

# Implementing the Mechanical Behaviour of a Series DC Machine in Traction Drive Systems with Inverter-fed Induction Machine

Grzegorz Skarpetowski

DIMOPEQ GmbH

Untersiggenthal, Switzerland

skarpetowski@hispeed.ch

Michèle Krättli

DIMOPEQ GmbH

Mastrils, Switzerland

kraettli@dimopeq.ch

Andreas Krättli

Mastrils, Switzerland

kraettli.skarpetowski@gmail.com

**Abstract**—Experience gained over the years of using induction machines in adhesion traction drives has shown that with the usage of "shunt" mechanical characteristics of the type  $|T_1| = |T_1^{MAX}| = \text{const.}$  causes significant problems in the controls during dynamic states. The destructive influence of dynamic states on traction drive systems can be significantly reduced by replacing "shunt" with "series" mechanical characteristics. This type of characteristics can be realised only in drives with inverter-fed induction machines supplied with three degrees of freedom of machine operating point. The article explains selected problems related to the creation of such serial characteristics.

**Keywords**— *traction drive, series characteristic, behaviour in "slip and slide" dynamic states*

## I. INTRODUCTION

In quasi-steady states operation of "shunt" mechanical characteristics-controlled drive, the induction machine operating point passes smoothly from one very steep shunt mechanical characteristic to another without causing any change in the torque developed by the machine.

In the dynamic states occurring in adhesion drives, caused by wheel "slip and slide", the operating point of the machine moves along the instantaneous shunt characteristic, causing significant changes in the drive torque, often up to a change in its sign.

The mechanical components of the power transmission channel are in this situation subjected to sudden additional dynamic loads. Dynamic overloading not only causes traction problems, but also leads to a significant reduction in the service life of the entire traction system.

## II. SERIES CHARACTERISTICS

It is advisable to read the contents of the articles [1], [2], [3], [4], [5] as they provide a good introduction to the topics covered further. The electrical operating point of the machine moves in the space defined by the coordinates  $(U_1, f_1, f_2)$  or  $(U_1, f_1, I_1)$  [3]. The position of the machine operating point described by the values of its coordinates must be controllable

according to the course of the implemented series characteristics.

In order to obtain the effect that causes this point to move in dynamic states along the series characteristics, the realisation of the dependence of the machine's phase voltage amplitude on the stator frequency  $U_1 = f(f_1)$  previously used to shape the shunt characteristics must be abandoned. Freeing the voltage coordinate from its relationship to the stator frequency equips the operating point of the induction machine with an additional degree of freedom allowing varying the value of the stator magnetic flux at any value of its other two coordinates.

In the dynamic states of a drive system with a converter-fed induction machine, the operating point of the machine should be controlled by an algorithm with a properties causing reaction similar to that observed in DC drives with a series machine. In this drive system, axis slip causes a reduction in motor current and flux and results in a decrease in the value of the driving torque developed by the motor.

Induction machines with three degrees of operating point freedom can be controlled along the series characteristics in two ways: either by reducing the flux and the current or by reducing only the flux. The authors have focused on the method based on the reduction of the stator magnetic flux. The stator flux modulus can be reduced by decreasing the phase voltage modulus at a constant value of the stator frequency or by increasing the stator frequency at a constant value of the phase voltage modulus according to the formula  $\Psi = \sim U/f$ .

The relation  $U_1/I_1$  [2] also explains the possibility of controlling the value of the stator current by adjusting the value of the rotor frequency accordingly to the change in the stator frequency.

A new limitation of the voltage coordinate value of the machine operating point will be introduced in the presented control method, limitation which forces a series course of the shaped mechanical characteristics of the drive. The control of the voltage value at the stator terminals becomes independently of its frequency a special task in traction drive systems and will be discussed in detail in this article.

The series character of the mechanical characteristics in multi-drive traction systems, where each induction

machine is powered by its own autonomous inverter, can be easily achieved by forcing all the inverters to generate the same phase voltage value, regardless of the stator frequency they produce.

This will be achieved by centrally supplying all independent drive systems, operating in motoring dynamic states at different frequencies, with the same phase voltage value, equal to the voltage on the machine operating at the lower stator frequency.

The lowest stator frequency occurs on the machine driving the axle operating with the least slip, i.e. at the speed closest to the actual vehicle speed.

The common value of the phase voltage amplitude corresponding to this frequency results from the relationship shown in equation (1).

$$U_{1W} = |U_1|(\text{MIN}(f_{11}, f_{12}, f_{13}, f_{14} \dots)) \quad (1)$$

This means that all machines operate with the same phase voltage, but with different stator frequencies, different stator fluxes and different torque values on different characteristics similar to those of series-excited DC motors.

