Journal of Multidisciplinary Engineering Science and Technology (JMEST)
ISSN: 2458-9403
Vol. 9 Issue 10, October - 2022

Effect of rotational Reynolds number on heat and mass transfer around a cone of revolution

BEZANDRY Germain
Laboratory of Mechanics Energetic and Environment
University of Fianarantsoa, Madagascar
bezandriantallah@gmail.com

RANDRIANARIVELO Raymond
Laboratory of Mechanics Energetic and Environment
University of Fianarantsoa, Madagascar
raymrandria2211@gmail.com

Belkacem ZEGHMATI
LAboratoire de Modélisation Pluridisciplinaire et Simulations. Groupe de Mécanique Énergétique
Université de Perpignan Via Domitia, France
zeghmati@univ-perp.fr

FANJIRINDRATOVO Tovondahiniriko
Laboratory of Physics and Environment
University of Toliara, Madagascar
fanjiry@gmail.com

Marcelin Hajamalala ANDRIANANTENAINA
Laboratory of Mechanics Energetic and Environment
University of Fianarantsoa, Madagascar
hajamalalaa@yahoo.fr

***Abstract*— The effect of rotational Reynolds number on heat and mass transfer around a cone of revolution in rotation to its axis of revolution and immersed in a Newtonian fluid as air at rest, has been investigated. The cone wall is impervious to heat and maintained at a constant and uniform temperature and mass concentration. The mathematical model that reflects the physical phenomenon of the studied model has been established. This phenomenon is modeled by partial differential equations. The heat, mass transfer, Navier-Stokes and continuity equations are approximated by boundary, permanent layers with constant physical properties. An implicit finite difference scheme is used to discretize the equations of the mathematical model. The discretized equations were solved by Thomas's algorithm associated with the boundary conditions. A calculation code has been developed. The results concern the distributions of the meridian, azimuth and normal components of velocity, temperature, mass concentration in the boundary layers and the Nusselt number, Sherwood number and friction coefficients. Results show that the rotation of the cone creates a longitudinal upward flow of fluid, and a suction of fluid towards the wall of the cone. There is an azimuthal detachment of the dynamic boundary layer. The influence of the rotational speed of the cone on heat and mass transfer rates is presented and discussed. Results have been validated with the study of Himasekhar et al. [16] and of Hering et al. [17] and presented a good agreement.**

Keywords—rotational Reynolds; heat transfer; mass transfer; cone of revolution.

I. INTRODUCTION

Heat transfer around symmetrical bodies of revolution has been the subject of many studies, given their practical interest, especially in machines. Examples include certain hydraulic structures, aircraft, turbo machinery, ship propulsion systems, rockets, projectiles, and metal vapor deposits. Most of the work concerns a simple geometry such as the plane plate, the cylinder, the sphere. To our knowledge, the only published works relating to the numerical study of combined natural and rotary convection Raymond [1] and of pure convection around a cone of revolution are those of Rakotomanga [2] and Ulrich [3].

