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Orthogonal Polynomial Technique For Achieving Optimum Stock Allocation In N-Deport Firms

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***Abstract*—Goods need to be kept in stock to meet demand timely. Stock control is a functional part of every manufacturing/distribution organization as goods need to move from the manufacturers to the consumers. The ability to keep goods in stock has its associated cost likewise the act of not keeping stock. The need to manage stock allocation makes for control so as to optimize returns. This research develops a polynomial approximation technique for optimizing stock allocation for multi-depot firms. It used polynomial as the basis for approximation based on the "Weiestrass theorem on polynomial approximation" to design an allocation technique that maximizes returns and minimizes costs. The number of warehouses is used on the polynomial to obtain a new allocation. The new allocation is weighted with the observed allocation by a normalizer to get the approximate (expected) allocation. The final allocations are then got by applying the stock approximation algorithm developed. Theorems showing that the cost associ- ated with the expected allocation is always lower than that of the observed and the algorithm is idempotent are proved. The technique is applied to a firm with eight depots and the outcome shows minimal cost and better returns. From the illustration shows the technique is suitable for multi-warehouse organizations. The results also show a better associated minimum costs and better returns. The technique yields an optimum allocation.**

Keywords—Optimization, Orthogonal polynomial, Approximation, Allocation.

1.1 The Background of the study

Stock can be defined to be goods being held for future use or sale. The ability to keep goods in a warehouse to make it available for sale or future use is called stocking. The need to manage stock requires control and the essence of stocking is to meet up with demand. Control is a process by which events are made to conform to a plan. Demand is dynamic and hence the pertinence to keep goods in stock. The act of maintaining stock has its associated costs likewise the act of not keeping stock. The later makes the manufacturer/distributor lose customers' goodwill thereby incurring shortage cost while the former may increase holding costs. It is therefore often necessary to stock physical goods in order to satisfy demand over a specified time period in a way that the firm minimizes cost and maximizes profit. The act of stocking goods to satisfy future demand gives rise to the problem of designing a very efficient allocation technique so as to minimize cost and maximize profit. Stock control is an important functional part of every manufacturing organization and needs proper attention

and management. This research work is aimed at looking into stock allocation system in a man- ufacturing company with widespread distribution outlets and developing a mathematical model using polynomial approximation. This is to ensure optimal allocation for the organization.

In this work, the polynomial approximation technique is used to obtain a new (approximate) allocation that optimizes stock allocation. The Weierstrass theorem states that for a contin- uous function f on an interval [a, b] , there exists a polynomial p(x) such the

$\text{Sup } |f(x)-p(x)| < \epsilon$ (Raul 2014). Jeffreys and Jeffreys (1998) added that any continuous

$x \in C[a,b]$

function on closed bounded interval can be uniformly approximated on the interval by polynomials to any degree of accuracy. The work would therefore consider some orthogonal polynomials in approximating stock allocation. The Chebyshev, Legendre, Laguerre and Hermite polynomials are applied to obtain approximate allocations with respect to the

Weierstrass theorem on polynomial approximations.

1.2 Review of Related Literature

Goods must be moved from their area of manufacture to the points of consumption. These goods would be stored in warehouses near the consumers. These products are therefore stocked in warehouses to meet the timely demand of customers hence the need for stock management. Demand in most cases is dynamic therefore policies must be put in place to ensure that the process of distribution is effective and costs less. Since 1957 when Bellman (1957) developed a functional equation with a necessary condition for optimality associated with the mathematical optimization method, the concept has continually drawn widespread attention. Burbridge (1998) defined stock allocation as the operation of continuously arranging receipts and issues to ensure that stock balances are adequate to support the current rate of consumption with due regard to economy. The researchers' area cut across several disciplines such as management, economics, engineering and sciences. This points to the importance and wide applicability of this concept. Kang and Gershwin (2005) pointed out that there is a conflict between the need to give a good service and the need to economize in stock holding. The more the stock held the easier it is to have required items readily available on demand. On the other hand the more the stocks held the greater the holding cost. It is therefore necessary to seek, find and operate a satisfactory compromise between these two opposing forces. Many of the work so far done focused on one or two warehouse stock allocation with the Economic Order Quantity (EOQ). Some of the works are; Optimal inventory model for items with imperfect quality Cardenas-Barron (2000), Eco- nomic Ordering Quantity model for items with imperfect quality, single period inventory model with two level of storage, Two warehouse inventory model with imperfect quality (Tien-Yu-Lin

2011), A two warehouse inventory model for deteriorating items with stock dependent rate and holding cost (Yang 2006), Deterministic inventory model for determining items with capacity constraint and backlogging rate (Erhan 2007), a generalized economic order quantity model with deteriorating items and time varying demand (Balkhi and Tudj 2008), Dynamic programming in minimum risk paths of stock allocation (Milovanic and Mastrion 2008) to mention but a few. Stock allocation is the act of distributing goods in stock to designated depots according to their needs (Chikwendu and Emenonye 2017). Stock allocation is an important part of any manufac- turing organization including wholesalers and retailers. (Chikwendu and Emenonye 2015) also asserted that maintaining inventory is very necessary for any company dealing with physical goods including manufacturers. Lucey (1988) stated the logical reasons for holding stock as; ensuring that sufficient goods are available to meet anticipated demand, absorb variations in demand and production, provide a buffer between production processes, absorb seasonal fluc- tuations in usage or demand, enable production processes flow smoothly and efficiently and as deliberate investment policy particularly in times of inflation or possible shortage.