In vehicle with single drive system, or in multi-drive system with slip and slide state on all axes, the amplitude of the stator voltage can be defined as depending on the vehicle actual speed according to the relation:

$$|U_1|(f) = f(vp). \quad (2)$$

Series characteristics can be realised in the whole area of possible operating points of an induction machine, regardless of the type of envelope used for shaping the traction characteristic waveform.[4]

### III. IMPLEMENTATION OF SERIES CHARACTERISTICS IN TRACTION DRIVES

In quasi-steady state drive operation, as long as the angular velocities of the axes vary only negligibly and the inverter generates the "same" stator frequency and the "same" stator voltage, induction machines drives with implemented series characteristic operate in a way corresponding to the their natural shunt characteristics.

The series nature of the control becomes apparent only in the dynamic states of the drives caused by a sudden change in load due to loss of adhesion at the wheel-rail contact.

In order to explain the principle of transition of the drive system operating point from the shunt characteristic to the series characteristic, an example of the dynamic state occurring during operation of the adhesion traction drive system is presented.

In the state of good adhesion in the range  $|\Psi_1| = \text{const.}$  all drive systems operate with the same frequency  $f_1$  and are supplied with the same phase voltage equal  $U_{1W} = |U_1|(\text{MIN}(f_{11}, f_{12}, \dots, f_{1N}))$ .

As a result of breaking adhesion on contact wheel-rail, the slip between the wheel and the rail increases and the induction machine current decreases rapidly in motoring and increases rapidly in the braking operation of the traction drive.

The control system using series characteristics in motoring operating conditions, increases the stator and rotor frequency in the slipping and sliding drive and tries to maintain the constant value of the stator current and in this way to prevent idling or even generator braking.

However, the central setting of the phase voltage amplitude which depends on the stator frequency of slowest drive does not allow adjusting the phase voltage to the dynamic stator frequency. This results in a reduction of the magnetic flux of the stator and the movement of the operating point of the relieved machine along the series characteristic.

Each autonomous drive works according to its own capabilities, which are determined by the difference in adhesion and the difference in wheel diameters, realizing a common stator current modulus value set by the controller, with common stator voltage amplitude for all the machines. The induction machines of such a system operate with different power, at different frequency and with different torque, but transfer to the rails the maximum driving power possible under these conditions.

The driving torque of the series characteristic decreases with increasing frequency and asymptotically approaches zero but cannot change its sign. The reduction of torque on a sliding vehicle axle continues until the forces at the wheel-rail contact are equalised. A positive side effect of this wheel slip is the cleaning or drying of the rail. The return of the machine operating points to their shunt character and the equalization of the load on all axles occurs only after a corresponding increase of the friction coefficient between wheel and rail.

Figure 1 shows the differences between the shunt characteristics  $|\Psi_1|$ ,  $|U_1|$  and  $|\Psi_2|$ , and the family of series characteristics starting at the nominal point. The steepness of the series characteristics is controllable and can be adjusted to the requirements of the dynamic state.

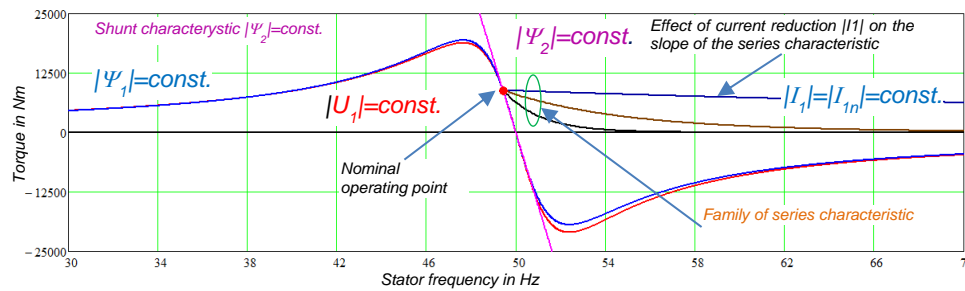


Figure 1. The figure shows the comparison of four types of induction machine mechanical characteristics. The characteristic curve  $|\Psi_1| = |\Psi_{1MAX}| = \text{const.}$  is shown by a blue line and the characteristic curve  $|U_1| = |U_{1MAX}| = \text{const.}$  is shown by a red line. The characteristic  $|\Psi_2| = |\Psi_{2MAX}| = \text{const.}$  which is linear in the whole range of used frequencies is shown in violet. Three series characteristics starting from the nominal operating point and showing the influence of the reduction of the stator current amplitude modulus on their slope are marked with a green ellipse. Three types of series characteristic curves are shown at figure 1. The curve with constant value of the stator current amplitude  $|I_1| = \text{const.}$  is shown in blue, the curve with variable, controlled current value  $|I_1| = f(s)$  in brown and the curve with constant value of the rotor frequency  $f_2 = \text{const.}$  in black

Figure 2 shows the usage of shunt characteristics  $|\Psi_1| = \text{const.}$  and examples of series characteristics of the type  $|I_1| = \text{const.}$  to control the converter traction drive.

