Design Of Computer Program For Satellite Visibility Time Computation For The Case Of Highly Eccentric Elliptical Orbit

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***Abstract*— The design of computer program for satellite visibility time computation for the case of highly eccentric elliptical orbit is presented. Specifically, the algorithm for the computer program is presented along with some sample numerical examples using Molniya 1-36 satellite as the case study. The algorithm enables the computation of the visibility time of the sample satellite when there are different values of minimum elevation angles. Also, the computation was conducted for different instantaneous values of the orbital eccentricity and orbital altitudes. The results show that by considering minimum elevation angle range of 0° to 15° and using the mean orbital eccentricity of 0.72625 and mean orbital altitude of 20194.6 km, the visibility time is highest with a value of 10.989 hours at 0° minimum elevation angle and the visibility time is lowest at a value of 9.157 hours at 15° minimum elevation angle. Also, the linear relationship between the minimum elevation angle and the visibility time was also derived. Furthermore, the other results from the study shows that instantaneous values of eccentricity and orbital altitude do affect the visibility time.**

***Keywords* — Molniya satellite, computer program, Elliptical Orbit, Satellite, Visibility Time, Highly Eccentric**

1. INTRODUCTION

Today, wireless communication have technologies have become part of our daily life [1,2,3,4,5,6,7,8,9,10,11,12]. Importantly, satellite communications which is a major component of wireless communication system has enable diverse form of global communications. Today, internet of things, wireless sensor applications, smart cities, smart homes, and many different smart technologies rely heavily of diverse wireless technologies [13,14,15,16,17,18,19,20,21,22,23]. In any case, when global coverage or wider portions of the earth needs to be covered, satellite communication technologies is key. With the geostationary satellites, the ability to cover a specific region depends on the location of the cite relative to the equator [24,25,26,27,28]. However, the use of highly eccentric elliptical orbit has be employed to address that limitation of the geo-satellites [29,30,31,32,33].

Importantly, as wireless communication technologies advances timely access and analysis of the various parameters of the wireless communication systems becomes paramount. As such in this study, the algorithm for analysing the visibility time of elliptical orbits with high values of orbital eccentricity is examined. The study presented the mathematical expressions and the algorithm for a computer program that can be used to analyse the visibility time of the Molniya 1-36 satellite [34,35] under different minimum elevation angle and different instantaneous orbital parameters. The details of the Molniya 1-36 satellite is also presented and the numerical computation examples based on the case study Molniya satellite are s presented.

2. METHODOLOGY

The eccentricity (e) and orbital period (T_o) of highly eccentric elliptical orbit is required to compute the visibility time (Δt_{VH}), as follows [36,37];

$$M(e) = 2\left(\tan^{-1}\left(\sqrt{\frac{(1-e)}{(1+e)}}\right)\right) - \left(e\left(\sqrt{(1-e^2)}\right)\right) \quad (1)$$

$$\Delta t_{VH} = \left(1 - \frac{M(e)}{\pi}\right) T_o \quad (2)$$

Where $M(e)$ is the mean anomaly of the orbit with the given eccentricity, e. According to [38], with a limit of elevation angle (ε_{min}) above the horizon visibility time will be reduced by a factor denoted as f_{min} which can be determined as;

$$f_{min} = 1 - \frac{2}{\pi}(\varepsilon_{min}) \quad (3)$$

Hence;

$$\Delta t_{VHR} \leq \Delta t_{VH}(f_{min}) \quad (4)$$

$$\Delta t_{VHR} \leq (f_{min})\left(1 - \frac{M(e)}{\pi}\right) T_o \quad (5)$$

In the situation where T_o is not given, it can be computed as follows;

$$T_o = 2\pi\sqrt{\frac{a^3}{\mu}} \quad (6)$$

Where $\mu = 398600\ Km^3/s^2$, a denotes orbital semi-major axis which is defined in terms of radius of periapsis (R_p) and radius of apoapsis (R_a) as follows;

$$a = \frac{R_a + R_p}{2} \quad (7)$$

The visibility time in minutes (Δt_{VHR_min}) and hours (Δt_{VHR_hr}) are computed as follows;

$$\Delta t_{VHR\_min} = \frac{\Delta t_{VHR}}{60} \quad (8)$$

$$\Delta t_{VHR\_hr} = \frac{\Delta t_{VHR}}{3600} \quad (9)$$

The visibility time can be computed using the algorithm given below.

