

Optimization of Biodiesel Production using Response Surface Methodology and Grey Wolf Optimizer

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Abstract— This study investigates the effect of response surface methodology (RSM) and Grey-wolf optimizer (GWO) in the optimization of biodiesel yield. Blends of palm kernel shell and cocoa pods oils were used for the production of biodiesel through transesterification process. Titanium oxide was used as a nano catalyst to increase the oil yield and reduce cost of production and time. The results obtained were in two sets – the actual (experimental) and the predicted values - using the design matrix. The matrix design was adopted to study the combined (predicted) effects of the process parameters in the production of the biodiesel. A two level five factor RSM full factorial composite response, which identified the various design points was employed to achieve the optimum process parameters for the produced biodiesel. For the single effect (experimental), maximum yield of 72.98% was obtained, whereas for the combined effect, the yield of 76.05% was gotten. A comparative analysis was carried out on the

biodiesel yield using the RSM and Grey-wolf optimization tools. The optimization process revealed that the biodiesel yield was closely related with a yield of 79.50 and 83.412% respectively. This result shows that the Grey-wolf optimizer (GWO) and the response surface methodology (RSM) are good optimization tools for biodiesel production. .

Keywords— *Biodiesel, Catalyst, Response Surface Methodology, Grey-wolf Optimizer, Oil Yield*

1. INTRODUCTION

Industrialization's need for energy, over the years, resulted in the transition from wood to coal and from coal to fossil fuels. Since then, fossil fuels have come to dominate the world's energy supply; and the global thirst for energy have gradually become unquenchable [1]. This rapid demand for energy is due to the increase in human population, advancement in technology, industrial applications and the limited availability of non-renewable energy resources in the world. The continuous use of fossil

products have resulted in depletion of the oil reserves, increase in cost of production and sales as well as depletion of the ozone layer due to emission of greenhouse gases (GHGs) generated from heavy-duty trucks, city transport buses, heavy-duty/small generators, power plants systems, quarry etc [2]. In addition, fossil fuel emissions will continue indefinitely [3]. Therefore, an alternative and renewable fuel resources that is capable of solving these current problems is very important. Consequently, biofuels (example, biodiesel) have been very promising in that regard. The International Energy Agency (IEA) wants biofuel to meet more than a quarter of the world demand for transportation fuels by 2050 in order to reduce dependency on petroleum [4]. Presently, the production and consumption of biofuels according to the report have not yet met the IEA's sustainable development scenario, but from 2020 to 2030, global biofuel output has to increase by 10% each year to reach the IEA's goal. Biodiesel is produced from edible and non-edible oils known as feedstock [5]. To meet the energy requirements of the future, different feedstock oils can be blended. Also, to avoid waste, increase oil yield, reduce time and cost of production, a catalyst is used through transesterification process. In this work a nano catalyst (Titanium oxide), which is highly environmental friendly, cost effective, safe and efficient [1, 6] was used to produce the biodiesel from hybrid of palm kernel and cocoa pods oils in the presence of alcohol (methanol) through transesterification process, which is easier, cheaper, faster and with a higher potential of increased yield [3, 7]. Before use, the catalyst was prepared and dried in accordance with the drying methods specified by [1, 6, 8, 9]. To further increase the biodiesel yield, the process parameters: reaction time, reaction temperature, catalyst concentration, agitation speed and methanol/oil ratio, were considered and varied individually and combined within different ranges to anticipate biodiesel yield in a matrix. The need to optimize the process parameters in a systematic way was necessary to achieve a higher output characteristic/responses using an optimization tool [10]. The response surface methodology (RSM) was used to find the relationships among process variables and response in an efficient manner using a minimum number of experiments. The Grey-wolf algorithm was also adopted to develop a model to analyse and predict the yield of cocoa pods and palm kernel shell oil methyl ester using the same process parameters; and the obtained biodiesel yield was set as response.

