to reduce the total computer design time for the system design. This generalization can be done on the basis of the control theory approach and includes the special control function to control the design process. This approach consists of the reformulation of the total design problem and generalization of it to obtain a set of different design strategies inside the same optimization procedure [17]. The number of the different design strategies, which appear in the generalized theory, is equal to 2^M for the constant value of all the control functions, where *M* is the number of dependent parameters. These strategies serve as the structural basis for more strategies construction with the variable control functions. The main problem of this new formulation is the unknown optimal dependency of the control function vector that satisfies to the time-optimal design algorithm.

However, the developed theory [17] is not the most general. In the limits of this approach only initially dependent system parameters can be transformed to the independent but the inverse transformation is not supposed. The next more general approach for the system design supposes that initially independent and dependent system parameters are completely equal in rights, i.e. any system parameter can be defined as independent or dependent one. In this case we have more vast set of the design strategies that compose the structural basis and more possibility to the optimal design strategy construct.

II. Problem Formulation

In accordance with the new design methodology [17] the design process is defined as the problem of the cost function $C(X)$ minimization for $X \in R^N$ by the optimization procedure, which can be determined in continuous form as:

$$\frac{dx_i}{dt} = f_i(X,U), \quad i = 1,2,\dots,N \qquad (1)$$

and by the analysis of the electronic system model in the next form:

$$(1-u_j)g_j(X) = 0, \quad j = 1,2,\dots,M \qquad (2)$$

where *N=K+M*, *K* is the number of independent system parameters, *M* is the number of dependent system parameters, *X* is the vector of all variables $X = (x_1, x_2, \dots, x_K, x_{K+1}, x_{K+2}, \dots, x_N)$; *U* is the vector of control variables $U = (u_1, u_2, \dots, u_M)$; $u_j \in \Omega$; $\Omega = \{0;1\}$.

The functions of the right part of the system (1) are depended from the concrete optimization algorithm and, for instance, for the gradient method are determined as:

$$f_i(X,U) = -b\frac{\delta}{\delta x_i}\left\{C(X) + \frac{1}{\varepsilon}\sum_{j=1}^{M} u_j g_j^2(X)\right\} \qquad (3)$$

for $i = 1,2,\dots,K$,

$$f_i(X,U) = -b\cdot u_{i-K}\frac{\delta}{\delta x_i}\left\{C(X) + \frac{1}{\varepsilon}\sum_{j=1}^{M} u_j g_j^2(X)\right\} + \frac{(1-u_{i-K})}{dt}\left\{-x'_i + \eta_i(X)\right\} \qquad (3')$$

for $i = K+1, K+2, \dots, N$,

where *b* is the iteration parameter; the operator $\delta / \delta x_i$ hear and below means $\frac{\delta}{\delta x_i}\varphi(X) = \frac{\partial\varphi(X)}{\partial x_i} + \sum_{p=K+1}^{K+M}\frac{\partial\varphi(X)}{\partial x_p}\frac{\partial x_p}{\partial x_i}$, x'_i is equal to $x_i(t-dt)$; $\eta_i(X)$ is the implicit function ($x_i = \eta_i(X)$) that is determined by the system (2), *C(X)* is the cost function of the design process.

The problem of the optimal design algorithm searching is determined now as the typical problem of the functional minimization of the control theory. The total computer design time serves as the necessary functional in this case. The optimal or quasi-optimal problem solution can be obtained on the basis of analytical [18] or numerical [19]-[22] methods. By this formulation the initially dependent parameters for $i = K+1, K+2, \dots, N$ can be transformed to the independent ones when u_j=1 and it is independent when u_j=0. On the other hand the initially independent parameters for $i = 1,2,\dots,K$, are independent ones always.

We have been developed in the present paper the new approach that permits to generalize more the above described design methodology. We suppose now that all of the system parameters can be independent or dependent ones. In this case we need to change the equation (2) for the system model definition and the equation (3) for the right parts description.

The equation (2) defines the system model and is transformed now to the next one:

$$(1-u_i)g_j(X) = 0 \qquad (4)$$

$$i = 1,2,\dots,N \quad \text{and} \quad j \in J$$

where *J* is the index set for all those functions $g_j(X)$ for which u_i = 0, *J* = {j_1, j_2, . . .,j_Z}, $j_s \in \Pi$ with *s* = 1, 2, . . ., *Z*, Π is the set of the indexes from 1 to *M*, Π = {1, 2, . . ., *M*}, *Z* is the number of the equations that will be left in the system (4), *Z* ∊{0, 1. . ., *M*}.

