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3D Model Construction Skills of Scutoid and Variant Scutoid

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***Abstract*—The scutoid is a new geometric shape discovered by scientists in recent years. This shape is unheard of for science and mathematics. But it has always been hidden in our epithelial cells. The uniqueness of this scutoid is that; Putting one scutoid together with another upside down scutoid, two stereos can be seamlessly connected, and an unlimited number of scutoids can be continuously folded and extended. In this paper, the parametric design tool is used to construct the 3D model of the scutoid, and then through the optimization analysis technique, the scutoid model with the smallest volume and accurate size is calculated within the required size range. At the same time, based on the modeling and analysis skills, the 3D model construction of various variant scutoids was deduced. Finally, 3D printing is used to print out all of the models to verify their correctness, hoping to further demonstrate the new geometric aesthetics.**

Keywords—Scutoid, parametric design

I. Introduction

Scutoid is a 3D geometric shape that exists between two parallel surfaces above and below, the sides of its two parallel surfaces (and all other parallel surfaces between them) each enclose a polygon, there is at least one vertex between the two surfaces, and the vertexes of the upper and lower polygons are connected by a curve or a Y-shaped line. There is at least one vertex between the two faces of the shield cylinder, but the faces of the shield cylinder are not necessarily convex, so multiple shield cylinders can be tightly joined together to fill all the space between the two parallel surfaces. So scutoids can generally be described in general terms as a hybrid geometry between a frustum and a quasi-cylinder.

This geometry was first described in July 2018 in a paper titled "Scutoids are a geometrical solution to three-dimensional packing of epithelia" by Gomez-Galvez et al. According to the authors of the paper, because the shape resembles the back and shield of beetles in the Scarabidae family, they coined the term "Scutoid" to describe this geometry.

Alan Burdick said that this geometrical shape, although bizarre, was the building block of multicellular tissue; without it, complex life would never have existed on Earth [1]. In epithelial tissue cells, the cells can take the shape of a scutellum to promote tissue bending, which is essential for the formation of developing organs. More specifically within the cells that make up the skin and line many organs, he believes that scutoids are the true shape of epithelial cells that protect organisms from infection and ingestion of nutrients.

Therefore, the scutoid is a quasi-cylinder with middle-level vertices added, and the extra vertices make part of the plane on the final geometry become a curved surface. This means that the scutoid is not a polyhedron because not all its faces are flat. For the computational biologists who created or discovered the scutoid, a key property of this geometry is that it can combine itself with other geometric objects (such as frustums) to form the 3D packing structure of epithelial cells [2].

The optimization algorithm is often used to find the most suitable parameter combination, so that its efficiency and results can meet the requirements of users, and find its suitable algorithm for different types of problems. The fields of application span mathematics, applied sciences, economics, statistics, and even medicine. The general optimization method is to infer the most suitable result through continuous numerical analysis. Therefore, for low-dimensional optimization problems, most of them can have good performance, but for high-dimensional problems with more than 20 or 30 variables, the results are often unsatisfactory. The current analysis optimization methods can be roughly divided into three types: Numerical Method, Enumerative and Random Search. Among them, the numerical method is the most important. It is based on the calculus of mathematics and searches for the best solution in a specific space by seeking extreme values. This is the spirit of the "Hill Climbing Algorithm". That is to say, it has only one or several starting points, and the next iteration value is generated according to the established mathematical model, and the calculation is repeated until the best solution is found. Therefore, it may converge to the local optimal solution, but cannot reach the global optimal solution [3].

Compared with numerical methods, the heuristic algorithm of artificial intelligence technology is to imitate different natural phenomena respectively, using the concept of random search method to randomly select many starting points, and search for the best solution at the same time. Each individual in the ethnic group has a search point. After the evolution of generations, each search point approaches the best solution direction [4]. In this paper, it is only necessary to deduce the relative length of various polyhedrons to dual polyhedrons through optimized analysis. Therefore, the "Hill Climbing Algorithm" is the most efficient way.

Parametric design is a design method that defines parameter rules and related processes as the design basis. Unlike traditional designers who are accustomed to directly determine the form and shape of design through experience based on specific design conditions, the design process of parametric design is not directly dealing with "shape", but the "logic" behind the design. The design method that controls the various parameter factors that affect the design and the correlation between each parameter to produce, evaluate, and adjust the geometric form of the design plan in real time [5].