Given the abundance of published work on convections and heat and mass transfers around a cone of revolution: Raymond et al. [1] focused their study to the natural and rotational three-dimensional mixed convective flow around an inclined cone of revolution immersed in a Newtonian fluid, with any opening and rotating with respect to a fixed axis with a constant angular speed. Authors highlighted the predominance of mass transfer against heat transfer and vice versa in pure mixed convection, and the predominance of natural convection over rotary convection and vice versa in the case where the transfers are comparable. Rakotomanga [2] studied transfers by laminar forced convection around a cone of revolution closed on its upper part by a spherical cap and inclined in relation to the vertical. Author determined the distribution of the outer velocity to the dynamic boundary layer using the singularity method. The conservation equations by an implicit finite differences method associated with Thomas' algorithm has been solved. It showed that the influence of the inclination of the cone results in a slight increase in the thickness of the boundary layer. Pop et al. [4] numerically studied natural convection around a sinusoidal wall cone of revolution. They dealt with the problem of natural convection flux and heat transfer induced by a vertically oriented cone with a constant surface temperature. The cone is assumed to have transverse corrugated configurations. The flow of the limit layer at rest is described by two coupled parabolic partial differential equations. These equations are solved numerically using the Keller's box method for a sinusoidal corrugated cone. The local Nusselt count was lower than the corresponding flat cone has been found. In addition, the local Nusselt number periodically varies in the direction of x and increases with x. The local Nusselt number is smaller for a corrugated cone than that corresponding to a flat cone, local Nusselt number values are higher for larger Prandtl number values. Hossain et al. [5] investigated the natural convection flow of viscous fluid around a truncated cone. They have treated in cases where the viscosity and thermal conductivity of the fluid are temperature dependent. Authors use the appropriate transformations to obtain the equations governing flow in a practical form and integrate them using an implicit finite difference method. Disturbance solutions are used to obtain the solution in diets near and far from the truncation point. Anilkumar et al. [6] treat unstable mixed convection flow on a rotating cone in a rotating fluid due to the combined effects of thermal diffusion and mass diffusion. The system of ordinary differential equations governing flow was solved numerically by using a schema of implicit finite differences in combination with the quasi-linearization technique. The prescribed wall temperature and heat flow conditions are both taken into account. Numerical results are reported for friction coefficients, Nusselt number and Sherwood number. The effect of various parameters on velocity, temperature and concentration profiles is also presented. Siabdallah et al. [7] treat, in a permanent state, the natural thermal and mass convection in the laminar boundary layer around a sinusoidal-walled cone trunk. It solved transfer equations by an implicit finite difference method associated with the Gauss Seidel iterative method. It shows that the increase in the amplitude of the sinusoid describing the shape of the cone trunk wall as well as the relationship between the volumic forces of thermal origin and those of mass origin is the cause of the decrease in the local Nusselt and Sherwood numbers and means. Rahman et al. [8] investigated the effect of temperature-dependent viscosity μ(T) on a two-dimensional natural convection flow along a vertical corrugated cone with a uniform surface heat flow. Viscosity is considered inversely proportional to temperature. The basic equations are transformed into adimensional boundary layer equations using suitable adimensional variables. Adimensional equations are discretized by the implicit finite differences method. It shown that the temperature inside the boundary layer at any fixed point decreases slightly as thermal conductivity increases. The results also show that the average heat transfer rate increases considerably with increases in thermal conductivity. Elbashbeshy et al. [9] investigated the influence of pressure on natural convection around a truncated cone. They solved digitally using the Mathematical technique. They showed that the speed and distribution of the temperature decrease with the increase in the pressure value. An increase in the value of the Prandtl number results in a decrease in the value of the coefficient of friction, but the local Nusselt number increases as the value of the Prandtl number increases.

This study focuses on the modeling of heat and mass transfers around a rotating cone of revolution. The effect of rotational Reynolds number on heat and mass transfer around a cone of revolution has been particularly studied. Heat and mass transfers within boundary layers as well as pulse transfer have been considered. The Couette effect caused by the rotation of the cone of heat and mass transfers bas been highlighted. Authors determine the distribution of velocity, temperature and concentration by solving systems of conservation algebraic equations using Thomas' algorithm. A detailed study of the mesh effect resulted in the selection of 150 x 250 knots as the most suitable mesh for this study. One analyzed the influence of the speed of rotation of the flow, heat and mass transfers.

II. Methodology

A. *Physical model*

A cone of revolution immersed in a Newtonian fluid at rest has been considered. It rotates around its axis of revolution at angular velocity ω (*Fig. 1*). The wall of the cone is maintained at a constant and uniform temperature T_p and mass concentration C_0. The temperature and concentration of the fluid outside the thermal boundary layer is set at T_∞ and C_∞ respectively. The difference between the temperature of the cone wall and that of the fluid at infinity is negligible when its speed of rotation is zero.