2. LITERATURE REVIEW

Various works have been carried out by different researchers in the production of biodiesel from different feedstocks and methods [7, 11, 12, 13, 14]. For optimization process, others have also employed different optimization tools like MINITAB, Taguchi, Artificial Neural Network, Box-Behnken fractional design, RSM etc, to obtain maximum result [7, 9, 13, 14, 15, 16]. It is worthy of note that these tools have been utilized in one way or the other to optimize biodiesel production data obtained from the corresponding experimental runs, which gave rise to improved results and production yield/outcomes. However, all the aforementioned software has notable drawbacks at one point or the other, such as the results obtained not being relative and not exactly indicating what parameter had the

highest effect on the performance characteristic, etc. In order to address these setbacks and obtain better results, this study focused on the utilization of another supportive tool, known as the Grey-wolf optimizer for the optimization of biodiesel production. Moreover, to accelerate the reaction time and oil yield, a nano catalyst was used. Records have shown that very limited research works have considered Grey-wolf optimizer for the optimization of the oil yields. Records have also shown that blends of palm kernel shell and cocoa pods oils have rarely been considered for the production of biodiesel, whereas the raw material for these non edible oils have always been wasted over the years, especially in Nigeria and other sub-Saharan African countries [1].

Review of previous studies demonstrate that the RSM could predict homogeneous, heterogeneous and nano catalysts based-biodiesel from various classes of feedstocks and their diverse engine characteristics [7, 12, 14]. RSM model demonstrates its capacity to enhance catalytic-based biodiesels from various oils. It has the ability to detect the correct quantity of catalyst in combination with other process parameters in order to increase the rate of methylic process. In addition, the RSM model's extraordinary development in efficiency and correlation of multi-input parameters and response [7].

The Grey-wolf optimizer on the other hand have also proven to be an excellent metaheuristic optimization algorithm. It draws inspiration from natural phenomena to guide search towards optimal solutions. It is a specific type of swarm intelligence metaheuristic optimization algorithm that mimics the social hierarchy and hunting behaviour of grey wolves, introduced by Seyedali Mirjalili et al., in 2014 [17]. Grey-wolf optimizer is highly recommended for its novelty, low number of parameters, fast convergence speed, high precision, balanced exploration and exploitation and simplicity [18, 19, 20, 21, 22].

For these and other reasons, the choice of the RSM and Grey-wolf optimizer was made for the modelling and optimization of biodiesel production from blends of palm kernel shell and cocoa pods oils using titanium oxide as catalyst in this study. This will go a long way to mitigating the adverse effect of fossil fuels, improve economy, improve biofuel production/yield and bridge the gap that existed in the production and optimization of biodiesel.

3. METHODOLOGY

3.1 Grey-Wolf Optimization

As an innovative swarm intelligence technique, the grey-wolf optimizer (GWO) is modest with differential evolution and gravitational search algorithm. As already stated, the GWO algorithm mimics the leadership hierarchy and hunting behaviour of grey-wolves in nature. The Grey wolf is known to reside at the top of the food chain and as a top-level predator. Also, they animate in groups that averagely consist of five to twelve wolves. In addition to that, they adopt the three (3) main steps of hunting prey - searching for prey, encircling prey, and attacking the prey. The GWO adopts same for implementation.

Four types of grey-wolves such as alpha (α), beta (β), delta (δ), and omega (ω) are employed for simulating the leadership hierarchy, according to fitness. According to the hierarchy of wolves as seen in Figure 1, the group is led by

the alpha wolves (α) – the best fit values found in the search space, followed by the beta wolves (β) that helps the alpha wolves (α) in decision making – the second best solution in the search space. The ' β ' augments the ' α ' commands in the group and gives feedback to the ' α '. The delta wolves (δ)

are the third best solution in the search space. Meanwhile, the minimum rank among the grey wolves (ω) are the last wolves that are allowed to eat the prey. The role of delta wolf (δ) is as a scout, hunter, caretaker, sentinel, and elder.