Parametric digital modeling through the acquisition and analysis of digital data, combined with CAM tools such as RP or CNC, can quickly view the design, greatly improving the efficiency and feasibility of non-traditional products design. This digital continuum produced by the integrated application of model software to manufacturing hardware has gradually driven the comprehensive innovation of contemporary digital design concepts to production methods [6]. In this paper, "Creo Parametric" modeling software is used as the construction software for all kinds of polyhedrons. At the same time, this software is also used as a tool for subsequent creative product creation.

II. GEOMETRY ANALYSIS OF SCUTOID AND VARIANT SCUTOID

A. *Scutoid Geometry*

A scutoid is a 3D geometric shape that exists between two upper and lower parallel surfaces. The two parallel surfaces are a regular pentagonal shape and a regular hexagon, which become the upper and lower sides of a cylinder, and then connect the vertices to form a 3D column. And one of the vertices of the regular pentagonal shape is divided into two paths in the middle of the connection, and the two vertices of the regular hexagon are respectively connected to complete the butt formation. The special feature of this scutoid is that by splicing one scutoid with another scutoid that is upside down and upside down, the two stereos can be seamlessly connected, and an infinite number of scutoids can be continuously closed and extended, so it has attracted attention. As shown in Figure 1.

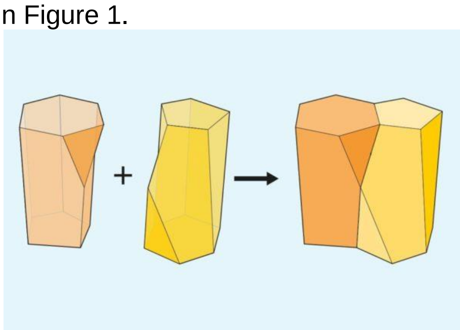


Figure 1 Scutoid model

(https://technews.tw/2018/08/04/new-member-of-the-geometric-world-scutoid/)

The main point of the geometric structure of this scutoid model is that the centroids of the regular pentagonal and regular hexagons of the upper and lower parallel surfaces are vertically projected on the middle plane of the two, and the relative offset of the X and Y directions position value. Because of the relative offset of these two directions, it will directly affect the main key dimension of whether the vertices of the two upside-down scutoids will be overlapped together. In this paper, the relative offset values in the X and Y directions after the vertical projection of the centroids of the regular pentagon and the regular hexagon of the upper and lower parallel surfaces on the middle plane of the two are used as the design variable, and two upside-down scutoids are put together, the relative distance between the vertices of their equal joints is "0" as the objective function, the correct relative offset value is deduced through the optimized design method - hill-climbing algorithm.

B. *Variant Scutoid Geometry*

As mentioned above the scutoid is a 3D geometric shape that exists between two upper and lower parallel surfaces. The two parallel surfaces are a regular pentagonal shape and a regular hexagon, which become the upper and lower sides of a cylinder, and then connect the vertices to form a 3D column. And one of the vertices of the regular pentagonal shape is divided into two paths in the middle of the connection, and the two vertices of the regular hexagon are respectively connected to complete the butt formation. The variant scutoid claimed in this paper is to extend the three-dimensional geometric shape existing between the upper and lower parallel surfaces from the regular pentagonal and regular hexagon to the regular n-shaped and the regular n+1-shaped A combination of models that form butt joints. For example, Figure 2 shows the variant scutoid model of the docking of a regular triangle and a regular quadrilateral. Figure 3 shows the variant scutoid model of the butt joint of the regular heptagon and regular octagon. The main point of the geometric structure of the same variant scutoid model is that the centroids of the regular n-square and the regular n+1 polygon of the upper and lower parallel surfaces are vertically projected on the middle plane of the two, X and Y The value of the relative offset position of the orientation. Because of the relative offset of these two directions, it will directly affect the main key dimension of whether the vertices of the two upside-down scutoids will be overlapped together. Similarly, in this paper, the correct relative offset value is deduced through the optimal design method - the hill-climbing algorithm.

