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Mathematical Model Describing The Technological Structures In Enrichment Factories

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***Abstract*—The present paper is devoted to a mathematical model describing the technological structures in enrichment factories. The base of our investigations is the reliability theory. This allows us to assess both the reliability of individual units and the operations of the entire factory. With this model for the productivity of a sequential technological production line, possible opportunities for achieving significant economic benefits are revealed, associated with reducing material and electricity expenses.**

Keywords—Enrichment factories; Mathematical models; Technological structures.

I. Introduction

Studying the reliability of technological structures in enrichment factories is one of the main sources of obtaining information necessary for managing technical systems. It is a fundamental prerequisite for their effective operation.

The issue of ensuring high reliability of large technical systems is extremely complex. It involves a combination of scientific, technical, and organizational issues, among which tasks related to developing and practically utilizing quantitative results to improve the operation of the facility hold a significant place. Achieving high reliability is of great importance in the development, production, and operation of technical devices of various types and purposes [1-3].

Indicators of reliability of material handling systems in enrichment factories form the basis for forming production plans for a respective calendar period [4]. They assess the production capabilities of the main units and the transportation mechanization.

From a production process perspective, the technological line is characterized by two main modes: operation and downtime. Essentially, this is a complex sequence of periods involving immediate work, troubleshooting, planned maintenance, interruptions due to organizational and technological reasons, etc.

II. Methods of Investigation

Every technological structure is characterized by the following quantitative reliability indicators [5, 6], both for individual machines and for the technological structure as a whole:

$P_i(t)$ - probability of failure-free operation of individual machines;

$P_{TS}(t)$ - probability of failure-free operation of the technological structure;

λ_i - failure intensity of individual machines;

λ_{TS} - failure intensity of the technological structure;

μ_i - recovery intensity of individual machines;

μ_{TS} - recovery intensity of the technological structure;

$f_i(t)$ - probability density function of failure-free operation of individual machines;

$f_{TS}(t)$ - probability density function of failure-free operation of the technological structure;

K_i - availability coefficient of individual machines;

K_{TS} - availability coefficient of the technological structure;

$\bar{T}_p$ - mean time between failures in the technological structure;

$\bar{T}_b$ - mean time to restore the structure;

$\bar{Q}_{TS}$ - average productivity of the technological structure between failures.

Establishing quantitative reliability indicators for technological structures and the machines involved in them is necessary for:

- determining operational productivity;
- developing an effective system for technical maintenance;
- defining guidelines for improving mechanization;
- identifying reserves for increasing net operating time;
- scientific justification of production plans.

For evaluating the operational modes of a given technological structure, the distribution of calendar time T_k during which it operates is of crucial importance [7, 8]. In the most general case, T_k is presented as:

$T_k = T_p + T_b + T_d$,(1)

where:

T_p - time for the operation of the technological structure;

T_b - time for recovery after failure;

T_d - time for technical maintenance.

The time for recovery is a fundamental indicator and is determined by:

$$T_b = \frac{\sum_{i=1}^{n}\left(t_i^{(1)} + t_i^{(2)} + t_i^{(3)} + t_i^{(4)}\right)}{n} ,(2)$$

where:

$t_i^{(1)}$ - time for failure detection;

$t_i^{(2)}$ - time for delivery of spare parts and organizational downtime before repair starts;

$t_i^{(3)}$ - time for repair;

$t_i^{(4)}$ - time for testing a machine after the failure has been resolved.

Recovery time T_b characterizes the ability to quickly detect failures and remedy them, the level of organization of repair activities, and the time required to procure necessary spare parts [9].

During the process, there are alternating periods of time for failure-free operation T_p and time for recovery T_b. It can be assumed that T_p and T_b are random and independent variables, and the density distribution of their sum $T_o = T_p + T_b$ is the convilution:

$$f_o(t) = \int_0^t f_p(x) f_b(t-x)\,dx ,(3)$$

i.e., this is the density distribution of the time between two consecutive recoveries.

