Design Of A Spatial Mechanism With Revolute-Spherical-Cylindrical-Revolute (RSCR) Kinematic Joints

José Luis Ramírez Cruz
Departamento de Energía
UAM Azcapotzalco
CdMx, México
rcjl@azc.uam.mx

Homero Jiménez Rabiela
Departamento de Energía
UAM Azcapotzalco
CdMx, México
hjr@azc.uam.mx

Pedro García Segura
Departamento de Energía
UAM Azcapotzalco
CdMx, México
pegase@yahoo.com.mx

Benjamín Vázquez González
Departamento de Energía
UAM Azcapotzalco
CdMx, México
bvg@azc.uam.mx

Adrian Gustavo Bravo Acosta
Departamento de Energía
UAM Azcapotzalco
CdMx, México
magba@yahoo.com.mx

***Abstract*—In this paper we calculated the principal dimensions of the tridimensional mechanism RSCR to generate a mathematical function. The mathematical formulation of the "design equation" is the result of using analytic geometry in three dimensions, adds with a condition that the bodies or links should be totally rigids. The "design equation" were solved using the software Maple. The simulation of the mechanism was done to check that it satisfies the input conditions and output for six positions.**

Keywords — Mechanism, design, kinematic

I. INTRODUCTION

A mechanism is defined as a system of rigid bodies connected together and moving with respect to a reference, according to the required motion. Mechanisms are the executors of movements in various sub-systems of automobiles, trucks of all types and other means of transporting cargo and people.

The synthesis (design) of mechanisms is classified according to the desired output motion. If the output motion is to be a specified function of the input motion, the task is called function generation [1]. For any synthesis task, there are two stages: type synthesis and dimensional synthesis.

Dimensional synthesis is the task of choosing the dimensions of the elements, once a specific bond type has been chosen. This task, at least for the simplest bonds, can be reduced to an analytical procedure and thus to a computational algorithm. However, it has been shown that the synthesis is an approximate task. There is no guarantee that the linkage can move continuously between design points without disassembly.

There are three-dimensional closed-chain and open-chain mechanisms, the mechanism with RSCR topology is a three-dimensional closed-chain mechanism, Fig. 1.

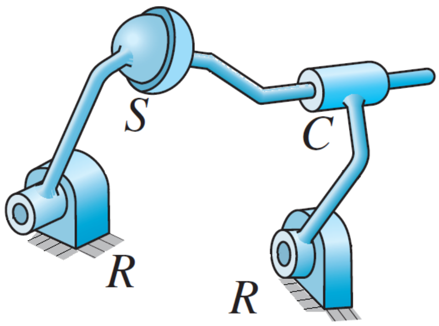

Fig. 1. Configuration of the RSCR Mechanism.

The three-dimensional RSCR mechanism with four articulated links could be used as a braking mechanism in buses and trucks [2] or in measuring instruments, Fig. 2.

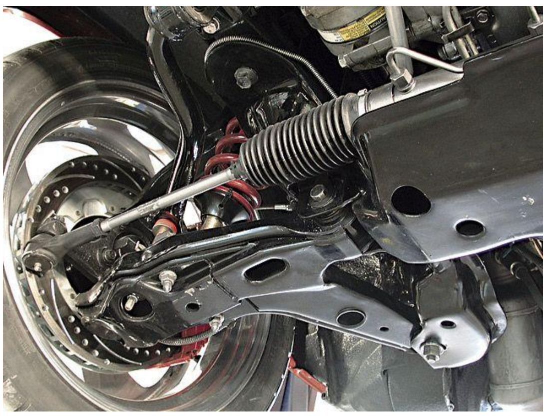

Fig. 2. Braking mechanism.

Revolute pair. Has one degree of freedom and permits a rotational motion about an axis.

Cylinder pair. Has two degrees of freedom and permits rotational and translational motions along collinear axes. The rotational motion is independent of the translational motion.

Spherical pair. Has three degrees of freedom and permits rotational motion about three independent axes.

The topology of the mechanism indicates that it has two fixed kinematic pairs of revolution and one degree of freedom, both forming joints with the frame. The kinematic pair "S" refers to a spherical type joint with three rotating degrees of freedom (see point "B" in Fig. 3).