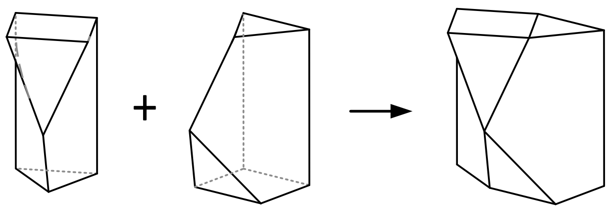


Figure 2 Variant scutoid model of triangle and quadrilateral docking

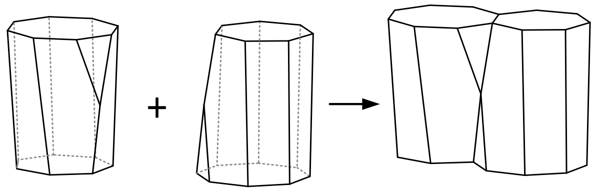


Figure 3 Variant scutoid model of the heptagon and octagon docking

III. APPLICATION OF OPTIMAL ANALYSIS TECHNIQUES

In the field of optimization research, many algorithms have been proposed, such as hill climbing algorithm, simulated annealing method, genetic algorithm, tabu search method, ant colony algorithm, particle swarm algorithm and so on. If the number of particles is used to distinguish, the above algorithms can be divided into two types: "single-particle type" and "multi-particle type". Among them, "hill-climbing algorithm", "simulated annealing method", and "taboo search method" belong to the "single-particle algorithm", while "genetic algorithm", "ant colony algorithm", "particle swarm algorithm", " The bee colony algorithm" are the multi-particle algorithm. Although most of the current academic research focuses on multi-particle algorithms, it is difficult to analyze the quality of these algorithms, which is caused by the inherent complexity of multi-particle systems. In this article, we only need to deduce the relative length of various polyhedrons to the side polyhedron through optimization analysis. Therefore, a well-known, simple and fast basic algorithm-"hill climbing algorithm" is adopted. As the optimized calculation tool in this article.

A. *Hill-Climbing Algorithm，HC*

The so-called "hill-climbing algorithm" is a simple regional search algorithm in the single-particle algorithm. Since its process is quite similar to the continuous upward movement of humans when climbing a mountain, it is called a hill-climbing algorithm. The hill-climbing algorithm can be said to be a heuristic method. The search strategy is to constantly find the best solution around, and then continue to move on to the best solution until it can no longer be improved. The implementation is quite easy and the execution speed is very fast. Therefore, it is often used as a benchmark for comparison of various optimization algorithms. And when the problem to be asked has multiple parameters, we can increase or decrease the value of a certain parameter by one unit in turn in the process of gradually obtaining the optimal solution through the hill climbing method. For example, the solution of a certain problem needs to use three integer-type arguments x1, x2, x3. At the beginning, set these three arguments to (2, 2, -2), increase/decrease x1 by 1, and get Two solutions (1,2, -2), (3, 2,-2); increase/decrease x2 by 1, and get two solutions (2,3, -2), (2,1, -2); Increase/decrease x3 by 1, and get two solutions (2,2,-1), (2,2,-3), so you get a solution set: (2,2,-2), (1, 2 ,-2), (3, 2,-2), (2,3,-2), (2,1,-2), (2,2,-1), (2,2,-3). Find the optimal solution from the above solution set, and then construct a solution set for this optimal solution according to the above method, and then find the optimal solution. In this way, the "hill-climbing" operation process ends until the optimal solution of the previous time and the optimal solution of the next time are the same. However, after this algorithm falls into the best solution in the area, it cannot jump out, that is, it cannot find a better solution. This is because the hill climbing algorithm only finds neighboring points for comparison, and does not allow walking in the worse direction, which makes climbing hills. In the more complex nonlinear programming problems, the algorithm is easy to fall into a poor area, and it is difficult to find the best solution in the whole domain.

Therefore, some experts have devised a jumping strategy that increases with the number of failures. This method is called Hill-Climbing with Jumping strategics (HCJ). The jumping mechanism makes it easier for the mountain climbing algorithm to leave the valley and find more good solution quickly. The difference between HCJ and the traditional hill-climbing algorithm is that the jumping steps of HCJ will increase with the number of failures. Therefore, when the particle is at the bottom of the valley, it will cause continuous jumping failure. At this time, HCJ will randomly increase the jumping range. By increasing the range, HCJ will have the opportunity to jump out of the current valley and move to a lower valley, allowing HCJ to find a better solution. In practice, in order to overcome the failure caused by the border area, then randomly select individuals from the range to make adjustments, and the size of the adjustment range is determined by random steps. When this neighbor selection method is used in the hill climbing algorithm, the individual can adjust in two directions, and there is a half chance that it will adjust in the correct direction. Therefore, the hill-climbing algorithm can be expressed by the following function:

```
Algorithm Hill-Climbing(pi)
    p = pi;
     while not isEnd( )
      pn = move(p);
      if pn.energy() <= p.energy()
      p = pn;
End Algorithm
```

The entire execution flow of the hill-climbing algorithm is shown in Figure 4.