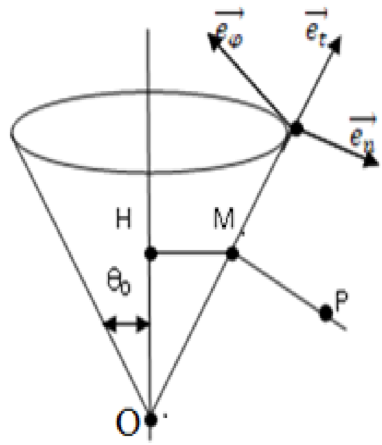

Fig. 1. *Diagram of the physical model*

or

x = OM: meridian coordinate
y = MP: normal coordinate
r = HM: normal distance from the projected M

B. *Mathematical formulation*

In order to develop our numerical model, we have adopted the following hypothesis:

- The flow is permanent and in laminar regime.
- The fluid is Newtonian and incompressible.
- The physical properties of the fluid are constant.
- The viscous dissipation function is negligible.
- The diffusion of species is to the mass basis.

We use the following adimensional variables:

$$x^+=\frac{x}{L},\quad y^+=\frac{y}{L},\quad r^+=\frac{r}{L} \tag{1}$$

$$V_x^+=\frac{V_x}{L\omega}Re_\omega^{\frac{1}{4}},\ V_y^+=\frac{V_y}{L\omega}Re_\omega^{\frac{3}{4}},\ V_\varphi^+=\frac{V_\varphi}{L\omega}Re_\omega^{\frac{1}{4}} \tag{2}$$

$$T^+=\frac{T-T_\infty}{\left(\frac{L^2\omega^2}{2Cp}\right)},\ C^+=\frac{C-C_\infty}{C_0-C_\infty} \tag{3}$$

These dimensionless equations in the boundary layers, boundary conditions, the coefficients of friction, Nusselt number and Sherwood number are written:

$$\frac{\partial V_x^+}{\partial x^+}+\frac{1}{\sqrt{Re_\omega}}\frac{\partial V_y^+}{\partial y^+}+\frac{V_x^+}{r^+}\frac{dr^+}{dx^+}=0 \tag{4}$$

$$V_x^+\frac{\partial V_x^+}{\partial x^+}+\frac{V_y^+}{\sqrt{Re_\omega}}\frac{\partial V_x^+}{\partial y^+}-\frac{V_\varphi^{+2}}{r^+}\frac{dr^+}{dx^+}=Re_\omega^{-\frac{3}{4}}\frac{\partial^2 V_x^+}{\partial y^{+2}} \tag{5}$$

$$V_x^+\frac{\partial V_\varphi^+}{\partial x^+}+\frac{V_y^+}{\sqrt{Re_\omega}}\frac{\partial V_\varphi^+}{\partial y^+}+\frac{V_x^+V_\varphi^+}{r^+}\frac{dr^+}{dx^+}=Re_\omega^{-\frac{3}{4}}\frac{\partial^2 V_\varphi^+}{\partial y^{+2}} \tag{6}$$

$$V_x^+\frac{\partial T^+}{\partial x^+}+\frac{V_y^+}{\sqrt{Re_\omega}}\frac{\partial T^+}{\partial y^+}=\frac{1}{Pr}Re_\omega^{-\frac{3}{4}}\frac{\partial^2 T^+}{\partial y^{+2}} \tag{7}$$

$$V_x^+\frac{\partial C^+}{\partial x^+}+\frac{V_y^+}{\sqrt{Re_\omega}}\frac{\partial C^+}{\partial y^+}=\frac{1}{Sc}Re_\omega^{-\frac{3}{4}}\frac{\partial^2 C^+}{\partial y^{+2}} \tag{8}$$

$$y^+=0: T^+=1, C^+=1,\ V_x^+=0, V_y^+=0, V_\varphi^+=r^+Re_\omega^{\frac{1}{4}} \tag{9}$$