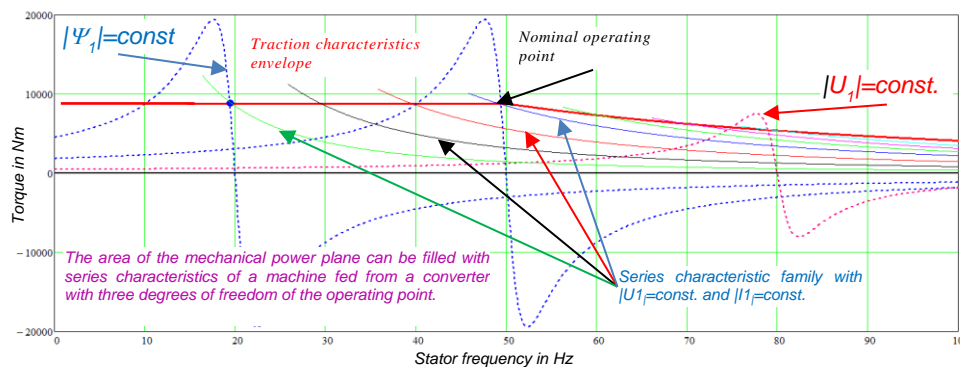


Figure 2. The first quadrant of the mechanical power plane ( $Mom, f_1$ ) shows the area bound by the envelope of traction characteristics (red). Examples of series characteristics (indicated by arrows) and shunt characteristics  $|\Psi_1| = \text{const.}$  and  $|U_1| = \text{const.}$  (dotted lines) indicate that they can be placed in this area

In the controllability area filled with series characteristic curves, the operating point of the traction machine in non-dynamic states in the control range  $|\Psi_1| = \text{const.}$  passes smoothly from one characteristic curve to the other without creating a torque jumps but in dynamic states it moves along the series characteristics of the machine.

In the stator frequency range higher than the rated frequency, the operating points of the traction system are already now arranged along a special type of series characteristic with critical value of rotor frequency.

The controllability of the series characteristic of the induction machine in this frequency range depends on whether the inverter pulsation mode allows the realisation of the third degree of freedom of the machine operating point.

When powered by 2-level inverters and operating with a full  $180^\circ$  voltage block at the terminals of the induction machine, the ability to reduce the phase voltage value is lost. In such a system, the waveform of the series characteristics can only be changed by varying the load with the value of stator current or with the rotor frequency.

#### IV. SERIES CHARACTERISTIC AS PART OF THE TRACTION CHARACTERISTIC

The controllability area of the traction drive is surrounded by the envelope called the traction characteristic. This area can be filled with both shunt and series characteristics. The use of series characteristics can help solving problems observed in the dynamic states of the traction drive systems with shunt characteristics. Series characteristics have only one point of contact with the envelope of the controllability area, but are able to produce operating points advancing along this envelope and leave at any stator frequency (travel speed) and reduce the driving torque. There are three sections in the traction characteristic curve using series mechanical characteristics.

The first section contains the set of operating points lying on the envelope of controllability, i.e. on the traction characteristic.

The second section contains the operating points between the frequency at which the dynamic state caused the transition to the series characteristic and the stator frequency of reaching the critical value of the driving torque.

The third section contains the operating points lying on the series characteristic controlled by the critical value of the rotor frequency.

V. FIRST SECTION OF SERIES CHARACTERISTIC WITH  $|\Psi_1| = \text{CONST.}$  AND  $|I_1| = \text{CONST.}$

The characteristic feature of the first section of series characteristic is quasi-stationary operation. All drive systems of traction vehicle operate with full magnetic flux of the induction machine. This operating state is described by the relationships between flux, current and rotor frequency shown in modular form in Figure 3. The value of one of these quantities can be determined from the values of the other two quantities. Thus, for example, the value of the module  $|\Psi_1|$  can be determined by measuring the values of  $f_2$  and  $|I_1|$ .