B. *Scutoid and Variant Scutoid Optimization Execution Process*

As mentioned above, this paper is to project the centroids of the regular n-shaped and regular n+1 polygons of the upper and lower parallel surfaces on the middle plane of the two. The offset value is used as a design variable, and when two upside-down scutoids are put together, the relative distance between the vertices of the two joints is "0" as the objective function, and in the set search area, the inference is searched here the relative offset value within the region that conforms to the objective function and minimizes the volume. Taking scutoid as an example, n = 5, the side length is N, the initial value Xini = 0.1N; Yini = 0.2N, the search area 1: Xmin = 0.5 Xini; Xmax = 1.5 Xini; Ymin = 0.5 Yini; Ymax = 1.5 Yini. If no suitable solution can be found within the range of search area 1, the search area is expanded by 0.5 Xini. That is; search area 2: Xmin = 1.0 Xini; Xmax = 2.0 Xini; Ymin = 1.0 Yini; Ymax = 2.0 Yini. And so on, continue to expand the search area until a suitable solution is found. So if side length N = 40, height = 100, then the initial value Xini = 4; Yini = 8, search area 1: Xmin = 2.0; Xmax = 6.0; Ymin = 4.0; Ymax = 12.0. After calculation by the hill-climbing algorithm, the suitable X and Y offset values closest to the initial value are: X = 5.9898, Y = 12. Figure 5 shows the initial state of the scutoid combination and the combination state after the calculation offset, it is obvious that it meets our needs. Figure 6 shows the convergence process for minimizing volume, and its volume value is 6.7883514E+05.

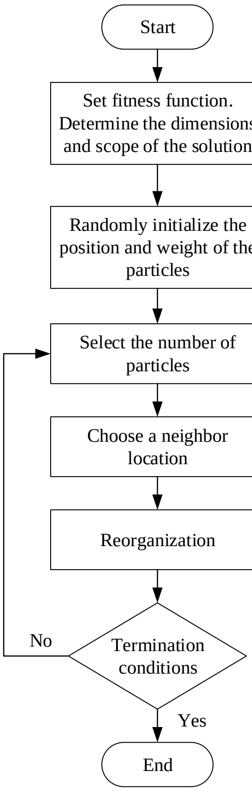
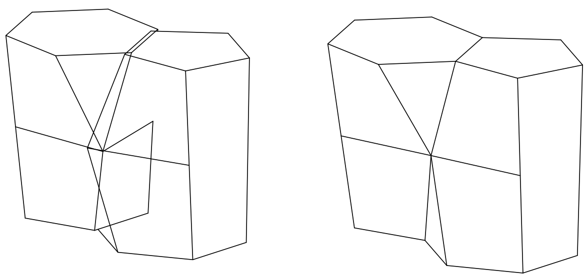


Figure 4 The entire execution flow of the hill-climbing algorithm



Initial state Exact state

Figure 5 The initial and exact combination state of scutoid models

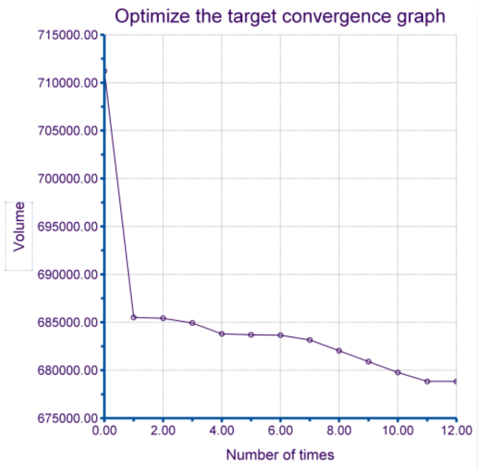


Figure 6 The convergence process for minimizing volume

IV. Modeling Steps Of Scutoid And Variant Scutoid

As mentioned above, this paper first uses the parametric design tool (Creo Parametric) to construct a scutoid-like 3D model, and then uses the optimization analysis technique - hill-climbing algorithm to calculate the minimum volume and accurate size of scutoid model within the required size range. Therefore, the modeling steps of any scutoid in this paper are as follows;

Step 1: Draw the bottom contour of the combination of two adjacent scutoid models (n and n+1 polygons) on the reference plane, where n = 5 is used as an example, as shown in Figure 7.

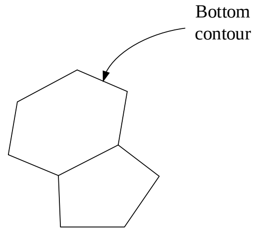


Figure 7 Bottom contour of the combination of scutoid models

Step 2: Construct the middle plane and the top plane with equal spacing, as shown in Figure 8.