In recent times according to Erhan (2007), customer service has become an important dimension

of competition along with price and quality. In order to retain a company's current customers and to acquire new customers, prompt service is always considered for which the first require- ment is to have service parts readily available. He concludes that the company therefore faces a problem of determining optimal stocking level of goods. Many of the work done on inventory and stock allocation have been on models for a single warehouse and that of two warehouses and these have been discussed under different conditions such as "with deteriorating items, with in- flationary effects, with/without allowing shortages, with price discount, with quantity discount" etc. This has given rise to a myriad of formulae.

1.3 Polynomial/ Approximation

Hirschfelder and Hirschfelder (1991) defined polynomial as a function of the form $f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_2 x^2 + a_1 x + a_0$ where n is a non-negative integer and the a_is the coefficients. Polynomials are expressions involving various degrees of variables x which may be sum together. They also posited that polynomial functions are particularly useful in approximating other func- tions. According to Jaggi (2006),approximation could be defined as the act of estimating a num- ber or an amount. Approximation arises due to the difficulty in obtaining the exact area/volume or dimension of some objects. In addition, the inability of man to foretell the future accurately as a result of his fallibility gives birth to estimation of quantities. Finding a good polynomial approximation to a given function $f(x)$ entails representing $f(x)$ in the form

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_2 x^2 + a_1 x + a_0 \quad (1)$$

Amparo,Javier and Nico (2008) stated the general approximation problem as follows;
If f is an element and S a subset of a normed linear space X ,then the approximation theory seeks to find an element $s \in S$ which is as close to f as possible; ie to find an element s^* of S such that

$$\|f - s^*\| \leq \|f - s\| \leq \epsilon \quad \forall s^* \in S. \quad (2)$$

According to Raul (2014),the Weierstrass theorem states that if f is a continuous function on an interval $[a, b]$ and is given, there exists a polynomial $p(x)$ such that

$$\mathrm{Sup}_{x \in C[a,b]} | f(x) - p(x) | \leq \epsilon \quad (3)$$

The Weierstress theorem therefore implies that "any continuous real-valued function f defined on a bounded interval $[a, b]$ of the real line can be approximated to any degree of accuracy using a polynomial .i.e. for any $\epsilon > 0$, there is a polynomial p such that

$$\|f - p\|^{\infty} \equiv \sup_{a \leq x \leq b} |f(x) - p(x)| < \epsilon$$

i.e. If $C[a, b]$ is the set of all continuous functions on $[a, b]$, then for all $f \in C[a, b]$ and $\epsilon > 0$, there exists a polynomial p for which

$$\sup_{a \leq x \leq b} | f(x) - p(x) | \leq \epsilon \quad (4)$$

Furthermore, if $f \in C^k[a, b]$, is a set of continuous functions of $[a, b]$ then there exists a sequence of polynomials, p_n, where the degree of p_n is such that

$$\lim_{n \to \infty} \max_{x \in C[a,b]} |f^k(x) - p^l(x)| = 0 \qquad l < k \quad (5)$$

In order words, there exists a polynomial that approximates any continuous function over a compact domain arbitrarily well. Pressman and Sethi (2004) therefore concluded that it is possible to use polynomial approximation to obtain an optimal stock allocation if the allocation is shown as a continuous function. The polynomial approximation is therefore stated as follows;
If P_n is the collection of all polynomials whose degree is at most n and f be a continuous function on the interval $[a, b]$ the polynomial P is a better approximation to f than q iff $\|f - p\| < \|f - q\|$. i.e. the polynomial P yields a smaller maximum error over $[a, b]$ than q. (Jenson and Roland 1975)

1.4 Chebyshev polynomial

Chebyshev's polynomial is a sequence of orthogonal polynomials which are defined recursively as

$$T_{n+1}(x) = 2xT_n(x) - T_{n-1}(x) \quad n = 1, 2, \ldots$$

with difference equation

$$(1 - x^2)\, y^{\prime\prime} - xy^{\prime} + n^2 y = 0, \tag{6}$$

. Theodore (1975) defined the Chebyshev polynomials by a three term recursion;
$T_0(x) = 1,\ T_1(x) = x$, and $T_{n+1}(x) = 2xT_n(x) - T_{n-1}(x); \quad n = 1, 2, \ldots$
According to Maihaila and Mihaila (2002),the Chebyshev polynomial is plausible for polynomial approximation because; it is recursive in nature, symmetric , orthogonal, has n-zeros in $[-1, 1]$ and has the minimax property.