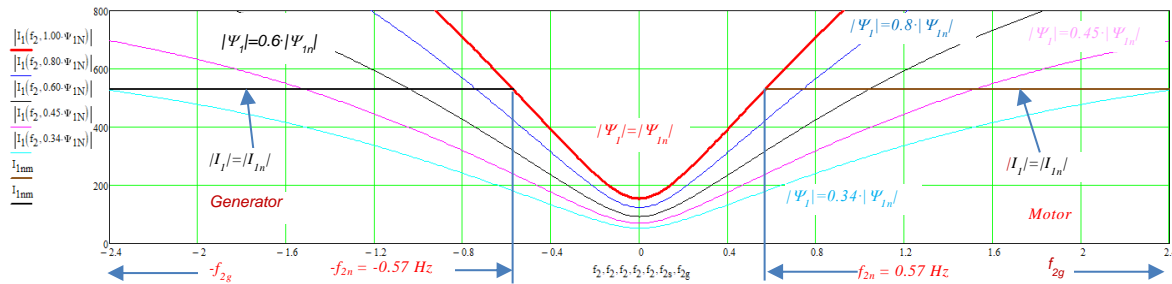


Figure 3. Maintaining a constant rated value of the rotor current modulus is possible in the range of rotor frequencies greater in modulus than those found in the rated curve  $|\Psi_1 n|$   $f_{2n} < |f_2|$  (red) and less than the limiting frequency  $|f_2| < f_{2g}$ . At a constant value of the stator current each change of the frequency in the rotor causes a change of the magnetic flux of the stator

VI. SECOND SECTION OF SERIES CHARACTERISTIC WITH  $|U_1| = \text{CONST.}$  AND  $|I_1| = \text{CONST.}$

In a converter drive system using series characteristics to control operating points of the machine not only in dynamic states but also in static states there is no fixed point of transition from the first to the second sector of the characteristics. The beginning of the range  $|U_1| = \text{const.}$  at  $f_1 < f_{1n}$  is determined by the occurrence of the "slip and slide" dynamic state causing the need to reduce the driving torque developed by the machine in motoring operation. The occurrence of a "slip and slide" at frequency  $f_{1z}$  causes the operating point of the drive system to leave the static states operating points curve and descend along the series characteristics.

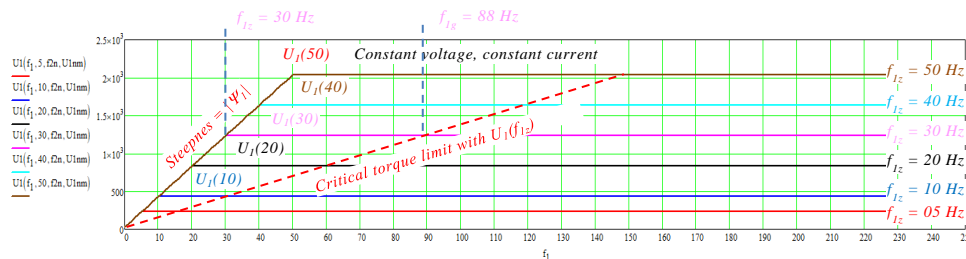


Figure 4: The figure shows the voltage-frequency characteristics for several selected values of the "slip and slide" frequency in a dynamic state. The term "constant stator voltage range" takes on a new meaning when dealing with series characteristics. This "constant voltage" range can start from any value of the stator frequency  $f_{1z}$ , and the voltage amplitude can take different values, depending on the lowest frequency generated by the other inverters of the vehicle. The voltage curves are valid for constant, dynamic states independent of vehicle speed. The usable range of phase voltage variation is limited by three quantities: the maximum value of the stator magnetic flux, the maximum value of the phase voltage and the value of the frequency at which the driving torque reaches its critical value. The frequency  $f_{1g}$  marking the end of the second segment of the characteristic at  $|U_1| = \text{const.}$  and  $|I_1| = \text{const.}$  also depends on the value of the "slip and slide" frequency  $f_{1z}$ . Example of the voltage range in the second sector of the series characteristic which starts at  $f_{1z} = 30$  Hz and ends at  $f_{1g} = 88$  Hz, is shown in pink

It will be assumed that at the moment of dynamic state occurring (slip and slide) the induction machine is at the operating point described by the coordinates  $f_{1z}$  and  $f_{2z}$  and the phase voltage modulus is equal to  $|U_1(f_{1z}, f_{2z})|$ . The converter supplying it, in order to maintain the current value at the level existing at the point  $|I_1(f_{1z}, f_{2z})| = \text{const.}$  raises the frequency, but does not increase the phase voltage. The drive system is in the range of a constant, but reduced phase voltage value, with  $|U_1(f_{1z}, f_{2z})| = \text{const.}$ , the stator frequency increases, the magnetic flux of the stator decreases and the drive torque moves along a curve approaching asymptotically to zero. At a stator frequency equal to  $f_{1g}$ , the range of the constant current module must be terminated because the rotor frequency reaches the critical value  $f_{2uk}(f)$ . For higher stator frequencies, the control principle has to be changed due to the critical torque value being reached.