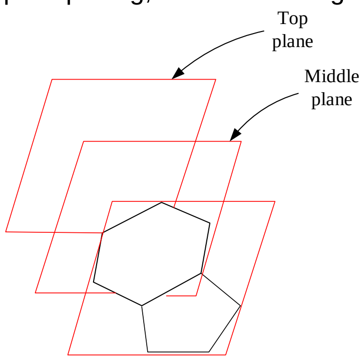


Figure 8 Bottom contour of the combination of scutoid models

Step 3: Draw a n+1 polygon on the top plane (here, a regular hexagon is taken as an example), and its centroid position relative to the centroid of the n polygon on the base plane (here, a regular pentagon is used as an example)), respectively offset △X and △Y on the horizontal plane, as shown in Figure 9.

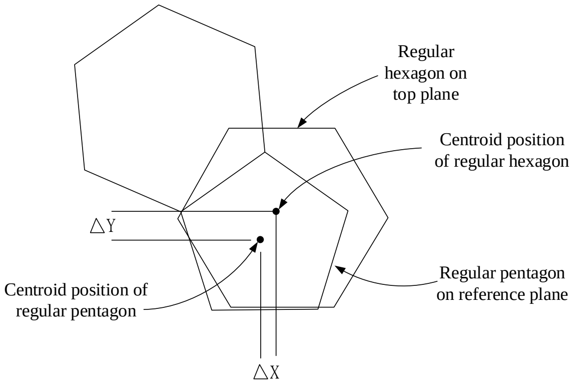


Figure 9 Initial offset status of upper and lower geometric figures

Step 4: Find the transition point P (ie line segment AP = line segment BP) where the two adjacent scutoid models combine with the same length on the middle plane, as shown in Figure 10.

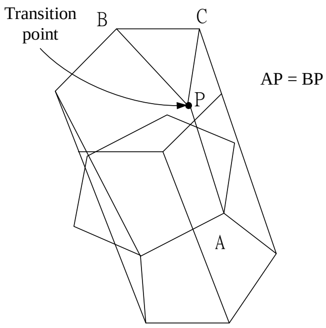


Figure 10 Transition point P on the middle plane

Step 5: The corresponding vertices are connected by straight lines, but one of the vertices of the regular pentagon needs to be connected to the transition point P, and then connected to the corresponding vertices of the regular hexagon. That is, A is connected to P, and P is connected to B and C respectively. This will complete the wire structure of the "scutoid-like model", as shown in Figure 11.

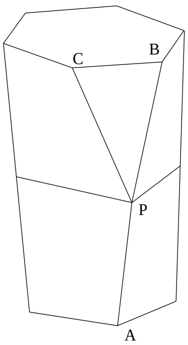


Figure 11 Wire frame of the scutoid-like model

Step 6: Construct a "scutoid-like solid model" using the "scutoid-like wirf frame model" in step 5, as shown in Figure 12.

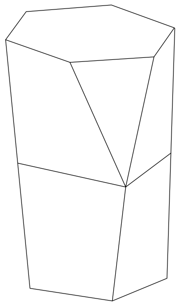


Figure 12 The scutoid-like solid model

Step 7: Using the two "scutoid-like solid models" completed in Step 6, and turn one of them upside down, and position them on the bottom contour line completed in Step 1, as shown in Figure 13.

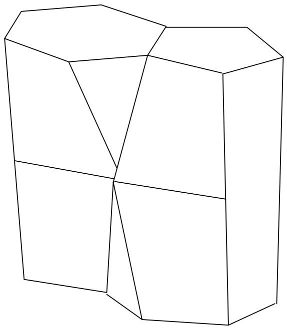


Figure 13 The state of the combination of upside-down scutoid-like models

Step 8: Measure the distance between the two transition points P-P' after the two "scutoid-like solid models" are positioned. And set this value to the target parameter "measure" as shown in Figure 14.

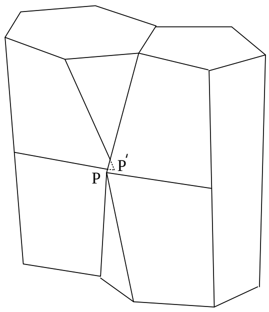


Figure 14 Schema of the objective function

Step 9: Taking the parameter measure = 0 as the objective function, adjust the regular hexagon on the top plane, its centroid position is relative to the centroid of the regular pentagon on the base plane, and the offsets △X and △Y on the horizontal plane are variation parameters, optimization analysis of feasibility and volume minimization, and completion of the correct scutoid solid model.