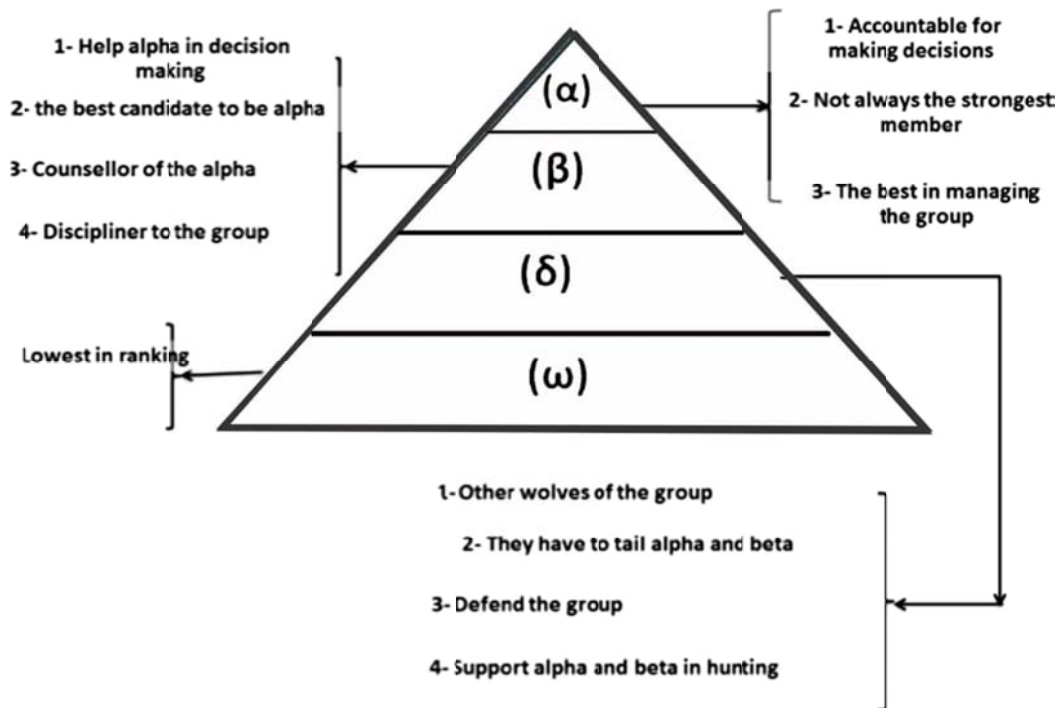


Figure 1: Grey-wolf predating position diagram

$$A = 2 * a * r \quad (4)$$

The best solution for the GWO algorithm can first be detected as ' α ' and then determined as ' β ', ' δ ' and ' ω ', respectively. During hunting time, the wolves incline to enclose their prey.

For optimum solution, the GWO utilizes the following formulas, equation 1 and equation 2, to simulate its encircling behaviour.

$$D = |C * X_p(t) - X(t)| \quad (1)$$

where D is the distance vector between a grey wolf ($X(t)$) and its prey ($X_p(t)$) at a particular iteration (t). It determines how close each wolf is to the prey.

C is the coefficient vector randomly generated within the range $[0, 2]$.

$$C = 2 * r \quad (2)$$

r is random vector in $[0, 1]$

$X_p(t)$ is the position vector of the prey (current best solution: alpha, beta, or delta)

$X(t)$ is the position vector of the wolf (candidate solution) at iteration t .

While hunting, the wolves update their position relative to the prey, thus:

$$X(t+1) = X_p(t) - A * D(t) \quad (3)$$

Where:

$X(t+1)$ - is the updated position vector of the wolf in the next iteration ($t+1$).

A - is a control parameter that linearly decreases from 2 – 0 over iterations.

α - decreases linearly from 2 – 0 over the course of iterations, helping to control the balance between exploration and exploitation.

@ $|A| > 1$, the wolves tend to diverge (explore), avoiding premature convergence

@ $|A| < 1$, the wolves converge (exploit), improving accuracy

To calculate the direction in which a specific wolf ($X(t)$) needs to move to get closer to the prey, the formula used is:

$$D(t) = C1 \times X_{alpha}(t) - X(t) + C2 \times X_{beta}(t) - X(t) + C3 \times X_{delta}(t) - X(t) \quad (5)$$

$C1, C2, C3$ are random coefficients generated within the range $[-1, 1]$. They introduce stochasticity,

preventing the wolves from converging too quickly or easily and potentially getting stocked in local optima.