$$y^+\to\infty: T^+=0\ ,\ C^+=0\ ,\ V_x^+=0\ ,\ V_\varphi^+=0 \tag{10}$$

$$\frac{1}{2}Re_\omega^{\frac{5}{4}}Cf_u=\left(\frac{\partial V_x^+}{\partial y_+^+}\right)_{y_+=0}\quad ,\ \frac{1}{2}Re_\omega^{\frac{5}{4}}Cf_w=\left(\frac{\partial V_\varphi^+}{\partial y_+^+}\right)_{y_+=0} \tag{11}$$

$$\frac{2}{Ec}Nu=-\left(\frac{\partial T^+}{\partial y^+}\right)_{y^+=0} \tag{12}$$

$$Sh=-\left(\frac{\partial C^+}{\partial y^+}\right)_{y^+=0} \tag{13}$$

The dimensionless parameters in these equations are defined as:

Prandtl number: $Pr=\frac{\mu Cp}{\lambda}$

Schmidt number: $Sc=\frac{\mu}{\rho D}$

Rotational Reynolds number: $Re_\omega=\frac{L^2\omega}{\nu}$

Eckert number: $Ec=\frac{(L\omega)^2}{Cp\Delta T}$

III. Numerical Methods

The equations of continuity, Navier Stokes, heat and mass transfer associated with boundary conditions are discretized using an implicit method of finite differences considering meshes of 150x250 knots. The conservation equations and the discretized mass transfer equation take the form of

$$A_j X_{j-1}^{i+1}+B_j X_j^{i+1}+C_j X_{j+1}^{i+1}=D_j \text{ for } 2\le j\le(J\max-1) \tag{14}$$

where X represents the quantities $T^+, C^+, V_x^+ \text{ et } V_\varphi^+$ and Jmax characterizes the thickness of the boundary layer. Systems of algebraic equations (14) associated with discretized boundary conditions are solved by Thomas's algorithm. The normal speed component V_y^+ is derived from the continuity equation:

$$V_y^+\binom{i+1}{j+1}=V_y^+\binom{i+1}{j}-\Delta y^+\sqrt{Re_\omega}\left(\frac{V_x^+\binom{i+1}{j}-V_x^+\binom{i}{j}}{\Delta x^+}+\frac{V_x^+\binom{i+1}{j}}{\Delta x^+}\left(1-\frac{r^+(i)}{r^+(i+1)}\right)\right) \tag{15}$$

The convergence of the iterative process within the boundary layer is assumed to be achieved when the following criterion is simultaneously verified on

$$T^+, C^+, V_x^+ \text{ et } V_\varphi^+ \text{ is: } \left|\frac{\max\left(X^{n+1}-X^n\right)}{\max\left(X^n\right)}\right|\le 10^{-3} \tag{16}$$

with n is the iteration number. Partial derivatives of the expressions of Nusselt, Sherwood and friction coefficients are approached by three-point discretization.

$$\frac{\partial X}{\partial y^+}=\frac{3X\binom{i+1}{j+1}-4X\binom{i+1}{j}+X\binom{i+1}{j-1}}{2\Delta y^+} \tag{17}$$

IV. Results and Discussion

Numerical results are obtained for values of: Pr = 0.71, Sc = 0.65, Ec = 0.0025, Δy^+ = 0.1 and

θ_0 = 20. Our calculation code was validated on a natural and rotary mixed convection problem around a vertical cone. We compared our results with those obtained by K. Himasekhar et al. [16]. The comparison is about the gradient of temperature for the Richardson number Ri = 0 and angular velocity ω = 50π rd/s. The analysis in Table I shows that our results are in perfect agreement with the results available in the literature, the relative difference not exceeding 2%.

TABLE I. EVOLUTION OF THE PARIETAL TEMPERATURE GRADIENT

	$-\left(\dfrac{\partial T^+}{\partial y^+}\right)_{y^+=0}$	Relative gap
Present model	0.4249	
Himasekhar et al. [16]	0.4299	0.0116
Hering et al. [17]	0.4285	0.0084

Fig. 2 shows the variation of the dimensionless meridian velocity as a function of y^+ for several values of ω. One notices that the maximum value of V_x^+ increases as a function of ω. Moreover, the thickness of the dynamic boundary layer decreases as a function of ω. Otherwise; it confirms the physical evidence that the meridian component arises following the rotation of the body: it increases even more as ω increases. In the limiting case of zero rotational speed, the meridian component is zero: there is no upward motion of the fluid in the case of pure rotation.