From the theory of queuing systems, it is known [10, 11] that when studying event streams, two characteristics are used - intensity and event stream parameter. For ordinary event streams, it is assumed that the two characteristics coincide. The relationship between the event stream parameter and the density distribution is:

$$w_o(t) = f_o(t) + \int_0^t w_o(\tau) f_o(t-\tau)\,d\tau .(4)$$

At a given moment of time t, the technological structure may be in an operational state under the presence of one of two mutually exclusive events:

1. The technological structure has not failed for the time interval $(0,t)$;

2. The structure has failed and is being recovered, and after the last recovery, it has not failed again.

The availability function $\Gamma(t)$, expressing the probability that the technological structure is operational, is equal to the sum of the probabilities of the two events occurring. The probability of the first event occurring is equal to the probability of the technological structure operating without failure $P_{TS}(t)$ for the time interval $(0,t)$.

To determine the probability of the second event occurring, a small time interval is considered $(\tau, \tau + \Delta\tau) < t$. The probability that within this interval the last recovery will be completed and the technological structure will not fail again until the end of the time interval is $f_{o_n}(\tau) P_{TS}(t-\tau)\,d\tau$.

Summing up by n, we obtain:

$$\sum_{n=1}^{\infty} f_{o_n}(\tau) P_{TS}(t-\tau)\,d\tau = w_o(\tau) P_{TS}(t-\tau)\,d\tau ,(5)$$

where $w_o(\tau) = \sum_{n=1}^{\infty} f_{o_n}(\tau)$.

After integrating over the interval $(0,t)$, the probability of the second event occurring is found, where the availability function will be equal to:

$$\Gamma(t) = P_{TS}(t) + \int_0^t P_{TS}(t-\tau) w_o(\tau)\,d\tau .(6)$$

The Smith theorem [10, 11] is applied to the obtained availability function of the technological structure, according to which:

$$\lim_{t\to\infty} \int_0^t Q(t-x)\,dE_1(x) = \frac{1}{E(t)} \int_0^{\infty} Q(x)\,dx ,$$

where:

$E(t)$ is the mathematical expectation of the time between two consecutive events;

$Q(x)$ - is a growing integrable function in the interval $(0,x)$;

$E_1(x)$- is the mathematical expectation of the number of failures in the interval $(0,x)$.

The mathematical expectation of the random variable $T_o = T_p + T_b$ is:

$E(T_o) = E(T_p) + E(T_b)$ and $\lim_{t\to\infty} P_{TS}(t) \to 0$;

$$\lim_{t\to\infty} w_o = \frac{1}{E(T_p) + E(T_b)} .$$

Under these conditions, it follows that:

$$\lim_{t\to\infty}\Gamma(t)=\frac{1}{E(T_p)+E(T_b)}\int_0^\infty P_{TS}(t)dt=\frac{E(T_p)}{E(T_p)+E(T_b)}=K_{TS}$$
.(7)

The availability function of the technological structure at $t\to\infty$ tends to a steady value K_{TS}, independent of the distribution laws of the random variables T_p and T_b. The availability coefficient determines the ratio of the time during which the structure is operational to the total operational time.

Analysis of statistical methods [12-14] for the operation of various technological structures shows that the times for failure-free operation and recovery follow the exponential distribution law:

$$F(t)=1-e^{-\lambda_{TS}t},\ G(t)=1-e^{-\mu_{TS}t}\ .(8)$$

The probability density functions are respectively:

$$f_{TS}(t)=\frac{dF(t)}{dt}=\lambda_{TS}e^{-\lambda_{TS}t},$$

(9)

$$g_{TS}(t)=\frac{dG(t)}{dt}=\mu_{TS}e^{-\mu_{TS}t}.$$

Solving equation (6) yields:

$$\Gamma(t)=\frac{\mu_{TS}}{\mu_{TS}+\lambda_{TS}}+\frac{\lambda_{TS}}{\mu_{TS}+\lambda_{TS}}e^{-(\lambda_{TS}+\mu_{TS})t},(10)$$

$$\lim_{t\to\infty}\Gamma(t)=\frac{\mu_{TS}}{\mu_{TS}+\lambda_{TS}}=K_{TS}.$$

Determining the availability coefficient of the technological line based on updating statistical information to a high degree reflects the operating conditions and serves to evaluate its performance [15-17].

The production ore-processing process in enrichment factories is realized through crushing technological lines [18-22]. The operational productivity Q of a crushing technological line with a sequential structure for a specific calendar period can be determined by the relationship:

$$Q=\sum_{i=1}^{M}(m_Q+n_i\sigma_Q)t_{p_i},(11)$$

where:

M is the number of states of the line, differing from each other with different productivity of the crushing unit;

t_{p_i} is the time for operation of the line.