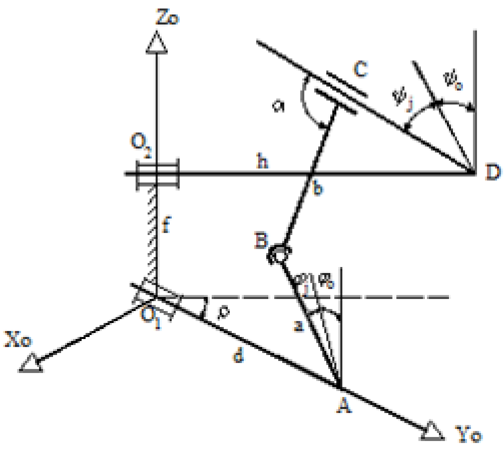

Fig. 3. RSCR three-dimensional mechanism.

The cylindrical pair "C" is a joint with two degrees of freedom, with a rotating and an independent sliding movement (see point "C" in Fig. 3).

Point "B" describes a circle perpendicular to the Oy axis. Point "C" moves in a plane perpendicular to the output axis of rotation, describing a complex trajectory in this plane. Point "C" describes a sphere with respect to point "B". There are several ways to determine the location of point "C"; for example, by finding the intersection of the sphere with the plane described by the link "DC".

For any given value of the input angle ϕ_j of the driving crank, the coordinates of the spherical pair "B" can be easily found, using three-dimensional rotation matrices, or by finding the coordinates of point "B" and "C" as can be easily seen in Fig. No. 3, using analytical geometry.

II. Motion Analysis

The present work focuses on obtaining a mathematical function, with the design equation F (ϕ , ψ) = 0 ; which contains the dimensions of the mechanism.

The design equation F (ϕ , ψ) = 0 is the objective function, no inequality constraints or any other constraints are specified as in the paper of G.K. Ananthasuresh and S.N. Kramer [3].

Our paper is obtained with a different methodology than the one mentioned in [3]. The RSCR mechanism has a degree of freedom, according to the Kutzback or Artobolevsky criteria, which are the existing equations, to define how many motors it will work with.

From Figure 3; the coordinates of the spherical pair B are defined by the position vector

$$\overline{O_1B} = \overline{O_1A} + \overline{AB} \quad (1)$$

Where

$\overline{O_1A} = d\hat{j};$

$\overline{AB} = aSin(\phi_o + \phi_j)\hat{i} + aCos(\phi_o + \phi_j)\hat{k}$

Substituting in (1)

$$\overline{O_1B} = aSin(\phi_o + \phi_j)\hat{i} + d\hat{j} + aCos(\phi_o + \phi_j)\hat{k}$$

The coordinates of the cylindrical pair C are defined by the position vector

$$\overline{O_1C} = \overline{O_1O_2} + \overline{O_2D} + \overline{DC} \quad (2)$$

Where

$\overline{O_1O_2} = f\hat{k};$

$\overline{O_2D} = -hSin\rho\hat{i} + hCos\rho\hat{j}$

To define the vector, a projection of the mechanism is considered, viewing it from the end of the Z-axis, and projecting it in the O1X_0Y_0 plane, see Fig. 4.

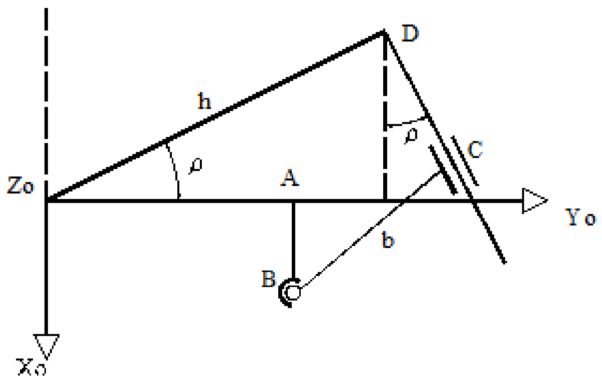

Fig. 4. Top view of the RSCR mechanism.