$X_{\alpha}(t)$, $X_{\beta}(t)$, and $X_{\delta}(t)$ represents the position vectors alpha, beta and delta wolves at current iteration (t).

The GWO commences by developing a random group of grey wolves, which can be displayed by candidates for the

answer; and throughout the modelling, α , β , and δ wolves govern the probable state of the hunt. Overall, the Grey-wolf application flowchart for iterating the respective search agents is presented in Figure 2.

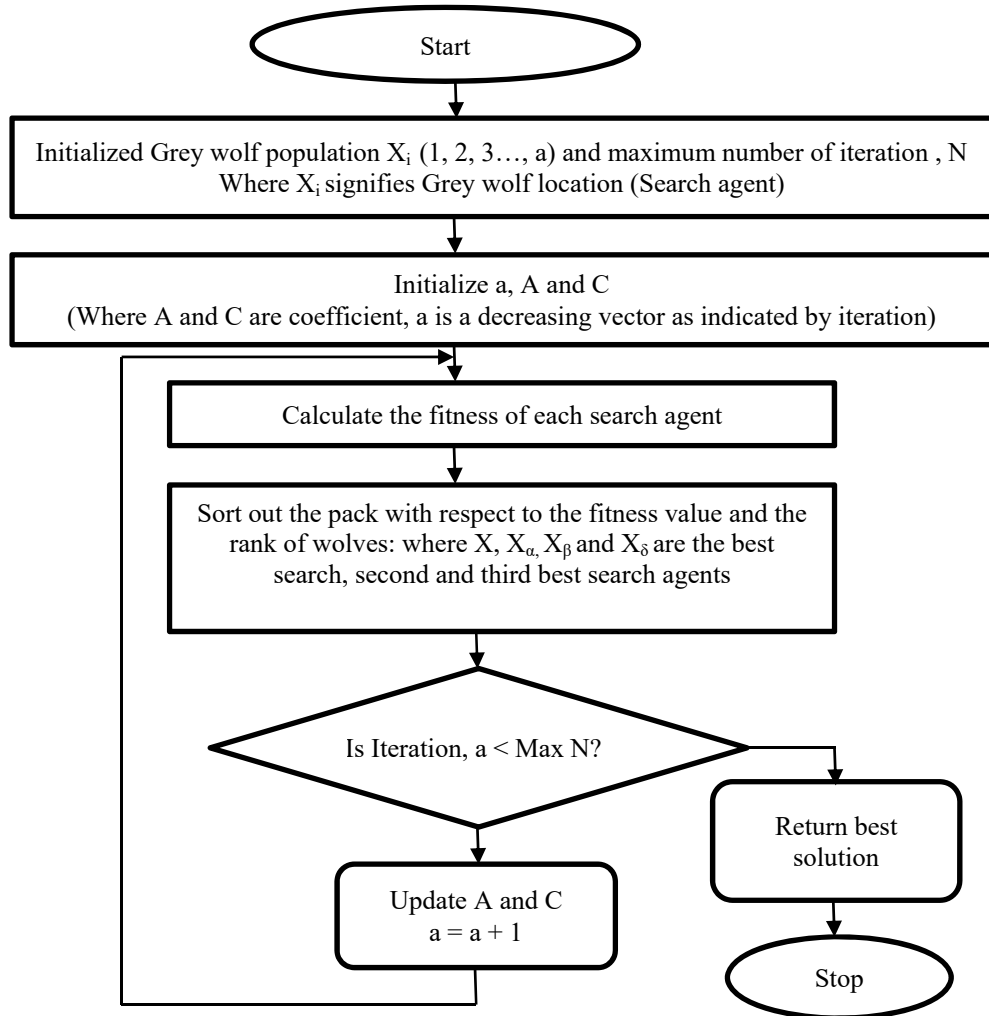


Figure 2: Grey-wolf application flowchart for iterating the respective search agents

3.2 Fractional Factorial Design of Experiment: Response Surface Methodology (RSM)

A two-level, five factor, factorial central composite design was used to study the combined effect of process parameters. To achieve this, design expert (DOE) was used to design the experimental study, which summed up to 32 experiments. Five study points were used in order to predict good estimation of errors and experiments performed in a randomized order. Five process conditions (independent variables) which included: effects of temperature, reaction

time, catalyst concentration, agitation speed and methanol / sample mole ratio were studied. The dependent variable was the biodiesel yield.