VII. THIRD SECTION OF SERIES CHARACTERISTIC WITH  $|U_1| = \text{CONST.}$  AND  $M1_1 = M_{KR}$

As soon as the stator frequency increases to the value  $f_{1g}$ , the force coordinate of the operating point of the machine, moving along the mechanical characteristic  $|U_1| = \text{const.}$  reaches its critical value of the driving torque



$M_{KR}$ , which depends on voltage and stator frequency. With further increase of the stator frequency due to the imbalance of forces at the wheel-rail contact, the torque decreases along the series critical torque curve with  $|U_1| = const.$  and  $M_{I1} = M_{KR}$  approaching asymptotically to zero.

VIII. LIMITS OF THE SECOND SECTION OF SERIES CHARACTERISTIC WITH  $|U_1| = CONST.$  AND  $|I_1| = CONST.$

Figure 5 shows the variety of realisable control ranges with  $|I_1| = const.$  and  $|U_1| = const.$  of the second section of the series characteristics for several selected frequencies of slip and slide states,  $f_{z1} = 5$  Hz,  $f_{1z} = 10$  Hz,  $f_{1z} = 20$  Hz,  $f_{1z} = 30$  Hz,  $f_{1z} = 40$  Hz and  $f_{1z} = 50$  Hz.

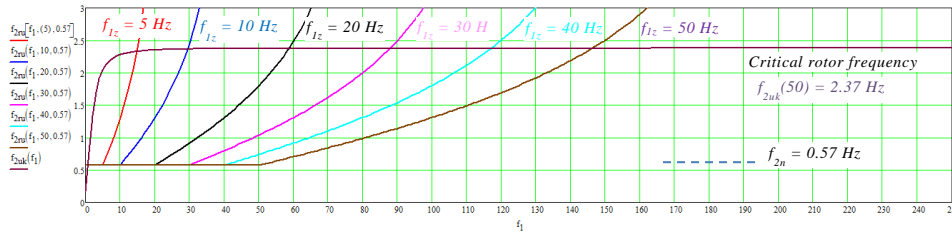


Figure 5. Curves showing the value of the rotor frequency  $f_{2ru}$  necessary to maintain the rated  $|I_1(f_{1z}, f_{2z})| = |I_1(f_{1n}, f_{2n})|$  value of the traction machine current for six examples of slip and slide frequencies:  $f_{z1} = 5$  Hz,  $f_{1z} = 10$  Hz,  $f_{1z} = 20$  Hz,  $f_{1z} = 30$  Hz,  $f_{1z} = 40$  Hz and  $f_{1z} = 50$  Hz without taking into account the effect of this frequency on the value of the torque developed by the machine i.e. without limiting the range of the constant current to the limiting rotor critical frequency  $f_{2uk}(f_1)$

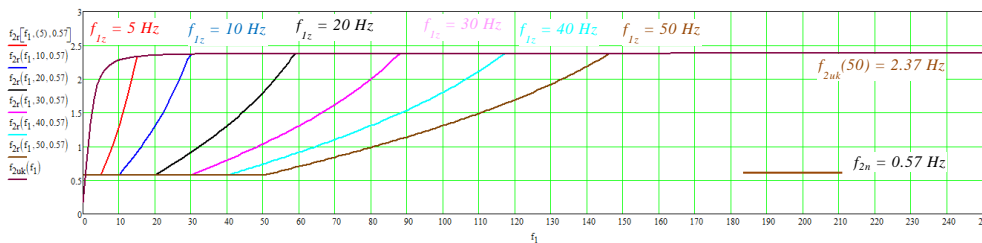


Figure 6: Modification of the rotor current frequency curves from the form  $f_{2ru}$  to the form  $f_{2r}$  taking into account the critical limit of the rotor frequency  $f_{2uk}(f_1)$