1.5 Legendre polynomial

The Legendre polynomial $p_n(x)$ is an nth degree orthogonal polynomial. The Legendre equation is a second order differential equation. The differential equation is given by

$$((1 - x^2)y^{\prime\prime} + n(n + 1)y^{\prime} = 0 \tag{7}$$

The Legendre polynomial has its roots from the Legendre equation

$$((1 - x^2)y^{\prime\prime} + n(n + 1)y^{\prime} = 0.$$

Koornwinder, Wong and Rene (2010),stated that the Legender polynomial has recursive for- mula

$$P_{n+1}(x) = \frac{(2n + 1 - x)p_n(x) - np_{n-1}(x)}{n + 1} \tag{8}$$

1.6 Laguerre

According to Koornwinder,Wong and Rene (2010), the Laguerre polynomial is an orthogo- nal polynomial which is obtained by using the recurrence relation for any $n \geq 1$

$$P_{n-1}(x) = \frac{(2n+1-x)P_n(x) - nP_{n-1}(x)}{n+1}$$

The generalised Laguerre polynomial are orthogonal over $[0, \infty)$,recursive with leading term $(-1)n^n$

It is recursive, orthogonal and symmetric in nature.

1.7 Hermite polynomial

The physicist's Hermite polynomial H_n has leading coefficient 2^n. .According to Suetin (2001),the nth order Hermite polynomial is a polynomial of degree n. The Hermite polynomial satisfies the recurrence relation

$$H_{n+1}(x) = 2xH_n(x) - 2nH_{n-1}(x) \tag{9}$$

with interval of stability $[-\infty, \infty]$ and difference equation $y'' - 2xy' + 2ny = 0$

2 Theorems

The theorems that assert the possibility of approximating functions using polynomials, the exis- tence of a best approximating polymial and the uniqueness of the best approximating polynomial are as follows.

Theorem 0.1 If $x_0, x_1, \ldots, x_n$ are distinct numbers (real or immaginary), then the interpola- tion polynomial

$$p(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_0 \quad \in P_n \tag{10}$$

has a unique solution. (Anderson 1974)

Theorem 0.2 The nth degree polynomial in a family of orthogonal polynomials associated with Weight function ω on the interval $[a, b]$ has n simple zeros, all of which lie in the interior of $[a, b]$ (Anderson 1974)

Theorem 0.3 For every continuous function f defined on a closed bounded interval, it holds that $\text{Lim}_{n\to\infty} E_n(f) = 0$ (Jeffreys and Jeffreys (1998))

3 Methodology

3.1 Model Development

In this work, an approximating technique is developed for stock allocation of a multi-warehouse firm. The allocation to the warehouses is modeled using polynomial approximating method. The polynomial approximation is applied on the observed allocations to produce a technique that optimizes stock allocation.

3.2 Algorithm

In what follows, we develop an algorithm for the polynomial technique for optimal stock allocation. We consider a multi-warehouse scenario.
Let $i = 1, 2, \ldots, m$ be the products manufactured or to be stocked by the firm. Let $j = 1, 2, \ldots, n$ be the warehouses used by the firm.
Let $x_{i,j}$ be the (observed) quantity of products i being held as stock in warehouse j. Let $y_{i,j}$ be the (expected) optimal quantity of product i held as stock in warehouse j. Let $C_{i,j}$ be the cost of holding one unit of product i as stock in warehouse j.
Let $\gamma_{i,j}$ be the return associated with one unit of product i held as stock in warehouse j that gets supplied out or sold.
Let $P_{n-1}(x)$ be the polynomial of degree $n - 1$ used to approximate the stock allocation for Product i
Let $P_{n-1,i}(j)$ be the polynomial of degree $n - 1$ used to approximate the stock allocation for warehouse j
Define $N_i = \sum_{j=1}^{n} x_{i,j}$, as the total stock of product i (i=1,2,. . .,m)
Define $D_i = \sum_{j=1}^{n} |P_{-1,i}(j)|$ (i=1,2,. . .,m); i.e D is the total allocation with respect to warehouse j
Define $q_{n-1,i}(x) = \frac{N_i}{D_i} |P_{n-1,i}(j)|$, the normalizing polynomial
The algorithm for obtaining the expected allocation $y_{i,j}$ is as follows:
$\forall\, i, (i = 1, 2, \ldots, m)$
$y_{i,j} = \min(q_{n-1,i}(j), x_{i,j})$ $\quad j = 1, 2, \ldots, n - 1$

$$y_{i,n} = N_i - \sum_{j=i}^{n-1} y_{i,j} \tag{11}$$

4 The Polynomial technique Theorems

Using the notations above, we show that the cost associated with the observed allocation is always higher than that incured by the approximate allocation and that the approximate allocation is optimal. These assertions are expressed in the following theorems.

4.1 Theorem (theorem 1)

$f_{i,o} \geq f_{i,e}$

Proof

Assume without loss of generality that $\forall$ i the warehouses have been arranged in such a way that $C_{i,j} \geq C_{i,j+1}$,
(j=1, . . ,m)

Let $p_{n-1,i}$, $q_{n-1,i}$, N_i D_i be as defined in the model.