The rotation of the cone also causes the creation of the normal component whose variation is represented in *Fig. 3*. It shows that the normal component decreases in the boundary layer with ω and takes the negative values, which means that the fluid particles are sucked by the axis of rotation, that is to say towards the wall. This suction is all the greater as the speed of rotation is high, and closer to the free fluid.

By Couette effect, the fluid particles in the immediate vicinity of the wall are driven by the movement of the cone, which is at the origin of the azimuthal component. *Fig. 4* shows the variation of the dimensionless azimuthal velocity as a function of y^+ for several values of ω. It is higher near the wall, decreases along the normal, and tends towards zero at infinity. One see that there is a privileged point along the normal for which V_φ^+ does not depend on ω, on either side of which is increasing and decreasing as a function of y^+.

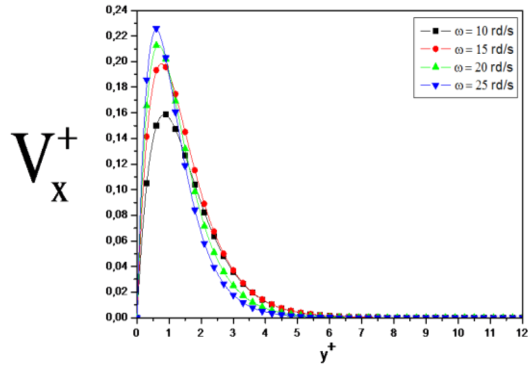

Fig. 2. *Reduced V_x^+ according to y^+ for several values of ω.*

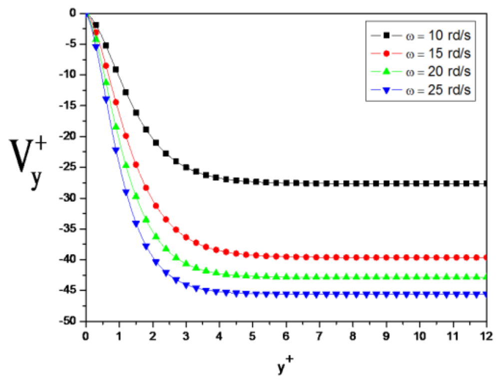

Fig. 3. *Reduced V_y^+ according to y^+ for several values of ω.*

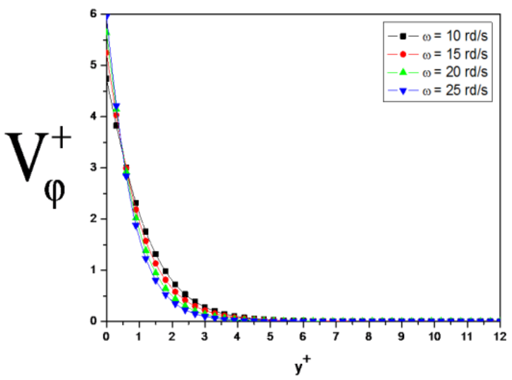

Fig. 4. *Reduced V_φ^+ according to y^+ for several values of ω.*

Fig. 5 represents the changes in the temperature of the fluid along the normal as a function of y^+ for the different values of studied ω. It shows that the particles of the fluids near the wall have retained the heat. Reading these values shows that the temperature of the fluid and the thickness of the thermal boundary layer decrease when the rotational Reynolds number increases. The increase in ω decreases the heat exchange between the cone wall and the fluid. It is observed that the thermal gradient remains closely localized near the wall. Quantitatively, this results in the existence of much larger temperature gradients in the *y* direction.