That is:

$$2^{5-1} + 2*5 + 6 = 32 \text{ experiments} \quad (6)$$

The factor levels shown in Tables 1 and 2. The matrix for the five variables were varied at two levels (-1 and +1). The lower level of variables was designated as -1 and the high level as +1. The coded values were designated by -2

(minimum), -1 (mid-minimum), 0 (centre), +1 (mid maximum) and +2 (maximum) respectively.

The optimization of transesterification using Central Composite Design (CCD), a Response Surface Methodology (RSM) was performed to determine the optimum values of the process variables.

The distance from the centre point, which can either be inside or outside the range, with the maximum value $2^{n/5}$, where n is the number of factors, is designated by alpha (α).

The empirical equation is represented as shown in equation (6).

$$Y = \beta_0 + \sum_{i=1}^5 \beta_i X_i + \sum_{i=1}^5 \beta_{ii} X_i^2 + \sum_{i < j} \beta_{ij} X_i X_j \quad (7)$$

Where:

Table 1: Studied range of each factor in actual and coded form for heterogeneous catalysts

Factors	Units	Low level	High level	$-\alpha$	$+\alpha$	0 level
Catalyst conc. (A)	Wt%	2(-1)	4(+1)	1(-2)	5(+2)	3
Methanol, (B)	Mol/mol	4(-1)	8(+1)	2(-2)	10(+2)	6
Temperature, (C)	°C	50(-1)	60(+1)	45(-2)	65(+2)	55
Reaction time (D)	Minutes	60(-1)	120(+1)	30(-2)	150(+2)	90
Agitation speed (E)	Rpm	200(-1)	300(+1)	150(-2)	350(+2)	250

It is noteworthy to point out that the software uses the concept of the coded values for the investigation of the significant terms, thus studying the effect of the variables on the response using coefficient equation, normal plot of

Y = predicted yield of biodiesel (%)
 X_i and X_j = the transesterification process variables

β_0 = the offset term

β_i = the coefficient of linear effect (single effect)

β_{ij} = the coefficient of interaction effect

β_{ii} = the coefficient of quadratic effect

The choice of the RSM for predicting and modelling the transesterification process was due to its simplicity and ability to correlate the input variables with responses; thereby achieving increased oil yield, and reduced production cost.

standardized effect, normal probability scatter plot, interaction plots such as 3D plots and contour plots.

Table 2: Experimental design matrix for transesterification studies catalyzed by activated titanium oxide catalyst

Run order	Catalyst conc. (wt %)		Methanol/Oil molar ratio		Temperature (°C)		Time (Minutes)		Agitation Speed (Rpm)	
	A		B		C		D		E	
	Coded	Real	Coded	Real	Coded	Real	Coded	Real	Coded	Real
1	-1	2	-1	4	-1	50	-1	60	+1	300
2	+1	4	-1	4	-1	50	-1	60	-1	200
3	-1	2	+1	8	-1	50	-1	60	-1	200
4	+1	4	+1	8	-1	50	-1	60	+1	300
5	-1	2	-1	4	+1	60	-1	60	-1	200
6	+1	4	-1	4	+1	60	-1	60	+1	300
7	-1	2	+1	8	+1	60	-1	60	+1	300
8	+1	4	+1	8	+1	60	-1	60	-1	200
9	-1	2	-1	4	-1	50	+1	120	-1	200
10	+1	4	-1	4	-1	50	+1	120	+1	300
11	-1	2	+1	8	-1	50	+1	120	+1	300
12	+1	4	+1	8	-1	50	+1	120	-1	200