Then

$$f_{i,o} - f_{i,e} = \sum_{j=1}^{n} C_{i,j} x_{i,j} - \sum_{j=1}^{n} C_{i,j} y_{i,j} \tag{12}$$

$$= \sum_{j=1}^{n} C_{i,j} x_{i,j} - y_{i,j})$$

$$\sum_{j=1}^{n-1} C_{i,j}(x_{i,j} - y_{i,j}) + C_{i,n}(x_{i,n} - y_{i,n})$$

$$= \sum_{j=1}^{n-1} C_{i,j}(x_{i,j} - \min\{q_{n-1,i}(j), x_{i,j}\}) + C_{i,n}(x_{i,n} - N_i + \sum_{j=1}^{n-1} y_{i,j})$$

$$\geq C \quad \sum_{j=1} (x_{i,j} - \min\{q_{n-1,i}(j), x_{i,j}\}) + C_{i,n}(\sum_{j=1}^{n-1} y_{i,j} - \sum_{j=1}^{n-1} x_{i,j})$$

$$> C_{i,n}(\sum_{j=1}^{n-1} x_{i,j} \sum_{j=1}^{n-1} x_{i,j} - \min\{q_{n-1,i}(j), x_{i,j}\} + C_{i,j}(\sum_{j=1}^{n-1} x_{i,j} - \min\{q_{n-1,i}(j), x_{i,j}\} - \sum_{j=1}^{n-1} x_{i,j}) = 0 \tag{13}$$

$= C_{i,n,} 0 = 0$

Hence $f_{i.o} \geq f_{i,e}$

4.2 Theorem (theorem 2) The polynomial technique is idempotent for the same polynomial.

proof

Using the family of polynomials, the first application yields $\{p_{n-1,i}\}_{i=1}^{m}$.

Recall that $N_i = \sum_{j=1}^{n} x_{i,j}$, $D_i = \sum^{n} |P_{n-1,i}(j)|$, $q_{n-1,i}(j) = \frac{N_i}{D}|P_{n-1,i}(j)|$,

Using the family of polynomials $\{P_{n-1,i}\}^{m}{}_{i=1}$, then the first application

$\forall$ i

$$\min\{\min\{x_{i,j}, q_{n-1,i}(j)\} \quad j = 1, 2, \ldots, n-1 \tag{14}$$

$$y_{i,n} = N_i - \sum y_{i,j}$$

Second application using the same family of polynomials $\{p_{n-1,i}\}_{i=1}^{m}$ yields

$N_{N_i^{(2)}}^{(2)} = \sum_{j=1}^{n} y_{i,j}$ (since $x_{i,j} = y_{i,j}$ now) $= N_i$.

$D_{D_i^{(2)}}^{(2)} = \sum_{j=1}^{m} |P_{n-1,i}(j)| = D_i$,

$q_{n-1}{}^{(2)} = \frac{N_i^{(2)}}{D_i^{(2)}}|p_{n-1,i}(j)| = \frac{N_i}{D_i}|p_{n-1,i}(j)| = q_{n-1,i}(j)$

so that

$$y_{i,j}^{(2)} = \min\{y_{i,j}, \; q_{n-1,i}(j)\}, \quad j = 1, 2, \ldots, n-1 \tag{15}$$

$$= \min\{\min\{x_{i,j}, \; q_{n-1,i}(j)\}, \; q_{n-1,i}(j)\} \quad j = 1, \ldots, n-1 \tag{16}$$

$$= \min\{x_{i,j}, \; q_{n-1,i}(j)\} = y_{i,j}; \quad j = i, \ldots, n-1 \tag{17}$$

also,

$$y_{i,n}^{(2)} = N_i - \sum_{j=1}^{n-1} y_{i,j}^{(2)} = N_i - \sum_{j=1}^{n-1} y_{i,j} = y_{i,n} \tag{18}$$

hence for any particular polynomial p_{n-1}, $PT(p)$
and so the PAT is idempotent.
⇒ each of PT_1, PT_2, PT_3, PT_4, PT_5,
and PT_6 is idempotent.

5 Remark

1. The cost associated with the approximate allocation is always lower than that of the observed.
2. The returns accruing to the expected allocation is better(higher) than that of the observed.
3. Theorem 2 shows that the technique is idempotent.
4. Theorem 3 shows that the technique is commutative for any two finite family of polynomials of degree $(P_{(n-1)})$

5 Illustration

The allocation to eight depots of a firm given below. The firm encures a cost of 0.01 for every 50 kilometers of operation and makes a return of 0.05 per unit supplied. The distances to warehouses are 198km,8km,67km,
98km,133km and 136km to warehouses $1, 2, \ldots, 6$ respectively.
The firms allocation with their associated costs are as follows:

	A	B	C	D	E	F	G	H
Stock(x_{ij})(000)	8	128	648	2048	5000	10368	19208	37790
Cost(c_{ij})	0.01	0.01	0.03	0.02	0.04	0.01	0.02	0.03

Fig 1 Table of allocations

5.1 Solution using Chebyshev polynomial

Re-arrange in descending order of cost to get;

	A	B	C	D	E	F	G	H	T
Stock(x_{ij})(000)	5000	37790	8	128	2048	19208	648	10368	65198
Cost(c_{ij})	0.01	0.01	0.03	0.02	0.04	0.01	0.02	0.03	-