Algorithm for Computing Satellite Visibility Time for the case of Highly Eccentric Elliptical Orbit

1. Define $\mu = 398600$ // unit $\frac{Km^3}{s^2}$
2. Input R_p, R_a // Radius of periapsis and apoapsis, unit Km
3. Input e // eccentricity, no unit
4. Input ε_{min} // Minimum elevation angle, unit radian
5. Compute mean anomaly
   $M(e) = 2\left(\tan^{-1}\left(\sqrt{\frac{(1-e)}{(1+e)}}\right)\right) - \left(e\left(\sqrt{(1-e^2)}\right)\right)$ // unit, radian
6. Compute $a = \frac{R_a + R_p}{2}$ // unit, Km
7. Compute $T_o = 2\pi\sqrt{\frac{a^3}{\mu}}$ // Orbital period, unit, seconds
8. Compute $\Delta t_{VS} = \left(1 - \frac{M(e)}{\pi}\right) T_o$ // Visibility time without restriction on elevation angle, unit, seconds
9. Compute $f_{min} = 1 - \frac{2}{\pi}(\varepsilon_{min})$ // Unit , Radian
10. Compute $\Delta t_{VS} = (f_{min})\left(1 - \frac{M(e)}{\pi}\right) T_o$ // Visibility time with restriction of minimum of ε_{min} elevation angle, unit, seconds
11. Compute $\Delta t_{VHR\_min} = \frac{\Delta t_{VS}}{60}$ // Visibility in minutes
12. Compute $\Delta t_{VHR\_hr} = \frac{\Delta t_{VS}}{3600}$ // Visibility in hours
13. Output e, R_p, R_a, ε_{min}, Δt_{VS}, t_{VHR_min}, Δt_{VHR_hr}
14. End

Note, if the orbital period, T_o is given, then R_p and R_a are not required. In this case, in step 2, Input R_p, R_a will be replaced with Input T_o while step 6 and step 7 will be omitted.

3. THE CASE STUDY SATELLITE: MOLNIYA 1-36

The case study highly eccentric orbit satellite is MOLNIYA 1-36. The launch details and orbital parameters of MOLNIYA 1-36 satellite is given in Table 1.The orbital view of the MOLNIYA 1-36 as simulated by https://celestrak.com/ online tool is shown in Figure 1. The graph plot of changes in orbital mean altitude with time is shown in Figure 2, the graph plot of changes in orbital inclination with time is shown in Figure 3 and the graph plot of changes in orbital mean altitude with time is shown in Figure 4. The graph of Figure 2, Figure 3 and Figure 4 are based on MOLNIYA 1-36 tracking data obtained from March 2019 to March 2022.

Table 1 Launch details and orbital parameters of MOLNIYA 1-36 satellite

Launched	24 March 1977
Category	Molniya
Launch site	Plesetsk Missile and Space Complex, Russia
Owner	Commonwealth of Independent States (former USSR)
NORAD ID	9880
COSPAR ID	1977-021A
Current orbital elements	**Parameter Value**
Inclination,	**63.288°**
Eccentricity,	**0.72625**
RA ascending node,	**18.753 hr**
Argument perihelion,	**289.477°**
Mean anomaly,	**9.694°**
Orbital period,	**718.188 min**
Epoch of osculation,	**17 Mar 2022m, 05:06**

Derived Quantities	
Current orbital elements	**Parameter Value**
Min Altitude	**901.4 km**
Mean Altitude	**20194.6 km**
Peak Altitude	**39487.7 km**

(Source: satellite https://in-the-sky.org/spacecraft.php?id=9880)

In order to examine how the changes in orbital parameters can possibly affect the visibility time, the time stamped changes in eccentricity and orbital altitude are matched in Figure 5 and four points are considered , namely,

i. Point A with the minimum orbital mean altitude
ii. Point B with the maximum orbital mean altitude
iii. Point C with the minimum eccentricity
iv. Point D with the maximum eccentricity

The values of the eccentricity and orbital altitude for the MOLNIYA 1-36 satellite at the four selected points along with the mean eccentricity and mean orbital attitude are shown in Table 2. The study computed the satellite visibility time for the five pairs of data presented in Table 5. Furthermore, the study also considered the effect of imposing a minimum elevation angle (ε_{min}) on the satellite visibility time.