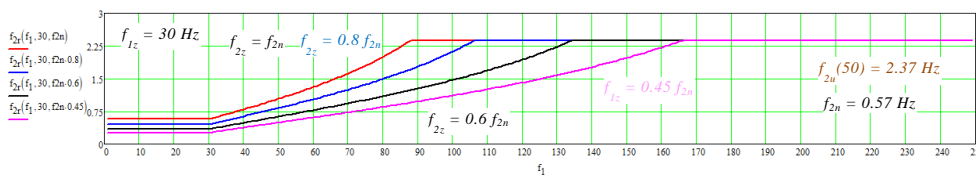


Figure 7: Effect of load reduction on the length of the range of constant current  $|I_1| = const.$  and constant voltage  $|U_1| = const.$  at example of frequency  $f_{1z} = 30$  Hz. Decreasing the current value by reduction of  $f_{2z}$  causes widening of this range

IX. MODELLING OF THE SERIES CHARACTERISTIC DRIVE SYSTEM PROPERTIES

The results of the modelling of traction drives with  $|U_1| = const.$  and  $|I_1| = const.$  control are presented at figures 8, 9, 10, 11.

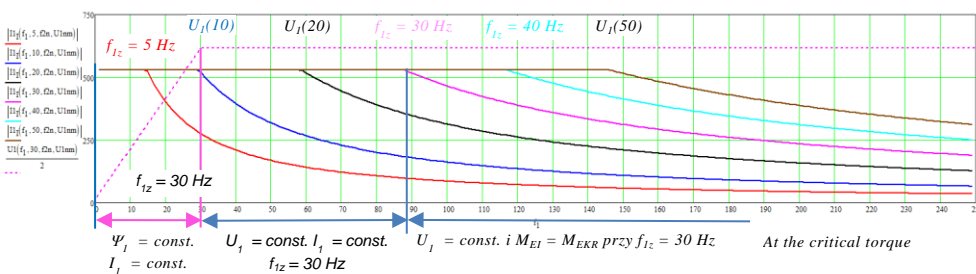


Figure 8: The figure shows the stator current moduli for selected slip and slide frequencies. The nature of these waveforms is explained using the example of frequency of  $f_{1z} = 30$  Hz. The voltage and current waveforms for this case are shown in pink. From  $f_1 = 0$  Hz to  $f_1 = 30$  Hz the drive system operates as if controlled with a "shunt characteristic". At  $f_{1z} = 30$  Hz, adhesion is broken. The controller maintains a constant current value by increasing the stator frequency up to a frequency of 88 Hz, at which the drive torque reaches a critical value. Starting from this stator frequency, the frequency in the rotor can only be reduced or maintained at a value  $[f_2] < f_{2uk}(f_1)$ . The current decreases as a result of the limitation of the torque developed

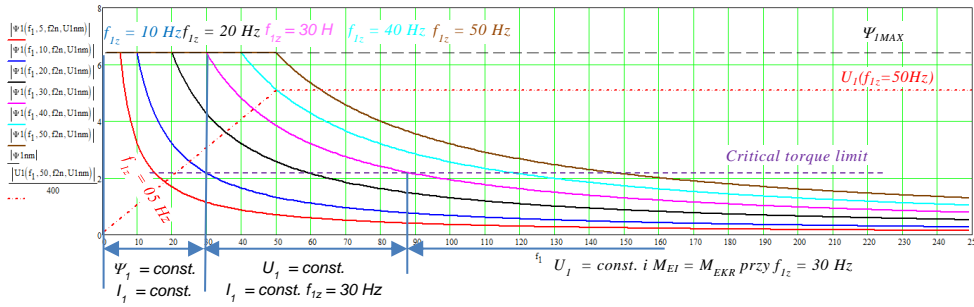


Figure 9: Variation of the stator magnetic flux for different values of the slip and slide frequency in the drive system controlled with series characteristics. The example of the flux curve shown in pink confirms that the reduction of the stator magnetic flux, in spite of keeping the machine current modulus constant, starts already at the "slip and slide" frequency equal in this case  $f_{1z} = 30$  Hz. The flux reduction in the range below the critical torque is consistent with the weakened field theory

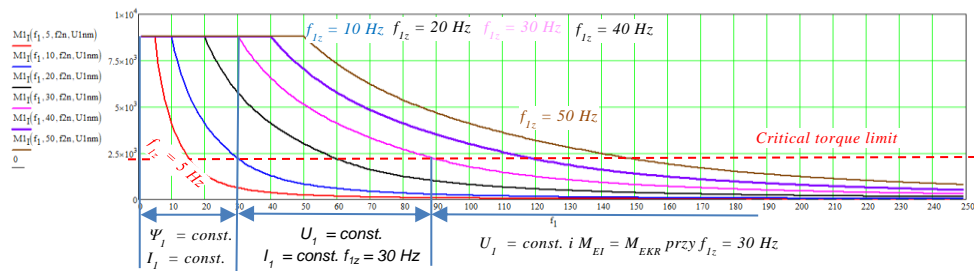


Figure 10: The shape of the family of series characteristics is produced by the decreasing flux and the phase change of the stator current complexor. In the course of pink curve the transition to the third segment of the series characteristic happens at  $f_1 = 88$  Hz