Fig 2 Re-arranged table of allocations -Chebyshev

Here $n = 8$ hence P_{n-1} is given by

$$P_7 = 64x^7 - 112x^5 + 56x^3 - 7x$$

Therefore $p_7(1) = 1$, $p_7(2) = 5042$, $p_7(3) = 114243$, $p_7(4) = 937444$, $p_7(5) = 4656965$, $p_7(6) = 17057046$, $p_7(7) = 50843527$, $p_7(8) = 130576328$ and the total is $D = 204190596$.
The normalizer $q(x)$ is;

$$q_7(x) = \frac{65198}{204189246}|(64x^7 - 112x^5 + 56x^3 - 7x)| \qquad (19)$$

Hence $q_7(1) = 0$, $q_7(2) = 2$, $q_7(3) = 36$, $q_7(4) = 299$, $q_7(5) = 1487$, $q_7(6) = 5446$,

$q_7(7) = 16234$, $q_7(8) = 41693$
Total $= 65198$.
Therefore the allocation table for the observed and expected allocation is:

	A	B	C	D	E	F	G	H	T
Stock(x_{ij})(000)	5000	37790	8	128	2048	19208	648	10368	65198
Stock($y_{i,j}$)(000)	0	2	36	299	1487	5446	16235	41693	65198
Cost(c_{ij})	0.01	0.01	0.03	0.02	0.04	0.01	0.02	0.03	-

Fig 3 Table of allocations for both observed and expected
$\Rightarrow \quad f_o = 1871.70$
Similarly $\gamma_o = 3259.90$ and $\alpha_0 = \gamma_o - f_o = 1388.20$
Now applying the algorithm;

$$y_{i,j} = \min(q_{n-1,i}(j), x_{i,j}) \quad j = i, 2, \ldots, n-1$$

$$y_{i,n} = N_i - \sum_{j=1}^{n-1} y_{i,j}$$

to the allocation above we obtain the following;

	A	B	C	D	E	F	G	H	T
Stock($y_{i,j}$)(000)	0	2	8	128	1487	5446	648	57479	65198
Cost(c_{ij})	0.04	0.03	0.02	0.02	0.02	0.02	0.01	0.01	-

Fig 4.Table of expected allocations-Chebyshev

$f_e = 722.71$
$\gamma_o = 3259.90$ $\gamma_e = 3259.90$
$\alpha_e = 2537.19$
Hence a better result is achieved with the expected allocation.
Clearly $\gamma_e > \gamma_o$ i.e. the net returns of the approximate(expected) allocation is higher than that of the observed allocation.

5.2 Solution using Legendre polynomial

	A	B	C	D	E	F	G	H	T
Stock(x_{ij})(000)	5000	37790	8	128	2048	19208	648	10368	65198
Cost(c_{ij})	0.01	0.01	0.03	0.02	0.04	0.01	0.02	0.03	-

Fig 5. Re-arranged table of allocations-Legendre

$$\frac{429x^7 - 693x^5 + 315x^3 - 35x}{16} \tag{20}$$

Here $n = 8$ hence P_{n-1} is given by

$$P_7(x)$$

$p_7(1) = 1$,, $p_7(2) = 563$, $p_7(3) = 48836$, $p_7(4) = 396195$, $p_7(5) = 1961826$, $p_7(6) = 7173225$, $p_7(7) = 2998686$, $p_7(8) = 54820687$,, Total $= 67,400,019$ The normalizer $q(x)$ is;

$$q_7(x) = \frac{65198}{67400019} \left| \left(\frac{429x^7 - 693x^5 + 315x^3 - 35x}{16} \right) \right|$$

$q_7(1) = 0$. $q_7(2) = 1$. $q_7(3) = 19$. $q_7(4) = 384$. $q_7(5) = 1898$. $q_7(6) = 6939$.

$q_7(7) = 2901$. $q_7(8) = 53030$.
Total $= 65198$
Hence the table

	A	B	C	D	E	F	G	H	T
Stock(x_{ij})(000)	5000	37790	8	128	2048	19208	648	10368	65198
Stock($y_{i,j}$)(000)	0	1	47	383	1898	6939	2900	53030	65198
Cost(c_{ij})	0.04	0.03	0.02	0.02	0.02	0.02	0.01	0.01	-

Fig 6. Table of observed/expected allocations-Legendre

$\Rightarrow\ f_o = 1871.70$
Similarly $\gamma_o = 3259.90$ and $\alpha_0 = \gamma_o - f_o = 1,388.2$
Now applying the condition

$$y_{i,j} = \min(q_{n-1,i}(j), x_{i,j}) \quad j = i, 2, \ldots, n-1$$

$$y_{i,n} = N_i - \sum_{j=1}^{n-1} y_{i,j}$$

	A	B	C	D	E	F	G	H	T
Stock($y_{i,j}$)(000)	0	1	8	128	1898	6939	648	55576	65198
Cost(c_{ij})	0.04	0.03	0.02	0.02	0.02	0.02	0.01	0.01	-

Fig 7. Table of expected allocations-Legendre

to the allocation above to obtain the following;
$f_o = 1871.70$ $f_e = 602.95$
$\gamma_e = 3259.90$
$\Rightarrow\ \alpha_e - f_e = 2656.95$
$\alpha_e > \alpha_0$
Hence a better allocation is achieved with the expected allocation. Clearly $\alpha_e > \alpha_o$ i.e. the actual returns of the approximate(expected) allocation is higher than that of the observed allocation.