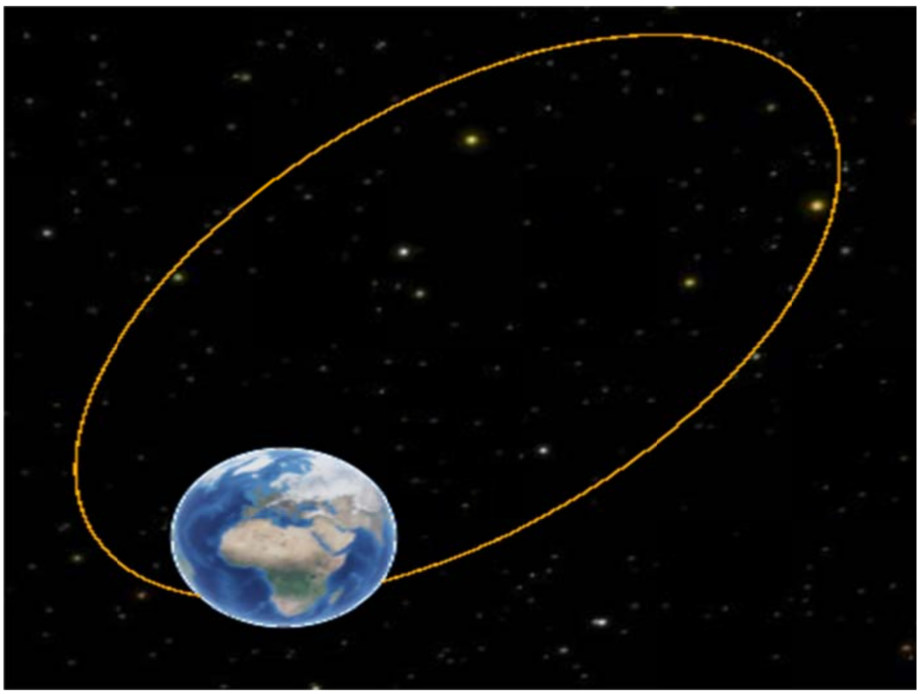

Figure 1 The orbital view of the MOLNIYA 1-36 as simulated by https://celestrak.com/ online tool ***(Source https://celestrak.com/cesium/orbit-viz.php?tle=/NORAD/elements/gp.php?INTDES=1977-021&satcat=/pub/satcat.txt&orbits=20)***

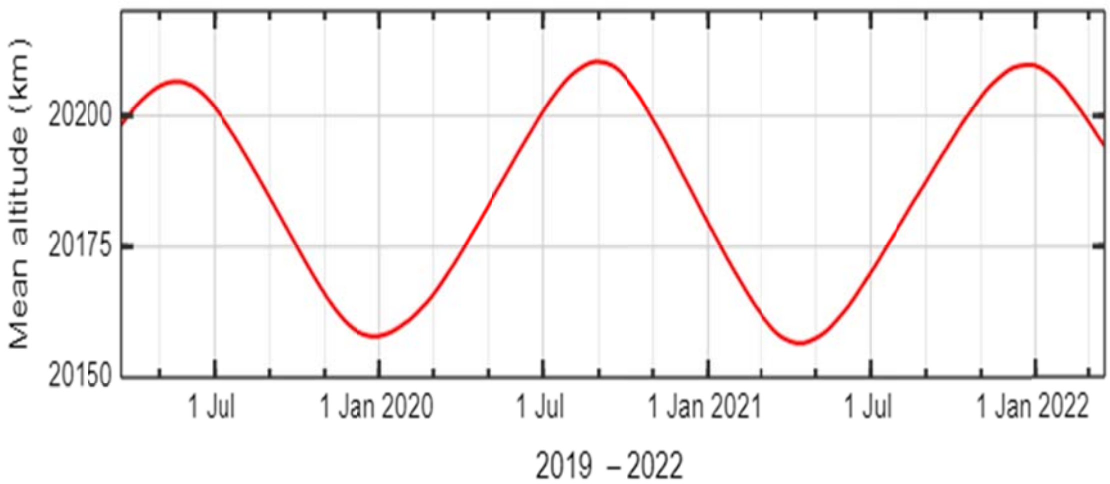

Figure 2 The graph plot of changes in orbital mean altitude with time for the MOLNIYA 1-36 satellite (Source: satellite https://in-the-sky.org/spacecraft.php?id=9880)