V. Results And Discussions

In this paper, the optimized initial conditions and the definition of the search area are; if the side length is N, the initial offset value $Xini = 0.1N$; $Yini = 0.2N$, the search area 1: $Xmin = 0.5\ Xini$; $Xmax = 1.5\ Xini$; $Ymin = 0.5\ Yini$; $Ymax = 1.5\ Yini$。 If no suitable solution can be found within the range of search area 1, the search area is expanded by 0.5 *Xini*. That is; search area 2: *Xmin* = 1.0 *Xini* ;*Xmax* = 2.0 *Xini* ; *Ymin* = 1.0 *Yini* ; *Ymax* = 2.0 *Yini*。And so on, continue to expand the search area until a suitable solution is found.

A. *The Scutoid Paired with a Regular Pentagon and a Regular Hexagon*

Here, the side length is 30 mm and the height is 100 mm as the benchmark. Therefore, the initial offset of the centroid between the regular pentagon and the regular hexagon during the model construction process is: △X = 3 mm; △Y = 6 mm, as shown in Figure 15. And Figure 16 shows the situation when two identical objects are assembled upside down. At this time, the distance between the two points P-P' is 6.80879 mm.

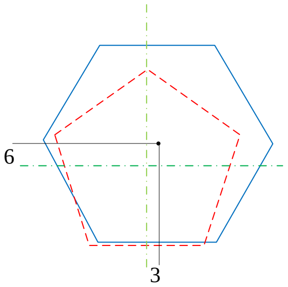


Figure 15 Initial offset value of centroid of regular pentagon and regular hexagon

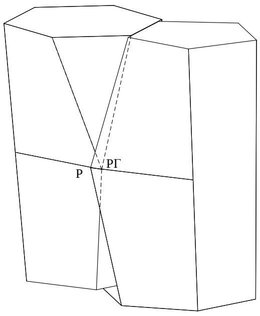


Figure 16 The situation when a regular pentagon and a regular hexagon are set upside down

According to the above search principle, X is in the range of 1.5 ~ 4.5 mm, Y is in the range of 3 ~ 9 mm, the distance between the two points P-P' is 0 mm, and the minimum volume is the target value to optimize the solution. The result is △X = 4.49235 mm; △Y = 9 mm, the volume is $3.843257{*}10^5$ mm3, and the solution goal is achieved. And Figure 17 shows the optimization convergence process, and Figure 18 shows the assembled state of the final result.

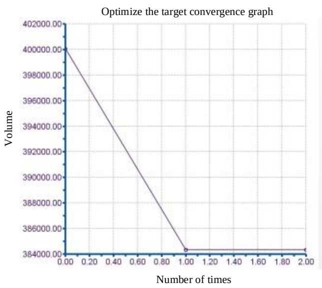


Figure 17 Convergence of optimization of regular pentagon and regular hexagon

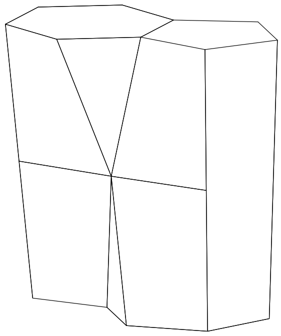


Figure 18 Optimal composition of regular pentagon and regular hexagon

B. *The Variant Scutoid Paired With a Regular Triangle and a Regular Quadrilateral*

Here, the side length is 50 mm and the height is 100 mm as the benchmark. Therefore, the initial offset of the centroid between the regular triangle and the regular quadrilateral during the model construction process is: △X = 5 mm; △Y = 10 mm, as shown in Figure19. And Figure 20 shows the situation when two identical objects are assembled upside down. At this time, the distance between the two points P-P' is 2.54866 mm.

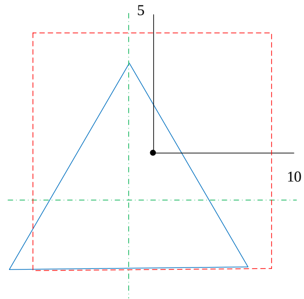


Figure 19 Initial offset value of centroid of regular triangle and regular quadrilateral

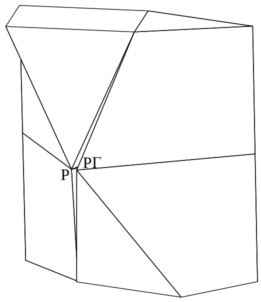


Figure 20 The situation when a regular triangle and a regular quadrilateral are set upside down

According to the above search principle, X is in the range of 2.5 ~ 7.5 mm, Y is in the range of 5 ~ 15 mm, the distance between the two points P-P' is 0 mm, and the minimum volume is the target value to optimize the solution. The result is △X = 7.49999 mm; △Y = 8.55663 mm, the volume is $3.8960275{*}10^5$ mm3, and the solution goal is achieved. And Figure 21 shows the optimization convergence process, and Figure 22 shows the assembled state of the final result.