So the vector

$$\overline{DC} = SjSin(\psi_o + \psi_j)Cos\rho\hat{i} + SjSin(\psi_o + \psi_j)Sin\rho\hat{j} + SjCos(\psi_o + \psi_j)\hat{k}$$

Where Sj is the displacement of the cylindrical pair C. And substituting in Eq. (2):

$$\overline{O_1C} = S\left[jSin(\psi_o + \psi_j)Cos\rho\hat{i} - hSin\rho\right]\hat{i} + \left[hCos\rho + SjSin(\psi_o + \psi_j)Sin\rho\right]\hat{j} + \left[f + SjCos(\psi_o + \psi_j)\right]\hat{k}$$

The position vector $\mathbf{\bar{b}} = \overline{\mathbf{BC}}$ is defined as the difference between $\overline{\mathbf{O_1C}}$ and $\overline{\mathbf{O_1B}}$; so $\overline{\mathbf{BC}} = \overline{\mathbf{O_1C}} - \overline{\mathbf{O_1B}}$, then:

$$\bar{b} = \left[SjSin(\psi_o + \psi_j)Cos\rho - hSin\rho - aSin(\phi_o + \phi_j)\right]\hat{i} + \left[hCos\rho + SjSin(\psi_o + \psi_j) - d\right]\hat{j} + \left[f + SjCos(\psi_o + \psi_j) - aCos(\phi_o + \phi_j)\right]\hat{k} \quad (3)$$

For to simplify the design equation of the RSCR mechanism with input angles ϕ_j ; or the RCSR mechanism with input angles ψ_j, it will be assumed that the angle ρ is constant; it has any value and in this case the value of ρ = 90° is considered, in the same way it is done for α = 90°.

Then the vector defining the coupling link $\bar{b}$ is:

$$\bar{b} = -\left[h + aSin(\phi_o + \phi_j)\right]\hat{i} + \left[SjSin(\psi_o + \psi_j) - d\right]\hat{j} + \left[f + SjCos(\psi_o + \psi_j) - aCos(\phi_o + \phi_j)\right]\hat{k} \quad (4)$$

Multiplying this vector in scalar form by itself we obtain

$$b^2 = a^2 + d^2 + f^2 + h^2 + S_j^2 + 2ahSin(\phi_o + \phi_j) + S_j\left[2fCos(\psi_o + \psi_j) - 2dSin(\psi_o + \psi_j) - aCos(\psi_o + \psi_j)Cos(\phi_o + \phi_j)\right] - 2afCos(\phi_o + \phi_j) \quad (5)$$

Since the cylindrical joint is a function of the displacement Sj and the angle of rotation; this does not matter for the case of function generation; what is required is a design equation F(ϕj,ψj) = 0 where there is no variable S_j. Then it is necessary to eliminate S_j from Eq. (4). For this purpose, the scalar product of the vector $\bar{b}$ by the unit vector along $\overline{DC}$ is performed.

The scalar product $\mathbf{\bar{b}} \bullet \frac{\overline{\mathbf{DC}}}{\left|\overline{\mathbf{DC}}\right|} = \mathbf{bCos}\alpha$, but by definition ρ=90º and α=90º, therefore, the unit vector along $\overline{DC}$ is:

$$\frac{\overline{DC}}{\left|\overline{DC}\right|} = Sin(\psi_o + \psi_j)\hat{j} + Cos(\psi_o + \psi_j)\hat{k}$$

$$\Rightarrow \bar{b} \bullet \frac{\overline{DC}}{\left|\overline{DC}\right|} = fCos(\psi_o + \psi_j) - dSin(\psi_o + \psi_j) + S_j - aCos(\phi_o + \phi_j)Cos(\psi_o + \psi_j) = 0 \quad (6)$$

From here:

$$S_j = -fCos(\phi_o + \phi_j) + dSin(\psi_o + \psi_j) + aCos(\phi_o + \phi_j)Cos(\psi_o + \psi_j) \quad (7)$$

and

$$S_j^2 = f^2Cos^2(\psi_o + \psi_j) + d^2Sin^2(\psi_o + \psi_j) + a^2Cos^2(\phi_o + \phi_j)Cos^2(\psi_o + \psi_j) - 2dfCos(\psi_o + \psi_j)Sin(\psi_o + \psi_j) - 2afCos(\phi_o + \phi_j)Cos^2(\psi_o + \psi_j) + 2adCos(\phi_o + \phi_j)Cos(\psi_o + \psi_j)Sin(\psi_o + \psi_j) \quad (8)$$