13	-1	2	-1	4	+1	60	+1	120	+1	300
14	+1	4	-1	4	+1	60	+1	120	-1	200
15	-1	2	+1	8	+1	60	+1	120	-1	200
16	+1	4	+1	8	+1	60	+1	120	+1	300
17	-2	1	0	6	0	55	0	90	0	250
18	+2	5	0	6	0	55	0	90	0	250
19	0	3	-2	2	0	55	0	90	0	250
20	0	3	+2	10	0	55	0	90	0	250
21	0	3	0	6	-2	45	0	90	0	250
22	0	3	0	6	+2	65	0	90	0	250
23	0	3	0	6	0	55	-2	30	0	250
24	0	3	0	6	0	55	+2	150	0	250
25	0	3	0	6	0	55	0	90	-2	150
26	0	3	0	6	0	55	0	90	+2	350
27	0	3	0	6	0	55	0	90	0	250
28	0	3	0	6	0	55	0	90	0	250
29	0	3	0	6	0	55	0	90	0	250
30	0	3	0	6	0	55	0	90	0	250
31	0	3	0	6	0	55	0	90	0	250
32	0	3	0	6	0	55	0	90	0	250

4.0 RESULTS AND DISCUSSION

RSM and GWO algorithms were adopted to develop a model predicting the yield of cocoa pods and palm kernel shell oil methyl esters. The reaction time, reaction temperature, catalyst concentration, agitation speed and methanol/oil molal ratio, were chosen as the process parameters on which the optimization was analysed; and the obtained biodiesel yield set as the response.

Table 3 presents the RSM boundary variables used for the GW optimization via MATLAB R2022 software.

Table 3: Values of the GWO Variables

Variables	GWO
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Number of particles	30
Number of iterations	100
Cognitive Acceleration	-
Inertia weight	-

Figure 3 shows the optimized biodiesel yield from the GWO and RSM models at the boundaries. The figure reveals that the yield of Grey-wolf optimization (YGWO) have closely related relationship compared to the yield of the response surface methodology (YRSM).

The result obtained for the 20 runs shown in Figure 3 are presented in Table 4.