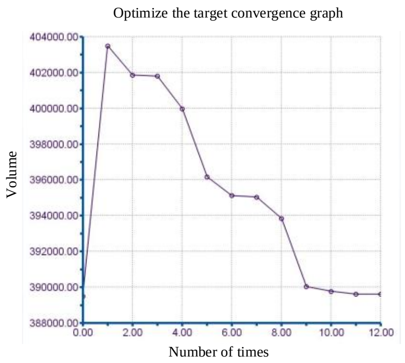


Figure 21 Convergence of optimization of regular triangle and regular quadrilateral

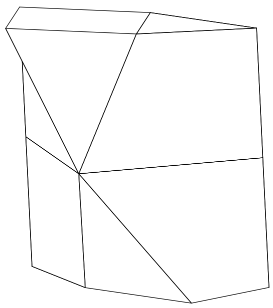


Figure 22 Optimal composition of regular triangle and regular quadrilateral

C. The Variant Scutoid Paired with a Regular Heptagon and a Regular Octagon

Here, the side length is 30 mm and the height is 100 mm as the benchmark. Therefore, the initial offset of the centroid between the regular heptagon and the regular octagon during the model construction process is: $\triangle X$ = 3 mm; $\triangle Y$ = 6 mm, as shown in Figure 23. And Figure 24 shows the situation when two identical objects are assembled upside down. At this time, the distance between the two points P-P' is 9.98742 mm.

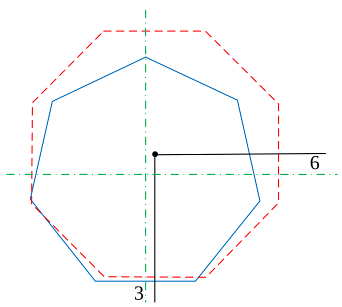


Figure 23 Initial offset value of centroid of regular heptagon and regular octagon

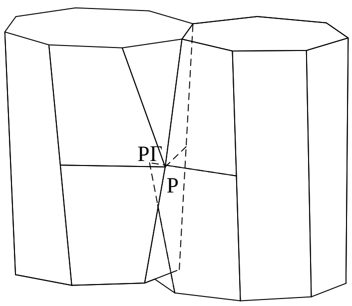


Figure 24 The situation when a regular heptagon and a regular octagon are set upside down

According to the above search principle, X is in the range of 1.5 ~ 4.5 mm, Y is in the range of 3 ~ 9 mm, the distance between the two points P-P' is 0 mm, and the minimum volume is the target value to optimize the solution. As a result, a suitable solution that satisfies the conditions cannot be found in this interval, as shown in Figure 25.

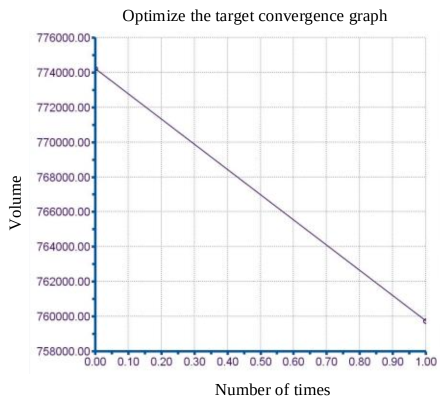


Figure 25 The first interval optimization fails to converge

Therefore, according to the above search principle, this solution space is changed to X is in the range of 3 ~ 6 mm, Y is in the range of 6 ~ 12 mm, still take the distance between the two points P-P' is 0 mm, and the minimum volume is the target value to optimize the solution. The result is $\triangle X$ = 6.0 mm; $\triangle Y$ = 11.0429 mm, the volume is $7.5295541*10^5$ mm3, and the solution goal is achieved. And Figure 26 shows the optimization convergence process, and Figure 27 shows the assembled state of the final result.