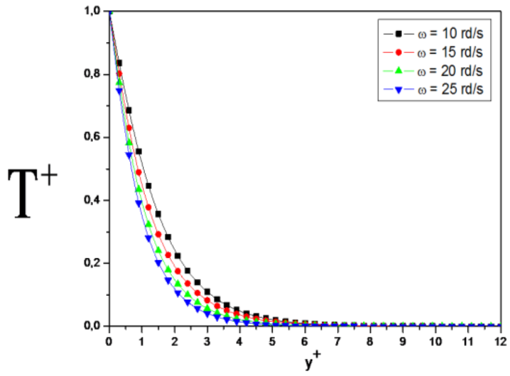

Fig. 5. *Reduced T^+ according to y^+ for several values of ω.*

Fig. 6 shows the evolution of the dimensionless mass concentration as a function of y^+ for several values of ω. The difference in concentration at the wall and at infinity creates a mass transfer. Mass is transferred from more concentrated regions to less concentrated regions. When the cone rotates, the fluid particles concentrate around the wall of the cone. It shows that C^+ decreases with the increase of ω. Increasing the rotational speed of the cone decreases the mass transfer.

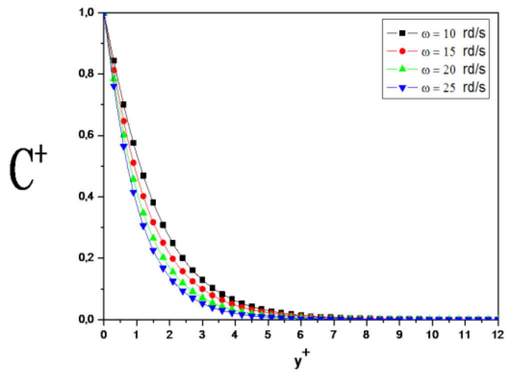

Fig. 6. *Reduced C^+ according to y^+ for several values of* ω

One represents, in *Fig. 7*, the variation of the coefficient of friction Cf_u for several values of ω as a function of x^+. One observes that the Cf_u increases along the wall of the cone and increases with ω. The growth of Cf_u when x^+ increases announces that the adherence of the fluid evolves along the wall of the cone. Increasing the rotational Reynolds number makes the fluid adhere better, and has the effect of increasing the coefficient of friction Cf_u. It also shows the dependence of friction and the abscissa x^+, in other words the further one moves downstream from the vertex O, the more the friction effect is damped.

Fig. 8 makes it possible to study the variation of the coefficient of friction Cf_w for several values of ω as a function of x^+. One observe that the Cf_w decreases along the wall of the cone, is a decreasing function of ω. The decrease of Cf_w when x^+ increases announces a separation of the boundary layer. It is seen that the separation of the fluid increases along the wall of the cone. The increase in the dimensionless rotation speed ω promotes the separation of the azimuthal dynamic boundary layer and increases the adhesion of the fluid to the wall.

Figs. 9 and *10* show the variation of Nu and Sh as a function of x^+. Near the stopping point, the heat and mass transfer rates increase abruptly and stabilize when the thermal equilibrium between the wall and the fluid particles is reached. They show that the intensity of heat and mass transfers between the wall and the fluid increase with longitude due to the progressively increasing difference in the wall temperature gradient. These dimensionless Nusselt and Sherwood numbers vary in the same way, and grow considerably along the wall but become stable from the point of fluid separation. One notices that the Sherwood and the Nusselt are more important when the cone rotates at high speed. The increase in ω promotes the transfer of mass and heat by mass rotary convection.