Substituting in Eq. (5) the values of S_j and S_j^2 is obtained:

$$b^2 = a^2 + d^2 + f^2 + h^2 + a^2Cos^2(\psi_o + \psi_j)Sin^2(\phi_o + \phi_j) - d^2Sin^2(\psi_o + \psi_j) - f^2Cos^2(\psi_o + \psi_j) + 2ahSin(\phi_o + \phi_j) - 2afCos(\phi_o + \phi_j) + 2dfCos(\psi_o + \psi_j)Sin(\psi_o + \psi_j) - 2afCos^2(\psi_o + \psi_j)Cos(\phi_o + \phi_j) - 2adCos(\phi_o + \phi_j)Cos(\psi_o + \psi_j)Sin(\psi_o + \psi_j) \quad (9)$$

Defining a new function $F(\phi_j, \psi_j) = 0$ we obtain:

$$F_j = a^2 - b^2 + d^2 + f^2 + h^2 + a^2Cos^2(\psi_o + \psi_j)Sin^2(\phi_o + \phi_j) - d^2Sin^2(\psi_o + \psi_j) - f^2Cos^2(\psi_o + \psi_j) + 2ahSin(\phi_o + \phi_j) - 2afCos(\phi_o + \phi_j) + 2dfCos(\psi_o + \psi_j)Sin(\psi_o + \psi_j) - 2afCos^2(\psi_o + \psi_j)Cos(\phi_o + \phi_j) - 2adCos(\phi_o + \phi_j)Cos(\psi_o + \psi_j)Sin(\psi_o + \psi_j) \quad (10)$$

The data entered in Eq. (10) are shown in Table 1.

TABLE I. Values of angles considered.

j	ϕ_j (radians)	ψ_j (radians)
1	0	0
2	0.5236	0.34907
3	1.0472	0.69813
4	1.5708	0.87266
5	2.0071	1.0472
6	2.5307	1.3963

The solution of the design equations was carried out for six points of precision, taking as known value for the link f equal to 5, these equations are shown below:

$$\begin{aligned} 0 &= a^2 - b^2 + d^2 + 5^2 + h^2 + a^2 Cos^2(\psi_o) Sin^2(\phi_o) - \\ & d^2 Sin^2(\psi_o) - 5^2 Cos^2(\psi_o) + 2ahSin(\phi_o) - \\ & 10aCos(\phi_o) + 10dCos(\psi_o) Sin(\psi_o) - \\ & 10aCos^2(\psi_o) Cos(\phi_o) - 2adCos(\phi_o) Cos(\psi_o) Sin(\psi_o) \end{aligned} \tag{10.1}$$

$$\begin{aligned} 0 &= a^2 - b^2 + d^2 + 5^2 + h^2 + a^2 Cos^2(\psi_o + 0.349) Sin^2(\phi_o + 0.523) - \\ & d^2 Sin^2(\psi_o + 0.349) - 5^2 Cos^2(\psi_o + 0.349) + 2ahSin(\phi_o + 0.523) - \\ & 10aCos(\phi_o + 0.523) + 10dCos(\psi_o + 0.349) Sin(\psi_o + 0.349) - \\ & 10aCos^2(\psi_o + 0.349) Cos(\phi_o + 0.523) - \\ & 2adCos(\phi_o + 0.523) Cos(\psi_o + 0.349) Sin(\psi_o + 0.349) \end{aligned} \tag{10.2}$$