5.3 Solution using Laguerre polynomial

	A	B	C	D	E	F	G	H	T
Stock(x_{ij})(000)	5000	37790	8	128	2048	19208	648	10368	65198
Cost(c_{ij})	0.01	0.01	0.03	0.02	0.04	0.01	0.02	0.03	-

Fig 8. Re-arranged table of allocations-Laguerre

Here $n = 8$ hence P_{n-1} is given by

$$P_7(x) = \frac{-x^7 + 49x^6 - 918x^5 + 180x^4 - 36600x^3 + 74520x^2 - 56880x + 5040}{720} \tag{21}$$

$\Rightarrow$ $p_7(1) = -14610$,, $p_7(2) = -93296$, $p_7(3) = -658080$, $p_7(4) = -2082192$, $p_7(5) = -5060110$, $p_7(6) = 10458000$,
$p_7(7) = -19350828$, $p_7(8) = -33015760$
Total $= 70732876$
The normalizer q(x) is;

$$q_7(x) = \frac{N}{D}\frac{(-x^7+49x^6-918x^5+180x^4-36600x^3+74520x^2-56880x+5040)}{5040} \qquad (22)$$

$P_7(1) = 14$. $q_7(2) = 86$. $q_7(3) = 67$. $q_7(4) = 1919$. $q_7(5) = 4664$. $q_7(6) = 9640$.

$q_7(7) = 17837$. $q_7(8) = 30432$.
Total $= 65198$
Thus the table of allocation;

	A	B	C	D	E	F	G	H	T
Stock(x_{ij})(000)	5000	37790	8	128	2048	19208	648	10368	65198
9. Table of	0	2	36	299	1487	5446	16235	41693	65197
observed/expect	0.01	0.01	0.03	0.02	0.04	0.01	0.02	0.03	-

Fig 9. Table of observed/expected allocations-Laguerre

$\Rightarrow$ $f_o = 1871.70$
Similarly $\gamma_o = 3259.90$ and $\alpha_0 = \gamma_o - f_o = 1,388.2$
Now applying the algorithm

$$y_{i,j} = \min(q_{n-1,i}(j), x_{i,j}) \quad j = i, 2, \ldots, n-1$$

$$y_{i,n} = N_i - \sum_{j=1}^{n-1} y_{i,j}$$

to the allocation above to obtain the following;

	A	B	C	D	E	F	G	H	T
Stock($y_{i,j}$)(000)	0	2	8	128	1487	5446	648	57479	22940
Cost(c_{ij})	0.04	0.03	0.02	0.02	0.02	0.02	0.01	0.01	-

10. Table of expected allocations-Laguerre

$f_o = 1871.70$ $f_e = 722.71$
$\gamma_o = 3259.90$ $\gamma_e = 3259.90$
$\alpha_e = 2537.19$
Hence a better allocation is achieved with the expected allocation.

Clearly $\alpha_e > \alpha_o$ i.e. the net returns of the approximate(expected) allocation is higher than that of the observed allocation.

5.5 Solution using Hermite polynomial

	A	B	C	D	E	F	G	H	T
Stock(x_{ij})(000)	5000	37790	8	128	2048	19208	648	10368	65198
Cost(c_{ij})	0.01	0.01	0.03	0.02	0.04	0.01	0.02	0.03	-

Fig 11. Re-arranged table of allocations-Hermite

Here $n = 8$ hence the corresponding polynomial P_{n-1} is given by

$$P_7(x) = 128x^7 - 1344x^5 + 33600x^3 - 1680x^2 - 56880x \tag{23}$$

$\Rightarrow$ $p_7(1) = 464$,, $p_7(2) = 13280$, $p_7(3) = 39024$, $p_7(4) = 929216$,
$p_7(5) = 6211600$, $p_7(6) = 26096544$, $p_7(7) = 83965616$, $p_7(8) = 226102144$
Total $= 343357888$
The normalizer $q(x)$ is;

$$q_7(x) = \frac{1}{-}\left|(128x^7 - 1344x^5 + 33600x^3 - 1680x^2 - 56880x)\right| \tag{24}$$

$q_7(1) = 0$. $q_7(2) = 3$. $q_7(3) = 7$. $q_7(4) = 176$. $q_7(5) = 1180$.
$q_7(6) = 4955$, $q_7(7) = 15944$. $q_7(8) = 42933$.
Total $= 65198$
Hence the table;

	A	B	C	D	E	F	G	H	T
Stock(x_{ij})(000)	5000	37790	8	128	2048	19208	648	10368	65198
Stock($y_{i,j}$)(000)	0	3	7	176	1180	4955	15944	42933	65198
Cost(c_{ij})	0.01	0.01	0.03	0.02	0.04	0.01	0.02	0.03	-