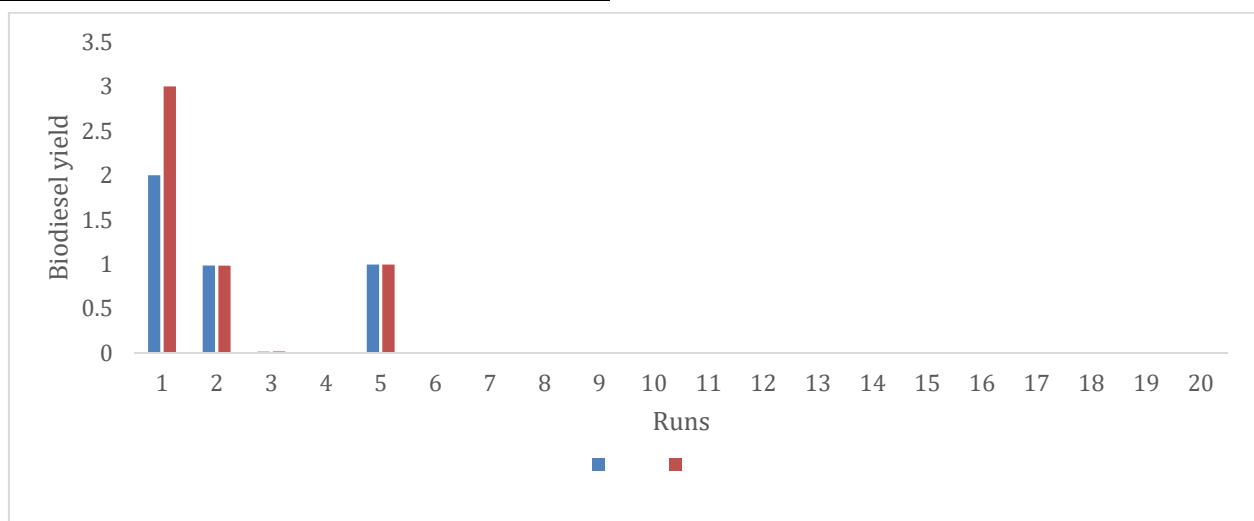


Figure 3: Optimized yield of GWO and RSM optimization

Table 4: Optimized yield from GWO and RSM

YRSM	48.203	38.690	63.916	69.540	50.984	61.806	48.945	71.895	83.412	49.908
YGWO	48.198	38.673	63.905	69.521	50.884	61.706	48.765	71.895	79.50	48.546
YRSM	59.629	45.747	51.287	54.038	43.547	66.618	39.007	73.137	59.873	48.751
YGWO	55.758	43.465	50.244	53.567	43.300	65.745	38.976	73.444	58.598	48.721

The optimized results shown in Figure 4 and 5 indicate the optimum value/optimized biodiesel yield from GWO and RSM with respect to the input process parameters, and the optimal conditions (desirability) for the cocoa pods and palm kernel shell oil methyl ester respectively. The optimized biodiesel yield and the optimum values were obtained through

iterations of one hundred (100) solutions and the best yield selected at iteration number seventy-seven (77), where catalyst concentration was 4.88715, methanol/molal ratio was 4.2253, temperature was 71.7772, reaction time was 141.83, and the agitation speed was 275.899. The optimized biodiesel value was 83.4125.

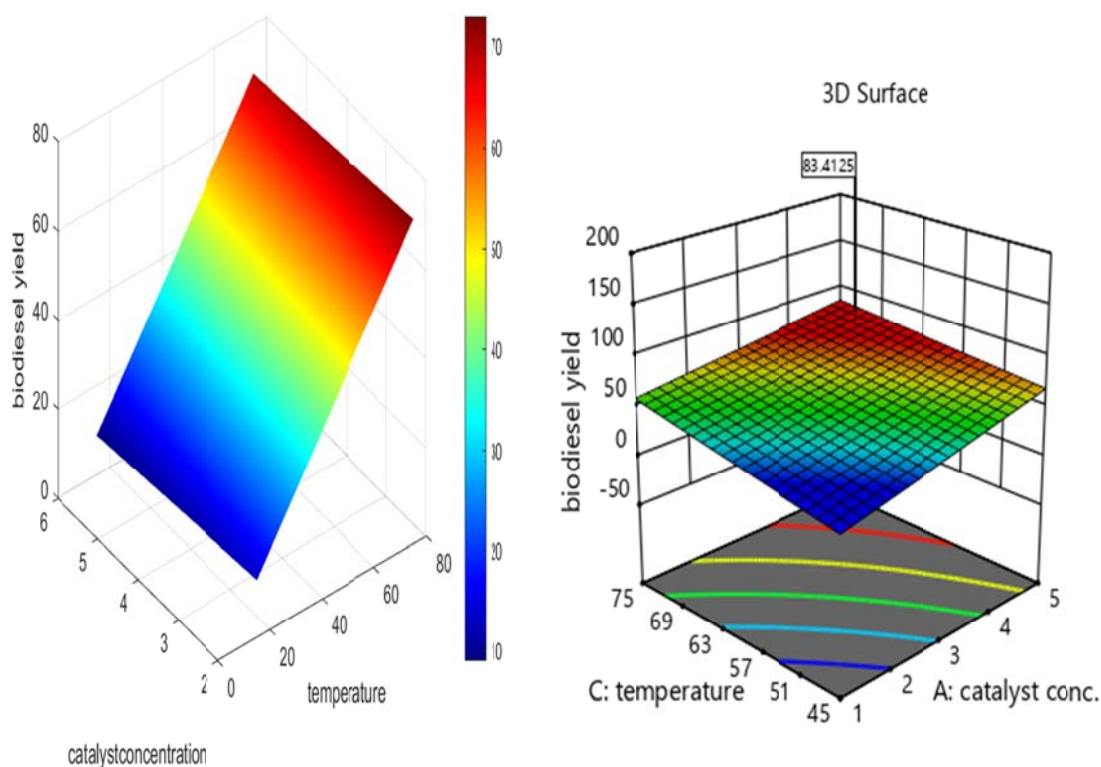


Figure 4: Optimum values and optimized biodiesel yield from GWO and RSM with respect to the process parameters obtained through iterations