The right hand side of the system (1) is defined now as:

$$f_i(X,U) = -b\cdot u_i\frac{\delta}{\delta x_i}F(X,U) + \frac{(1-u_i)}{dt}\left\{-x_i(t-dt) + \eta(X)\right\} \qquad (5)$$

for $i = 1,2,\dots,N$,

where *F(X,U)* is the generalized objective function and it is defined as:

$$F(X,U) = C(X) + \frac{1}{\varepsilon}\sum_{j\in\Pi\setminus J} g_j^2(X) \qquad (6)$$

This new definition of the design process is more general than in [17]. It generalizes the methodology for the system design and produces more representative structural basis of different design strategies. The total number of the different strategies, which compose the structural basis, is equal to $\sum_{i=0}^{M} C_{K+M}^{i}$. We expect the new possibilities to accelerate the design process.

III. NUMERICAL RESULTS

Some non-linear electronic circuits have been analyzed. The numerical results correspond to the integration of the system (1) with variable optimized step. The cost function *C(X)* has been defined as a sum of squares of differences between before defined and current value of some node voltages.

A. *Example 1*

A simple two-node nonlinear passive circuit is presented in Fig. 1. The design procedure was realized by means of the new generalized methodology.

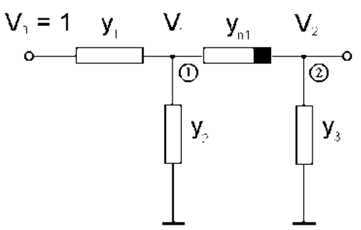

Fig. 1. *Two-node circuit topology.*

The nonlinear element has the following dependency: $y_{n1} = y_0 + b(V_1 - V_2)^2$. The vector *X* includes 5 components: $x_1^2 = y_1$, $x_2^2 = y_2$, $x_3^2 = y_3$, $x_4 = V_1$, $x_5 = V_2$. The model of this circuit (4) includes two equations (*M*=2) and the optimization procedure (5) includes five equations. There are four different design strategies in the structural basis according to the generalized methodology of the first level. Nevertheless, in the limits of the second level of generalization, the total structural basis contains $\sum_{i=0}^{2} C_5^i = 16$ different design strategies. The system (4) is solved by the Newton-Raphson method. The cost function *C(X)* is defined by the following form: $C(X) = (x_4 - k_1)^2 + (x_5 - k_2)^2$. The results for some strategies of full structural basis are presented in Table 1.

Four last strategies of the table are the same that had been defined inside the previously (first level) formulated methodology. We can name these strategies as the "old" ones. It is very interesting that some new strategies have the computer time significantly lesser than all the "old" strategies. The strategy number 1 with the control vector (01011) has the minimal computer time among all the strategies and it has the maximum time gain 12.2 with respect to the traditional design strategy (TDS) that corresponds to the control vector (11100). At the same time the modified traditional design strategy (MTDS) that corresponds to the control vector (11111) is the best among all "old" strategies and has the time gain 1.67 only. Strategy 1 has an additional time gain 7.3 times.

TABLE I. SOME STRATEGIES OF THE STRUCTURAL BASIS FOR TWO-NODE CIRCUIT

N	Control functions vector U (u1,u2,u3,u4,u5)	Calculation results: Iterations number	Total design time (sec)
1	(0 1 0 1 1)	5	0.000851
2	(0 1 1 1 1)	178	0.016671
3	(1 0 0 1 1)	201	0.026235
4	(1 0 1 1 1)	3162	0.300012
5	(1 1 0 0 1)	23	0.002205
6	(1 1 0 1 0)	49	0.100011
7	(1 1 0 1 1)	49	0.002405
8	(1 1 1 0 0)	107	0.010365
9	(1 1 1 0 1)	1063	0.170011
10	(1 1 1 1 0)	143	0.013115
11	(1 1 1 1 1)	243	0.006215

B. *Example 2*

The four-node circuit is analyzed below (Fig. 2) by means of the new generalized methodology.

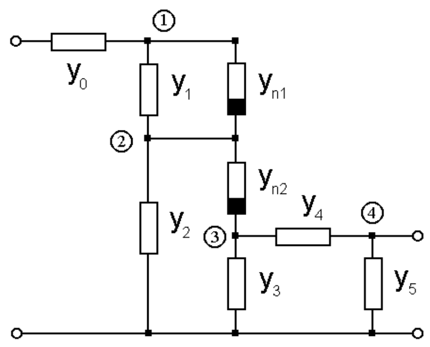

Fig. 2. *Four-node circuit topology.*

The design problem includes five parameters as admittances $(x_1, x_2, x_3, x_4, x_5)$, where $x_1^2 = y_1$, $x_2^2 = y_2$, $x_3^2 = y_3$, $x_4^2 = y_4$, $x_5^2 = y_5$, and four parameters as nodal voltages (x_6, x_7, x_8, x_9), where $x_6 = V_1$, $x_7 = V_2$, $x_8 = V_3$, $x_9 = V_4$. The control vector *U* includes nine components $(u_1, u_2, ..., u_9)$. The nonlinear elements were defined by the same dependencies like in the previous example. The circuit model includes four equations of the system (4) and the optimization procedure includes nine equations (1), (5). The system (4) is solved by the Newton-Raphson method. The cost function *C(X)* of the design process is defined by the following form:

$$C(X)=(x_9-k_0)^2+(x_6-x_7-k_1)^2+(x_7-x_8-k_2)^2.$$

The total number of the different design strategies that compose the structural basis of the generalized theory is equal $\sum_{i=0}^{4} C_9^i = 256$. At the same time the structural basis of the previous developed theory includes 16 strategies only (2^4). The results of the analysis of some strategies of structural basis that include all the “old” strategies (from 10 to 25) and new strategies (from 1 to 9) are shown in Table 2.

TABLE II. SOME STRATEGIES OF THE STRUCTURAL BASIS FOR TWO-NODE CIRCUIT

N	Control vector U (u1,u2,...,u9)	Calculation results Iterations namber	Total CPU time (s)
1	(1 1 1 0 1 0 0 0 1)	5	0.0031
2	(1 1 1 0 1 1 0 0 1)	5	0.0029
3	(1 1 1 1 0 0 1 0 1)	101	0.0232
4	(1 1 1 0 1 0 0 1 1)	15	0.0134
5	(1 1 1 0 1 1 1 0 1)	5	0.0009
6	(1 1 1 0 1 1 1 1 1)	101	0.0243
7	(1 1 1 1 0 0 1 1 1)	185	0.0324
8	(1 1 1 1 0 1 0 0 1)	74	0.0102
9	(1 1 1 1 0 1 1 1 1)	159	0.0127
10	(1 1 1 1 1 0 0 0 0)	33	0.0263
11	(1 1 1 1 1 0 0 0 1)	397	0.4317
12	(1 1 1 1 1 0 0 1 0)	6548	7.1392
13	(1 1 1 1 1 0 0 1 1)	76	0.0122
14	(1 1 1 1 1 0 1 0 0)	456	0.5113
15	(1 1 1 1 1 0 1 0 1)	24	0.0052
16	(1 1 1 1 1 0 1 1 0)	3750	4.3661
17	(1 1 1 1 1 0 1 1 1)	90	0.0095
18	(1 1 1 1 1 1 0 0 0)	68	0.0354
19	(1 1 1 1 1 1 0 0 1)	596	0.6213
20	(1 1 1 1 1 1 0 1 0)	5408	6.2191
21	(1 1 1 1 1 1 0 1 1)	78	0.0255
22	(1 1 1 1 1 1 1 0 0)	238	0.2104
23	(1 1 1 1 1 1 1 0 1)	77	0.0227
24	(1 1 1 1 1 1 1 1 0)	139	0.0131
25	(1 1 1 1 1 1 1 1 1)	131	0.0103

Strategy 10 is the TDS. There are seven different strategies in the “old” group that have the design time less that the TDS. These are the strategies 13, 15, 17, 21, 23, 24 and 25. The strategy 15 is the optimal one among all the “old” strategies and it has the time gain 5.06 with respect to the TDS. The best strategy among all the strategies (number 5) of the Table 2 has the time gain 29.2. So, we have an additional acceleration in 5.77 times. This effect is obtained on the basis of a more extensive structural base and serves as the main result of the new generalized methodology. The optimization of the control vector *U* can increase this time gain as shown in [23]-[24].

C. Example 3

Next example corresponds to the active network in Fig.3. The Ebers-Moll static model of transistor has been used [25].

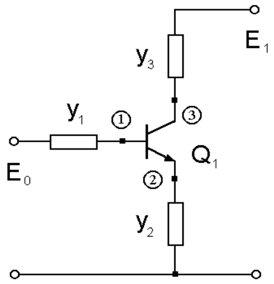

Fig. 3. *One-stage transistor amplifier.*

The vector *X* includes six components: $x_1^2 = y_1$, $x_2^2 = y_2$, $x_3^2 = y_3$, $x_4 = V_1$, $x_5 = V_2$, $x_6 = V_3$. The model of this network (2) includes three equations (*M*=3) and the optimization procedure (1) includes six equations (*K*+*M*=6). The “old” structural basis contains eight different design strategies. The total number of the design strategies that compose the new structural basis of the second level of generalized theory is equal $\sum_{i=0}^{3} C_6^i = 42$. The strategy that has the control vector (111000) is the TDS in terms of the first level of generalized methodology. Only three first equations of the system (1) are included in optimization procedure to minimize the generalized cost function *F(X,U)*. The model of the circuit includes three equations too. The cost function *C(X)* was defined by the next formula:

$$C(X)=[(x_4-x_5)-m_2]^2+[(x_6-x_5)-m_1]^2 \qquad (7)$$

where m_1, m_2 are the necessary, before defined voltages on transistor junctions.The strategy 16 that corresponds to the control vector (111111) is the MTDS. All six equations of system (1) are involved in the optimization procedure, but the model (2) has been vanished in this case. Other strategies can be divided in two parts. The strategies that have units for three first components of the control vector define the subset of “old” strategies in limits of the first level of generalized methodology.

TABLE III. SOME STRATEGIES OF THE STRUCTURAL BASIS FOR ONE-STAGE TRANSISTOR AMPLIFIER

N	Control functions	Calculation	results
	vector	Iterations	Total design
	U (u1,u2,u3,u4,u5,u6)	number	time (sec)
1	(0 1 1 1 0 0)	12850	10992.33
2	(0 1 1 1 0 1)	47	19.73
3	(0 1 1 1 1 0)	30015	10998.24
4	(1 0 1 1 1 0)	55992	25094.21
5	(1 0 1 1 1 1)	1195	170
6	(1 1 0 0 1 1)	174	60.01
7	(1 1 0 1 0 1)	606	220.21
8	(1 1 0 1 1 1)	778	139.11
9	(1 1 1 0 0 0)	9311	7977.01
10	(1 1 1 0 0 1)	7514	4989.11
11	(1 1 1 0 1 0)	75635	43053.12
12	(1 1 1 0 1 1)	324	60.11
13	(1 1 1 1 0 0)	25079	10970.12
14	(1 1 1 1 0 1)	243	40.11
15	(1 1 1 1 1 0)	10232	2398.53
16	(1 1 1 1 1 1)	2418	196.21

We can see that two strategies 12 and 14 have the total computer time lesser that others. Strategy 14 corresponds to the optimal one in this case and it has time gain 198 times with respect to the TDS. Strategies numbered from 1 to 8 are the "new" strategies of the second level of generalization. Strategy 2 has the shortest design time among all strategies and has more than double the time gain of the best strategy 14. The time gain achieves 404 times in this case. However, more impressive results were obtained analyzing more complex networks.

D. Example 4

Other example corresponds to the circuit in Fig.4.

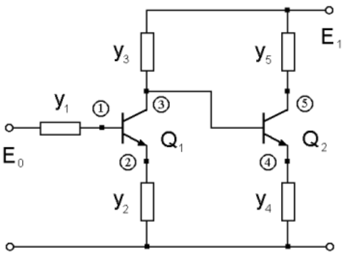

Fig. 4. *Two-stage transistor amplifier.*

The cost function *C(X)* for the design problem was defined by the formula similar to (7). This network is characterized by five independent parameters and five dependent parameters in accordance with the traditional approach. Accordingly the first level of generalized methodology the control vector includes five control functions, but the same control vector has 10 components following to the second level of generalized methodology. The structural basis consists of 32 design strategies accordingly the first level of generalization. On the other hand the total number of the different design strategies, which compose the new structural basis is equal to $\sum_{i=0}^{5} C_{10}^{i} = 638$. This structural basis can provide significantly better results for the time minimization. The results of analysis of some design strategies are presented in Table 4.