Since during the respective calendar period the crushing unit will operate with different productivities, the ratio

$$\frac{t_{p_i}}{T_k}=p(t_p)_i$$

will be the probability of being in the i - th state of productivity. Then the time t_p will be:

$$t_p=T_kK_{TS}\frac{l_1}{l_1+l_2}.(12)$$

In accordance with formula (7), the sum of the average times between failures T_p and T_b is represented as:

$$T_p+T_b=\frac{T_p}{K_{TS}}.(13)$$

During the analysis of the operation of sequential technological structures, the coefficient of technical maintenance K_d is also used:

$$K_d=\frac{T_k-T_d}{T_k}=\frac{l_1}{l_1+l_2},(14)$$

where:

l_1 - the number of shifts worked on the technological line;

l_2 - the number of maintenance shifts.

Taking into account expressions (1), (13), and (14), the average time of the technological line for the calendar period can be determined as:

$$T_p=T_kK_{TS}\frac{l_1}{l_1+l_2}.(15)$$

Then

$$Q=T_kK_{TS}\frac{l_1}{l_1+l_2}\sum_{i=1}^{M}Q_i^{(T)}p(t_p).(16)$$

From the analysis of the operation of individual technological lines in enrichment factories, it is concluded that the probability density distributions of the material flows follow the Gaussian law, i.e.

$$f(Q)=\frac{1}{\sigma_Q\sqrt{2\pi}}e^{-\frac{(Q-m_Q)^2}{2\sigma_Q^2}}.$$

The mathematical expectation $E(Q)$ turns out to be significantly smaller than the productivity of the main technological machine, as indicated in its technical characteristics. There are cases where the productivity $m_Q+3\sigma_Q$ is lower than the nominal one. Taking into account the fact that the material flows in these structures follow the Gaussian law, when determining the parameters of the rubber-belt conveyor (drive power, belt width), the following expression should be used for the calculated load:

$$V=\left(1-\frac{t_{\Sigma_{np}}}{T_k}\right)\sum_{i=1}^{M}Q_i^{(T)}\ .$$

Since in practice determining the coefficients K_{TS} and K_d is relatively imprecise, it is more appropriate to determine the predicted workload performed by the crushing unit from the expression:

$$V=\frac{T_k-t_{\Sigma_{np}}}{T_k}\sum_{i=1}^{M}Q_i^{(T)}\ .$$

III. Conclusion

With the described model for the productivity of a sequential technological production line, possible opportunities for achieving significant economic benefits are revealed, associated with reducing material and electricity expenses.

References

[1]. Goodfellow, R., Dimitrakopoulos, R. (2015). Stochastic global optimization of open pit mining complexes with capital expenditures: Application at a copper mining complex. *Les Cahiers du GERAD*, pp. 1-26, 0711–2440.

[2]. Del Castillo, M. F., Dimitrakopoulos, R. (2018). Stochastic optimization of mining complexes integrating capital investments and operational alternatives. *Les Cahiers du GERAD*, pp. 1-19, 0711–2440.

[3]. LaRoche-Boisvert, M., Dimitrakopoulos, R. (2021). An Application of Simultaneous Stochastic Optimization at a Large Open-Pit Gold Mining Complex under Supply Uncertainty. *Minerals*, 11, 172, pp. 1-16, https://doi.org/10.3390/min11020172.

[4]. Iliev, I. (2025). Optimal technical high - efficiency solutions in power supply. *Print factor*, pp. 142, 978-619-7427-38-7, (in Bulgarian).

[5]. Drzymala, J. (2007). Mineral processing: foundations of theory and practice of minerallurgy. *Wroclaw University of Technology*, pp. 510, 978-83-7493-362-9.

[6]. Montgomery, D. C, Runger, G. C. (2011). Applied Statistics and Probability for Engineers. *John Wiley & Sons*, pp. 821, 978-0-470-05304-1.

[7]. Cisternas, L. A., Lucay, F. A., Botero, Y. L. (2020). Trends in Modeling, Design, and Optimization of Multiphase Systems in Minerals Processing. *Minerals*, 10, 22, pp. 1-28, https://doi.org/10.3390/MIN10010022.