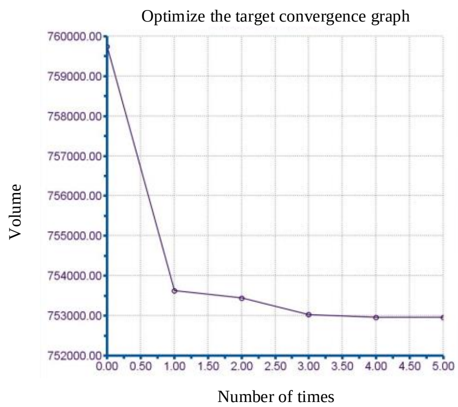


Figure 26 The second interval optimization convergence situation

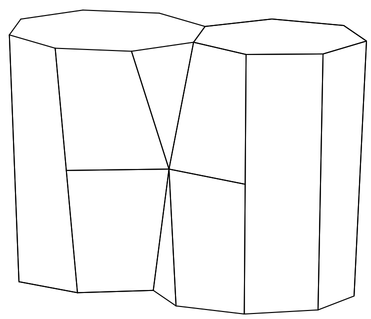
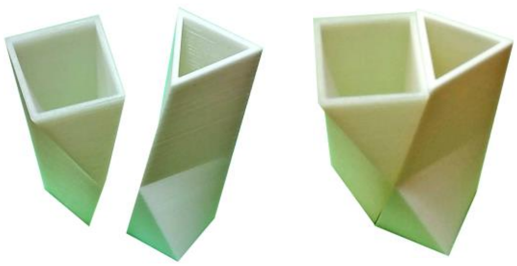


Figure 27 Optimal composition of regular heptagon and regular octagon

VI. CONCLUSION

The scutoid is a new geometric shape discovered by scientists in recent years. This shape is unheard of for science and mathematics. But it has always been hidden in our epithelial cells. The uniqueness of this scutoid is that; Putting one scutoid together with another upside down scutoid, two stereos can be seamlessly connected, and an unlimited number of scutoids can be continuously folded and extended. In this paper, the parametric design tool is used to construct the 3D model of the scutoid, and then through the optimization analysis technique, the scutoid model with the smallest volume and accurate size is calculated within the required size range. At the same time, based on the modeling and analysis skills, the 3D model construction of various variant scutoids was deduced. Finally, 3D printing is used to print out all of the models to verify their correctness, hoping to further demonstrate the new geometric aesthetics. Figure 28 ~ Figure 36 are the 3D printed products of the scutoid and variant scutoid deduced in this paper.



Separation status Combination status

Figure 28 3D printed product of regular triangle and regular quadrilateral variant scutoid

Separation status Combination status

Figure 29 3D printed products of regular pentagon and regular hexagon scutoid

Separation status Combination status

Figure 30 3D printed products of regular heptagon and regular octagon variant scutoid

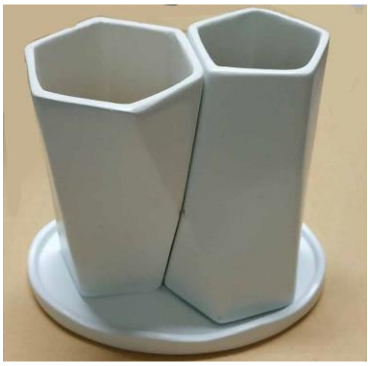


Figure 31 Pentagonal and hexagonal scutoid 3D printed tea set

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REFERENCES

[1] Burdick, Alan. We Are All Scutoids: A Brand-New Shape. Explained. the new yorker,2018, pp.07-30.

[2] Gómez-Gálvez, Pedro; Vicente-Munuera, Pablo; Tagua, Antonio; Forja, Cristina; Castro, Ana M.; Letrán, Marta; Valencia-Expósito, Andrea; Grima, Clara; Bermúdez-Gallardo, Marina. Scutoids Are A Geometrical Solution To Three-Dimensional Packing Of Epithelia. Nature Communications ,pp. 07-27.2018.

[3] A. Juels, A., M. Watenberg, Stochastic Hill-Climbing as a Baseline Method for Evaluating Genetic Algorithms. Tech. Report, University of California at Berkeley, 1994.

[4] Zhang Zhanjia. Operational Case Demonstration of Parametric Design and Digital Manufacturing. National Taiwan University of Science and Technology Institute of Architecture, Master Thesis, 2011.

[5] You Shengbo. Natural Mode: The Application of Parametric Design in Product Form Creation. National Chiao Tung University Institute of Applied Arts, Master Thesis, 2012.

[6] Qiu Haoxiu, The New Poeticness of Architecture Under the Digital Construction Method. Architecture Research Center, School of Creative Arts, Tunghai University,2016.