Fig 12. Table of allocations of observed/expected-Hermite

$\Rightarrow$ $f_o = 1871.70$
Similarly $\gamma_o = 3,259.90$ and alpha$_0 = \gamma_o - f_o = 1,388.2$
Now applying the condition;

$$y_{i,j} = \min(q_{n-1,i}(j), x_{i,j}) \quad j = i, 2, \ldots, n-1$$

$$y_{i,n} = N - \sum_{j=1}^{n-1} y_{i,j}$$

to the allocation above to obtain the following;

	A	B	C	D	E	F	G	H	T
Stock($y_{i,j}$)(000)	0	3	7	128	1180	4955	648	58, 277	65198
Cost(c_{ij})	0.04	0.03	0.02	0.02	0.02	0.02	0.01	0.01	-

Fig 13. Table of expected allocations-Hermite

$f_o = 1871.70$ $f_e = 714.74$
$\gamma_o = 864.45$ $\gamma_e = 3259.90$
$\alpha_e = 2545.16$
Hence a better allocation is achieved with the expected allocation.
Clearly $\alpha_e > \alpha_o$ i.e. the net returns of the approximate(expected) allocation is higher than that of the observed allocation. Hence a better allocation is achieved with the expected allocation.
Clearly $\alpha_e > \alpha_o$ i.e. the actual returns of the approximate(expected) allocation is higher than that of the observed allocation. The costs and net returns of the allocations are compared in the tables below.The different polynomials are represented by CH, LE, LA, HM, LG and NE for the Chebyshev, Legendre, Laguerre, Hermite, Lagrange and Newton polynomial respectively.

Polynomial	CH	LE	LA	HM	LG	NE
f_0	48.64	48.64	48.64	48.64	48.64	48.64
f_e	25.99	25.81	33.38	25.55	29.27	24.39
γ_0	98.65	98.65	98.65	98.65	98.65	98.65
γ_e	98.70	98.69	98.80	98.65	98.70	98.50
α_0	50.01	50.01	50.01	50.01	50.01	50.01
α_e	72.66	73.84	65.27	73.10	63.38	74.26

Fig 14. Table of summary of costs/returns of allocations-6-warehouses

Polynomial	CH	LE	LA	HM	LG	NE
f_0	1871.7	1871.7	1871.7	1871.7	1871.7	1871.7
f_e	722.71	602,95	722.71	714.74	679.61	849.49
γ_0	3259.9	3259.9	3259.9	3259.9	3259.9	3259.9
γ_e	3259.85	3258.6	3259.95	3259.90	3259.85	3269.95
α_0	1388.20	1388.20	1388.20	1388.20	1388.20	1388.20
α_e	2537.19	2566.95	2537.19	2545.16	2580.29	2410.41

Fig 15. Table of summary of costs/returns of allocation-8-warehouses

We proceed to further compare the observed and approximate allocations by their per unit attributes. λ and β represent per unit cost and per unit returns of stock respectively. The per unit cost comparison table are as follows;

Polynomial	CH	LE	LA	HM	LG	NE
λ_0	0.025	0.025	0.025	0.025	0.025	0.025
λ_e	0.013	0.013	0.017	0.013	0.015	0.012

Fig 16.Table of cost/unit of allocations-(6-waewhouses)

Polynomial	CH	LE	LA	HM	LG	NE
λ_0	0.029	0.029	0.029	0.029	0.029	0.029
λ_e	0.011	0.009	0.0011	0.011	0.010	0.013

Fig 17. Table of cost/unit of allocations=8-waewhouses)

The net returns per unit stock are shown in the tables below;

Polynomial	CH	LE	LA	HM	LG	NE
β_0	0.025	0.025	0.025	0.025	0.025	0.025
β_e	0.037	0.037	0.033	0.037	0.032	0.040

Fig 18. Table of net returns/unit of allocations-(6-waewhouses)

Polynomial	CH	LE	LA	HM	LG	NE
β_0	0.025	0.025	0.025	0.025	0.025	0.025
β_e	0.039	0.041	0.039	0.039	0.040	0.037

Fig 19. Table of net returns/unit of allocations-8-waewhouses)

6. Summary

Relevant attributes of the polynomials used have been stated earlier. The tables above give the summary of costs and returns of the polynomials. It is clearly observed that the cost expended on the original allocation is higher than that of the expected. The net returns of the expected allocation is generally higher than that obtained with the observed allocation in the polynomials used. The advantage of the stock allocation by the orthogonal polynomial technique is attested to as it shows a lower per unit cost and higher per unit returns as shown in tables above. It is therefore a fact that this new technique minimizes cost and maximizes returns.

7 Conclusion

In recent times, rules have been made by relevant agencies to standardize research, specify prod- ucts and ensure quality control of products. Due to advanced technology in communication, the issue of time wastages and unnecessary delay have been reduced. The answer to the question "how best can stock be managed to minimize losses and which allocation technique would be suitable to minimize cost and optimize returns?" has been provided by this work. The research has shown that polynomials can be used to approximate any continuous function as asserted by the Weierstrass theorem. This work has shown that;

1. The orthogonal polynomial technique is more cost effective as the cost associated with its allocation is less.
2. The final returns accruing to the firm through the approximate allocation is higher than that of the observed stock.
3. The method solves stock allocation problem for n-warehouses, ($n \geq 2$). The illustration has shown its use in six and eight warehouses.
4. It has a wide area of applicability as it could solve stock allocation problem for n-warehouses ($n \geq 2$). The illustration above has shown its use in eight warehouses of a firm.
5. In line with Kopechy's condition, the method is relatively easy to evaluate.