[8]. Rakhmangulov, A., Burmistrov, K., Osintsev, N. (2021). Sustainable Open Pit Mining and Technical Systems: Concept, Principles, and Indicators. *Sustainability*, 13, 1101, pp. 1-24, https://doi.org/10.3390/su13031101.

[9]. Wills, B. A., Napier-Munn, T. J. (2006). Mineral Processing Technology: An Introduction to the Practical Aspects of Ore Treatment and Mineral Recovery. *Elsevier Science & Technology Books*, pp. 450, 0750644508.

[10]. Cox, D. R., Snell, E. J. (1981). Applied Statistics. Principles and Examples. *Department of Mathematics, Imperial College, University of London, Chapman and Hall, New York*, pp. 196, 0-412-16570-8.

[11]. Hines, W. W., Montgomery, D. C. (1990). Probability and statistics in engineering and management Science. *John Wiley & Sons*, pp. 752, 9780471600909.

[12]. Semmar, A., Machkour, N., Amalik, H., Elouazguiti, M., Medromi, H. (2022). Production System Modelling Methodologies of an Open Pit Mine. *The International Conference on Artificial Intelligence and Engineering*, vol. 43, pp. 1-11, https://doi.org/10.1051/itmconf/ 20224301001.

[13]. Mukhitdinov, D. P., Boybutayev, S. (2021). Problems of Modeling and Control of the Technological Process of Ore Grinding. *Chemical Technology, Control and Management*, Iss. 4, pp. 49-62, https://doi.org/10.51346/tstu-02.21.4-77-0030.

[14]. Castillo-Villagra, R., Icarte, G., Thoben, K. (2023). Modelling the Make Process of the Mineral Supply Chain Upstream Segment. *MDPI AG*, pp. 31, https://doi.org/10.3390/ resources12110132.

[15]. Sánchez, F., Hartlieb, Ph. (2020). Innovation in the Mining Industry: Technological Trends and a Case Study of the Challenges of Disruptive Innovation. *Mining, Metallurgy & Exploration*, vol. 37, pp. 1385-1399, 2524-3470.

[16]. Radu, S. M., Bărbulescu, A., Coandreş, C., Mvongo Mvodo, Ch. R., Scutelnicu, I. P., Khamis, J., Lihoacă, A., Burian, A. A. (2023). The Complex System of Mining and Energy Resources: Premises for Clusterization. *Revista Minelor – Mining Revue*, vol. 29, issue 1, pp. 101-110, 2247-8590, DOI: 10.2478/minrv-2023-0008.

[17]. Nunes, R. A., de Tomi, G., Allan, B., Bezerra, E. B., Silva, R. S. (2019). An integrated pit-to-plant approach using technological models for strategic mine planning of copper and gold deposits. *REM - International Engineering Journal*, 72 (2), pp. 307-313, 2448-167X, http://dx.doi.org/10.1590/0370-44672018720060.

[18]. Liu, S. Q., Kozan, E. (2017). Integration of mathematical models for ore mining industry. *International Journal of Systems Science: Operations & Logistics*, pp. 1-14, 2330-2682, http://dx.doi.org/10.1080/23302674.2017.1344330.

[19]. King, R. P. (2012). Modeling and simulation of mineral processing systems. *Published by the Society for Mining, Metallurgy, and Exploration, Inc.*, pp. 457, 978-0-87335-345-8.

[20]. Grzná, P., Gregor, M., Krajĉoviĉ, M., Mozol, Št., Schickerle, M., Vavrí, V., Durica, L., Marschall, M., Bielik, T. (2020). Modeling and Simulation of Processes in a Factory of the Future. *Applied Sciences*, 10(13):4503, ISSN: 2076-3417, DOI:10.3390/app10134503.

[21]. Parashkevova D. Daniela. On the Qualitative Analysis of Certain Flotation Processes. 9(11), International Journal of Engineering Sciences & Research Technology, 2020, 2277-9655, DOI:https://doi.org/10.29121/ijesrt.v9.i11.2020.12, 91-99.

[22]. Parashkevova D., Grebchev V. Condition for Effective Functioning of Enrichment Plants. 12(6), American Journal of Engineering Research, 2023, 2320-0847, 147-152.