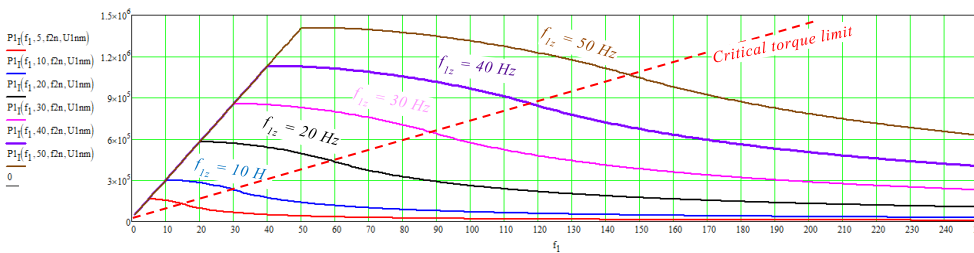


Figure 11: Power curves at the stator terminals of an induction machine operating with the series mechanical characteristic. Maintaining a constant value of the stator current modulus does not lead to the creation of a range of constant values of the power consumed

#### X. EXTENDING THE OPERATING AREA OF THE MACHINE BEYOND THE CRITICAL TORQUE

If the operating point of the machine, while trying to maintain a constant value of the stator current modulus, moves beyond the critical torque of the characteristic curve to be realised, this will cause a fairly rapid reduction of the driving torque developed, but will not result in a loss of controllability of the machine current.

The results of the modelling of the waveforms occurring after the "slip and slide" at the stator frequency  $f_{1z} = 30$  Hz presented in Fig. 12 and Fig. 13 show that in this case it is possible to maintain a constant value of the current modulus up to the stator frequency  $f_1 = 125$  Hz, i.e., up to the almost complete disappearance of the driving torque of the induction machine.

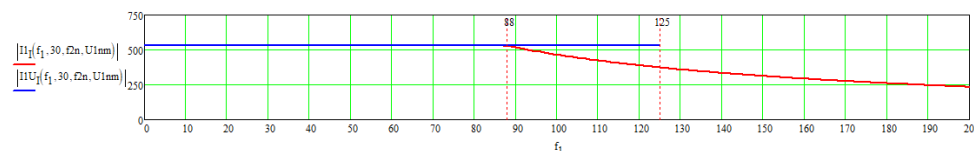


Figure 12: The maintenance of a constant value of the current modulus, by increasing the rotor frequency, after "slip and slide" at  $f_{1z} = 30$  Hz is possible up to a frequency of 126 Hz. However, the rotor critical torque frequency  $f_{2uk}$  is already reached at  $f_1 = 88$  Hz

When entering the range of rotor frequencies higher than the critical torque frequency  $f_{2uk}$ , there is a decrease in the torque developed by the machine and a decrease in the power consumption despite the constant value of the stator current modulus.

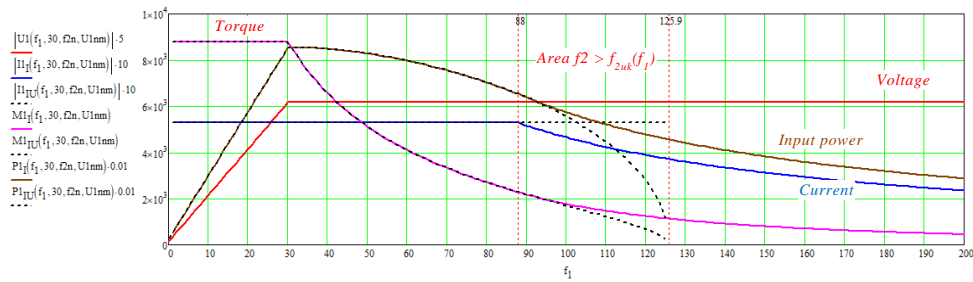


Figure 13. Exceeding the critical rotor torque frequency can be caused, for example, by the controller not responding to the command to change the algorithm from controlling the current amplitude module to controlling the rotor frequency at a level lower than the critical value  $f_{2uk}$ . This will lead to a decrease in the torque developed by the machine and a reduction in the power consumed despite the stator current remaining constant. When the supply frequency of the machine used in the calculation example reaches  $f_1 = 126$  Hz, the frequency in the rotor will increase to  $f_2 = 24$  Hz, which corresponds to a mechanical frequency of  $f_m = 102$  Hz, the driving torque will drop to 14 Nm and the power consumption to 114 kW