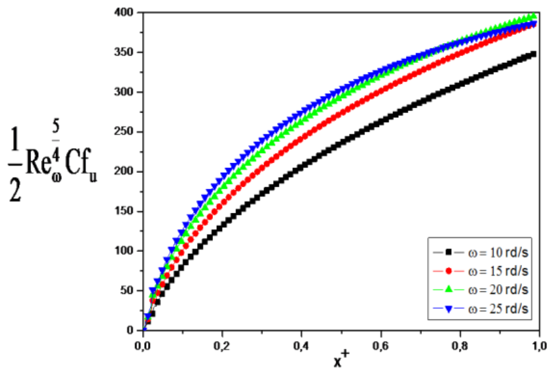

Fig. 7. *Coefficient of friction according to x as a function of x^+ for several values of ω*

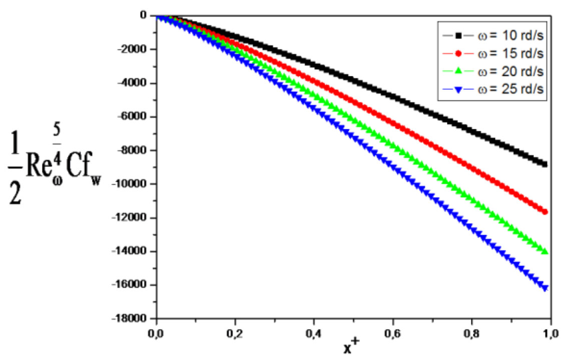

Fig. 8. *Coefficient of friction according to φ as function of x^+ for several values of ω*

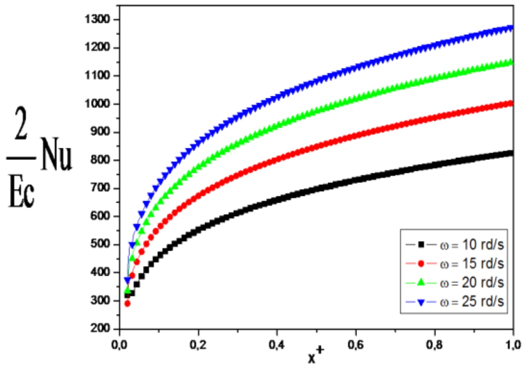

Fig. 9. *Evolution of the Nusselt number as a function of x^+ for several values of ω.*

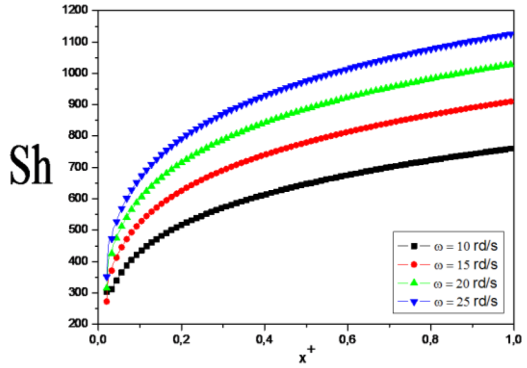

Fig .10. *Evolution of the Sherwood number as a function of x^+ for several values of ω.*

V. Conlusion

The effect of rotational Reynolds number on heat and mass transfer around a cone of revolution has been modeling. Authors developed the mathematical model based on the heat, mass transfer, Navier-Stokes and continuity equations coupled implicit finite difference scheme technique. THOMAS algorithm has been used to solve the algebraic equations system associated with boundary conditions. A numerical code has been developed. The results show that, by Couette effect, the rotation of the cone around its axis of revolution creates a meridian velocity component which causes an upward flow inside the dynamic boundary layer. It also creates a fluid flow in which a boundary layer of concentration develops. The fluid particles are exposed to suction. The thickness of the thermal boundary layer is smaller than that of the mass boundary layer. Heat and mass transfer rates increase along the wall. At the stopping point, there are small Nu and Sh. In the higher meridians, near the point of detachment, the Nu and Sh are considerable. The transfer rates evolve in the same direction, as do the temperature and the mass concentration. The heat transfer rate is greater than the mass transfer rate. Heat transfer is more dominant than mass transfer. The rotational speed of the cone has a positive influence on heat and mass transfer.

Acknowledgment

Author would like to thank the team of the Laboratory of Mechanics Energetic and Environment for their collaborations and particularly Pr. Dr. Marcelin Hajamalala ANDRIANANTENAINA for agreeing as my Doctoral Thesis/PhD supervisor and corresponding author.

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