TABLE IV. SOME STRATEGIES OF THE STRUCTURAL BASIS FOR TWO-STAGE TRANSISTOR AMPLIFIER

N	Control functions	Calculation	results
	vector U (u1,u2,...,u10)	Iterations	Total design
		number	time (sec)
1	(0 0 0 0 0 1 1 1 1 1)	55	0.159
2	(0 0 0 0 1 1 1 1 1 0)	7912	23.985
3	(0 0 0 0 1 1 1 1 1 1)	209	0.429
4	(0 0 0 1 1 1 1 1 0 0)	57245	229.963
5	(0 0 0 1 1 1 1 1 1 1)	420	0.561
6	(0 0 1 1 1 1 1 0 1 1)	25884	52.022
7	(0 0 1 1 1 1 1 1 0 1)	232	0.309
8	(0 0 1 1 1 1 1 1 1 0)	138426	230.014
9	(0 0 1 1 1 1 1 1 1 1)	381	0.319
10	(0 1 0 1 0 1 0 1 1 1)	201	0.401
11	(0 1 0 1 1 1 0 1 0 0)	47186	190.979
12	(0 1 0 1 1 1 0 1 1 1)	242	0.329
13	(0 1 0 1 1 1 1 1 1 1)	371	0.319
14	(0 1 1 0 1 1 0 1 1 1)	338	0.441
15	(0 1 1 0 1 1 1 1 1 1)	414	0.341
16	(0 1 1 1 0 1 0 1 1 1)	156	0.209
17	(0 1 1 1 0 1 1 1 1 1)	480	0.409
18	(0 1 1 1 1 1 0 1 1 0)	8511	11.998
19	(0 1 1 1 1 1 0 1 1 1)	68	0.082
20	(0 1 1 1 1 1 1 0 1 1)	22381	26.012
21	(0 1 1 1 1 1 1 1 0 0)	31525	55.061
22	(0 1 1 1 1 1 1 1 1 0)	9264	8.961
23	(0 1 1 1 1 1 1 1 1 1)	205	0.091
24	(1 0 0 0 0 0 1 1 1 1)	98	0.291
25	(1 0 0 0 0 1 1 1 1 1)	150	0.309
26	(1 0 0 1 1 0 1 1 0 0)	40121	165.003
27	(1 0 0 1 1 0 1 1 1 1)	286	0.379
28	(1 0 0 1 1 1 1 1 0 1)	170	0.239
29	(1 0 1 1 1 1 1 1 0 0)	35624	63.014
30	(1 0 1 1 1 1 1 1 1 1)	691	0.342
31	(1 1 0 0 0 0 0 1 1 1)	4557	22.019
32	(1 1 1 0 1 1 1 1 1 1)	976	0.945
33	(1 1 1 1 0 0 0 0 0 1)	79079	326.941
34	(1 1 1 1 0 1 1 1 1 1)	542	0.271
35	(1 1 1 1 1 0 0 0 0 0)	83402	333.601
36	(1 1 1 1 1 0 0 0 1 1)	6695	8.991
37	(1 1 1 1 1 0 0 1 1 1)	3395	4.007
38	(1 1 1 1 1 0 1 1 1 1)	253	1.292
39	(1 1 1 1 1 1 0 0 0 1)	70887	125.994
40	(1 1 1 1 1 1 0 1 1 1)	588	2.701
41	(1 1 1 1 1 1 1 0 0 1)	148299	158.038
42	(1 1 1 1 1 1 1 0 1 1)	24678	15.945
43	(1 1 1 1 1 1 1 1 0 0)	56464	57.015
44	(1 1 1 1 1 1 1 1 0 1)	496	2.402
45	(1 1 1 1 1 1 1 1 1 0)	5583	2.007
46	(1 1 1 1 1 1 1 1 1 1)	614	1.699

The design strategies numbered from 35 to 46 belong to subset that appears in limits of the first level of generalization. The MTDS that corresponds to the control vector (1111111111) has the minimum computer time among this subset. The time gain is equal to 196 times in this case. However, there are 21 others strategies that appear among the subset of new design strategies that have the computer design time lesser that MTDS.

The best strategy 19 that corresponds to the control vector (0111110111) has the time gain 4068 with respect to TDS and has an additional gain 20.7 with respect to the MTDS. Other strategies, for instance 1, 7, 9, 12, 13, 16, 23, 24, 25, 28 and 34 have a significant value of time gain, more than 1000 times. So, the second level of generalization of design methodology includes more perspective strategies to minimize the CPU time.

IV. CONCLUSION

The traditional approach to designing analog circuits is not time-optimal. The problem of constructing an optimal algorithm can be solved more adequately by applying optimal control theory. The algorithm of time-optimal design is formulated as a functional optimization problem in optimal control theory. In this case, it is necessary to select one optimal trajectory from a quasi-infinite number of different design strategies that are produced. The new and more complete approach to the electronic system design methodology has been developed now. This approach generates more broadened structural basis of the different design strategies. The total number of the different design strategies, which compose the structural basis by this approach, is equal to $\sum_{i=0}^{M} C_{K+M}^{i}$. This new structural basis serves as the necessary set for the optimal design strategy search. This basis includes very perspective strategies that can be used for the time-optimal design algorithm construction. Some new strategies have better convergence and lesser computer time than the strategies that appeared in before developed methodology. This approach can reduce considerably the total computer time for the system design. Analysis of the different problems of the electronic system design shows a significant potential of the new level of generalized design methodology. The potential gain in computational time that can be obtained based on the extended structural base is significantly greater than for the previously developed methodology, and reaches several thousand already for small examples and continues to increase for more complex systems.

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