### XI. FEATURES OF SERIES CONTROL WITH FIXED F2

In order to indicate the possibility of steepness control of the series characteristics, a comparison of the already discussed constant-current characteristic with  $|U_1| = const.$  and  $|I_1| = const.$  with the characteristic  $|U_1| = const.$  and  $f_2 = const.$  maintaining a constant value of the frequency in the rotor in in the dynamic state was carried out.

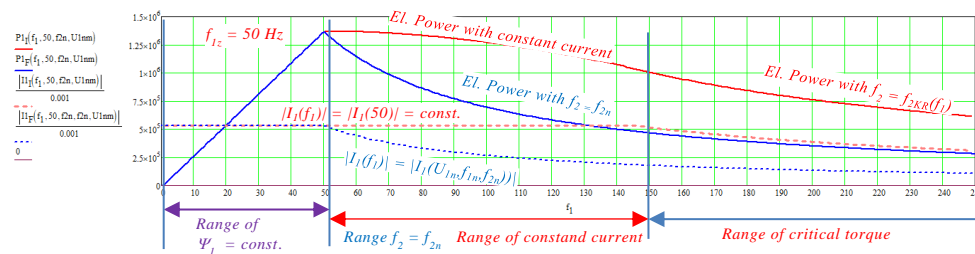


Figure 14: Comparison of power and current curves for two types of series characteristics produced in converter drives with induction machines. The red colour shows the characteristics with the rated value of the machine current modulus and the blue colour shows the characteristics with the rated value of the rotor frequency at the "slip and slide" frequency  $f_{1z} = 50$  Hz

Figures 14 and 15 show a comparison of the characteristics of these control methods, indicating a difference in the active power consumed, a difference in the value of the stator current modulus and a difference in the driving torque developed. The characteristic realised with a constant rated stator current has much less steepness than the characteristic realised with a constant rated frequency value in the rotor. This makes it possible to respond flexibly to the occurrence of different dynamic states in adhesion traction drives.

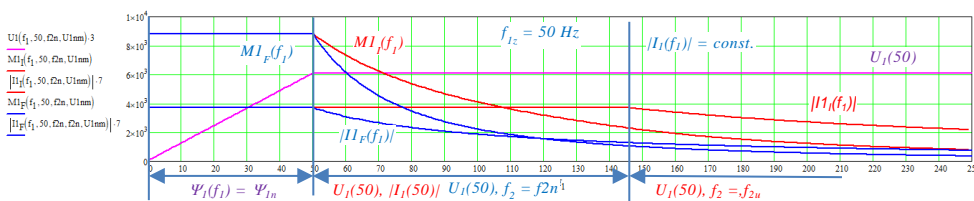


Figure 15: Comparison of the torque and stator current amplitude modulus curves for two types of series characteristic control. The curves corresponding to the control with constant stator current  $M1_f(f_1, f_2, U_1)$  and  $I1_f(f_1, f_2, U_1)$  are shown in red and the curves with constant rotor frequency  $M1_F(f_1, f_2, U_1)$   $I1_F(f_1, f_2, U_1)$ , i.e. with correspondingly reduced current, in blue. The comparison was made for "slip and slide" frequency  $f_{1z} = 50$ Hz

### XII. CONCLUSION

By adding a third degree of freedom to the operating point of an inverter-fed induction machine, the possibility of creating the series course of its mechanical characteristics was achieved.

The value of the phase voltage, which is this released coordinate, is not completely arbitrary and depends on three following quantities: on magnetic stator flux, on

the "slip and slide" stator frequency and on the critical rotor frequency.

Comparing the waveforms shown in Figures 14 and 15, it can be seen that the power of the drive system and its torque are much better utilised when using a constant-current control algorithm compared to controlling with a fixed value of the rotor frequency. The same applies to the current capability of the converter, which is fully used over a wider range of

stator frequencies if control with constant-current is implemented.

The steepness of the series characteristics can be significantly modified by varying the value of the stator current modulus. The influence of the current value on the gradient of the series characteristic is shown in Figure 1.

The results of all calculations presented in this paper were performed on a modified induction machine model [2] with the equivalent data introduced in [5].

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