References

[1] Amparo,G; Javier,S. and Nico,T.(2007). Numerical methods for special functions. Society for Industrial and Applied Mathematics.

[2] Anderson,T(1974) Numerical methods of special functions. Siciety for Industrial and appled mathematics.

[3] Bellman R.E (1957). Dynamic Programming (Republished by Dover). Princeton Press.

[4] Burbridge, J.I (1998) Principles of Production Control, London. McDonald and Evans Ltd.

[5] Cardenas-Barron,L.E.(200) Observation on Economic production quantity Model for items with imperfect quality. *International Journal of Production Economics*.67,201

[6] Chikwendu,C.R and Emenonye,C.E(2013)A two warehouse stock control model for perfect quality goods with quantity discount.*Elixir Journal of Applied mathematics,Elixir International journal.*

[7] Chikwendu,C.R. and Emenonye, C.E. (2015)Dynamic Programming method for optimizing stock allocation using Chebyshev polynomial approximation. *British Journal of Applies Sciences and Technology.* 7(3): 316-324

[8] Emenonye,C.E.and Chikwendu, C.R.(2017)Hybrid Technique for optimizing stock alloca- tion(HTFOSA). *International Journal of Mathematical Analysis and Optimization: Theory and Applications.* Vol 2017, pp 125-138

[9] Erhan K (2007). Determinate Inventory Model for deteriorating item with capacity construct and time-proportional backlogging rate. *European Journal of Operational Research,* 198, 789 − 807

[10] Erhan,K(2008)Deterministic Inventory model for determining items with capacity constraint and time-proportional backlogging rate. *European Journal of Operations Research,*178,789-807

[11] Hieschfeld,A and Hielschfeld,N (1991) Introduction to Discrete Mathematics, California. Brooks/Cole publishing company.

[12] Jaggi, C.K.(2006) Two warehouse partial backlogging inventory model for deteriorating items with linear trend in demand under inflationery conditions. *International Journal of System science.* 42,1185-1196.

[13] Jeffreys,S.H. and Jeffreys,B.S. (1998) Weierstrass theorem on approximation of polynomials and extension of Weierstrass approximation theory. Method of Mathematical physics. 3rd Edition, Cambridge University press. (446-448)

[14] Jensen, J.A.and Rowland,J.H.(1975) Methods of computation. A linear space approach to Numerical analysis. Illinois, Scott,Foresman and company.

[15] Kang Y and Gershwin S. (2005) Information Inaccuracy Systems: Stock loss and stock out. IIE Trans 37, 843－859.

[16] kirkpahcks (1993). Optimization by Simulated and reaching science 220 : 671－680.

[17] Koonwinder,T,H,Wong,R.S. Koekoek,R.S. and Rene,F. (2010) Orthogonal polynomials- Handhook of Mathematical functions. London, CambridgenUniversity press.

[18] Kopecky, K.H. (2007)Function Approximation. Applied Computational Economics. Cambridge, MIT press.

[19] Lucey,T(1988) Quantitative Techniques: An International Manual. 3rd edition, Hampshire publications Ltd.

[20] Maddeh B. and Jabber M.I (2008). Economic Order Quantity for item with imperfect quality. Revisited *International Journal of production Economic* 12, 808－815

[21] Mihaila,B and Mihaila,I (2002)Numerical approximations using Chebyshev polynomial expansions.Journal physics A:Mathematics and general. 35(2002),731-746

[22] Milovanic G and Mastrion G (2008) Basic theory of interpolation (orthogonal polynomials and weighted applications).http://www.springer.com

[23] Powel, P and Belcerzek, J.(2009) Application of the Chebyshev polynomial to approximation and construction of map projections. Warsaw University of Technology, Poland

[24] Pressman E and Sethi S. P. (2004). Stochastic Inventory Model with continuous and pernous demand and University of Texas at Dellas Richardson TX.

[25] Raul, S.L.(2014) Function approximation, Quantitative macroeconomics. Washington University,St Louis

[26] Rowland,J.H.(1975)Methods of Computation; the linear space approach to numerical analysis. Glenview Scott,Foresman and Company

[27] Suetin, P.K. (2001)Encyclopedia of mathematics. Springer Science + Business media B.V./Kluwer Academic publishers.

[28] Theodore, J(1974) The Chebyshev polynomial. Pure and Applied mathematics. Viley-Interscience, New York; John Wiley and sons. 56-123

[29] Tien-Yu Lin (2011). A Two-warehouse Inventory model for items with imperfect quality and quantity accounts. *Asian Pacific Journal of Operation Research .28.No.2 (20011)147－161*

[30] Yang H.L (2006) Two warehouse Parties backlogging Inventory Models for deteriorating items under inflation. *International Journal of Production Economics* 103, 362－370.

[31] Yuan Xu (1991)Approximation of functions. Department of Mathematics, University of Oregon. yuan@math.uoregon.edu. National Science foundation