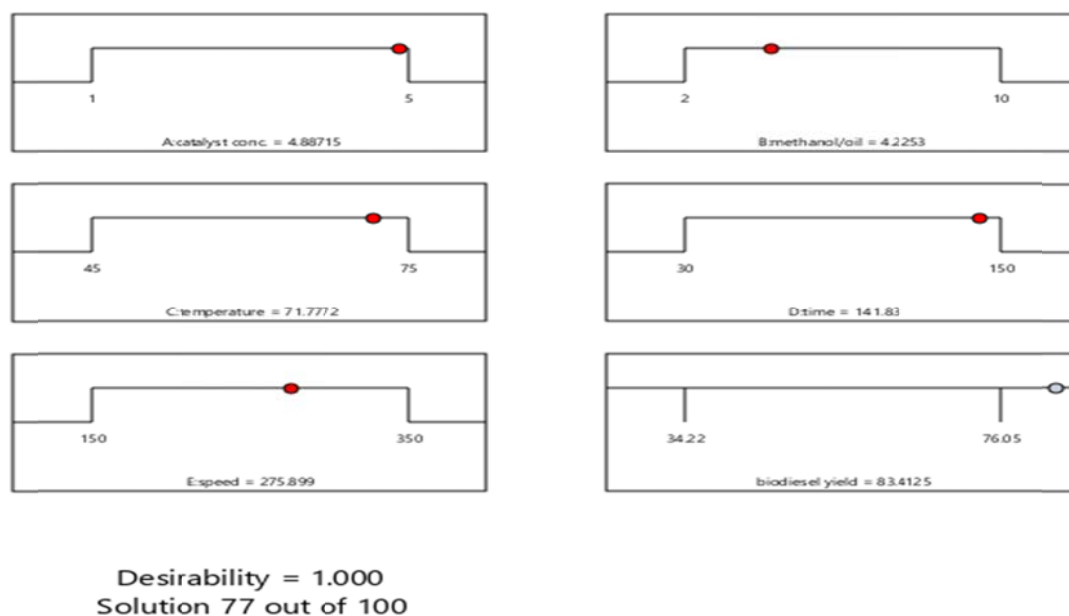


Figure 5: Optimal conditions for cocoa pods and palm kernel shell oil methyl ester

5. CONCLUSION

An optimal design in RSM full fractional factorial which identified the various design points being numerical and discrete was employed to achieve the optimum process parameters for the production of biodiesel using titanium oxide as catalyst. Five process parameters (catalyst concentration, reaction temperature, reaction time, methanol/oil molal ratio and agitation speed) were considered and varied individually within different ranges to anticipate biodiesel yield in a matrix.

The process parameters were independent variables while the biodiesel yield was the response, which is known as the dependent variable. A matrix design was adopted to study the combined effects of the process parameters in the production of the biodiesel.

There were two sets of values: the actual (experimental) values and the predicted values for the biodiesel yield using the design matrix for the titanium oxide catalyst. An ANOVA statistics was carried out to predict the biodiesel yield and to understand the combined or interactive effects of the process parameters. The output model equation was established using coefficient in terms of coded factors and statistical plots. This was employed to identify the impacts of the process parameters by comparing the factor coefficients.

This model equation was used to make predictions about the response (biodiesel yield) while the combined effects of

the parameters' interaction were presented in 3D plots. The condition for optimization of the biodiesel production using RSM and GWO was established. A comparative analysis of the results obtained was carried out. It showed that the optimized biodiesel yield from the RSM was higher when compared with that of the Grey-wolf optimization. However, it was revealed that the two results were closely related; with total yield of 79.50 and 83.412% for GWO and RSM respectively. This indicates that the GWO and RSM are good optimization tools for biodiesel production.

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