$$\begin{aligned} 0 &= a^2 - b^2 + d^2 + 5^2 + h^2 + a^2 Cos^2(\psi_o + 0698) Sin^2(\phi_o + 1.047) - \\ & d^2 Sin^2(\psi_o + 0698) - 5^2 Cos^2(\psi_o + 0698) + 2ahSin(\phi_o + 1.047) - \\ & 10aCos(\phi_o + 1.047) + 10dCos(\psi_o + 0698) Sin(\psi_o + 0698) - \\ & 10aCos^2(\psi_o + 0698) Cos(\phi_o + 1.047) - \\ & 2adCos(\phi_o + 1.047) Cos(\psi_o + 0698) Sin(\psi_o + 0698) \end{aligned} \tag{10.3}$$

$$\begin{aligned} 0 &= a^2 - b^2 + d^2 + 5^2 + h^2 + a^2 Cos^2(\psi_o + 0.872) Sin^2(\phi_o + 1.57) - \\ & d^2 Sin^2(\psi_o + 0.872) - 5^2 Cos^2(\psi_o + 0.872) + 2ahSin(\phi_o + 1.57) - \\ & 10aCos(\phi_o + 1.57) + 10dCos(\psi_o + 0.872) Sin(\psi_o + 0.872) - \\ & 10aCos^2(\psi_o + 0.872) Cos(\phi_o + 1.57) - \\ & 2adCos(\phi_o + 1.57) Cos(\psi_o + 0.872) Sin(\psi_o + 0.872) \end{aligned} \tag{10.4}$$

$$\begin{aligned} 0 &= a^2 - b^2 + d^2 + 5^2 + h^2 + a^2 Cos^2(\psi_o + 1.047) Sin^2(\phi_o + 2.007) - \\ & d^2 Sin^2(\psi_o + 1.047) - 5^2 Cos^2(\psi_o + 1.047) + 2ahSin(\phi_o + 2.007) - \\ & 10aCos(\phi_o + 2.007) + 10dCos(\psi_o + 1.047) Sin(\psi_o + 1.047) - \\ & 10aCos^2(\psi_o + 1.047) Cos(\phi_o + 2.007) - \\ & 2adCos(\phi_o + 2.007) Cos(\psi_o + 1.047) Sin(\psi_o + 1.047) \end{aligned} \tag{10.5}$$

$$\begin{aligned} 0 &= a^2 - b^2 + d^2 + 5^2 + h^2 + a^2 Cos^2(\psi_o + 1.396) Sin^2(\phi_o + 2.53) - \\ & d^2 Sin^2(\psi_o + 1.396) - 5^2 Cos^2(\psi_o + 1.396) + 2ahSin(\phi_o + 2.53) - \\ & 10aCos(\phi_o + 2.53) + 10dCos(\psi_o + 1.396) Sin(\psi_o + 1.396) - \\ & 10aCos^2(\psi_o + 1.396) Cos(\phi_o + 2.53) - \\ & 2adCos(\phi_o + 2.53) Cos(\psi_o + 1.396) Sin(\psi_o + 1.396) \end{aligned} \tag{10.6}$$

III. RESULTS

The design equations were solved using Maple software. The dimensions of the RSCR mechanism obtained by solving the system of equations were:

h = 1.199257161
b = 9.123878266
a = 5.346838510
d = 4.411014114
ϕ_0 = -4.454717731
ψ_0 = -1.625891902

The angles obtained are indicated in radians. Figure 5 shows the starting position, where the links d and h reach critical positions.

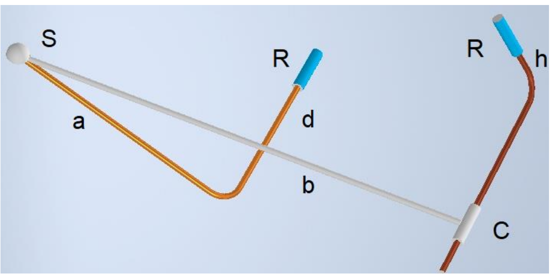

Fig. 5. Simulation of the RSCR mechanism.

IV. CONCLUSIONS

With the dimensions obtained from the solution approach of the design equation, the mechanism was simulated in Inventor software, verifying that the mechanism assembles perfectly with the obtained dimensions and the given angular displacement ranges. This work can be extended by including Chebyshev polynomials to reduce the rounding error, the mechanism can be designed to generate curved surfaces such as cylinders, cones, hyperboloids, etc. or to drive a rigid body placed in the coupler link.

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