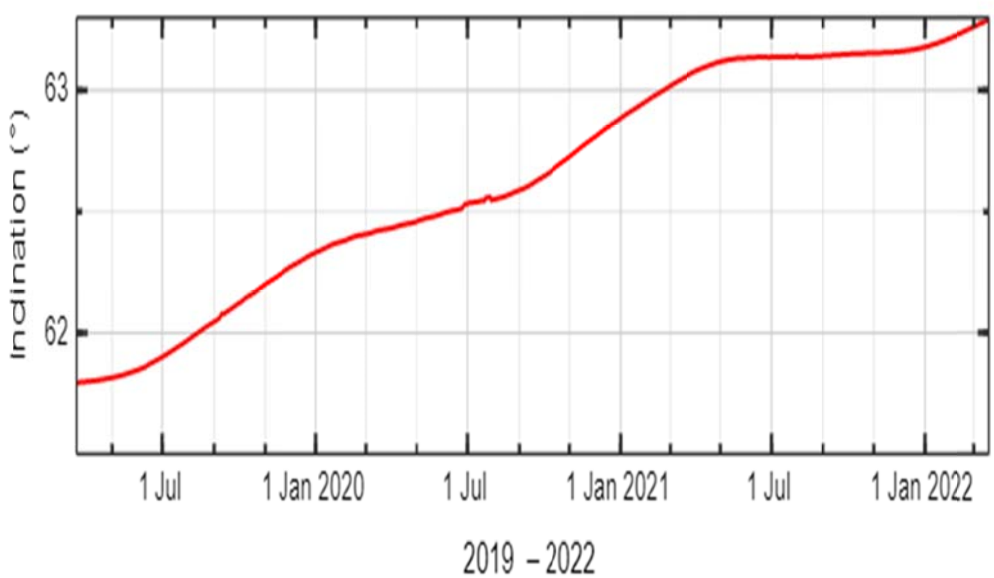

Figure 3 The graph plot of changes in orbital inclination with time for the MOLNIYA 1-36 satellite (Source: satellite https://in-the-sky.org/spacecraft.php?id=9880)

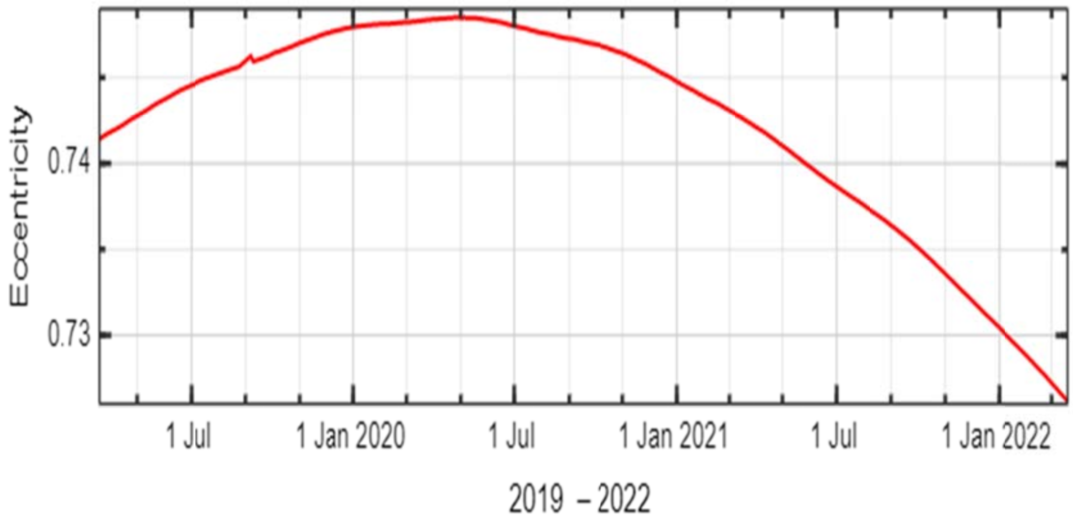

Figure 4 The graph plot of changes in orbital mean altitude with time for the MOLNIYA 1-36 satellite (Source: satellite https://in-the-sky.org/spacecraft.php?id=9880)

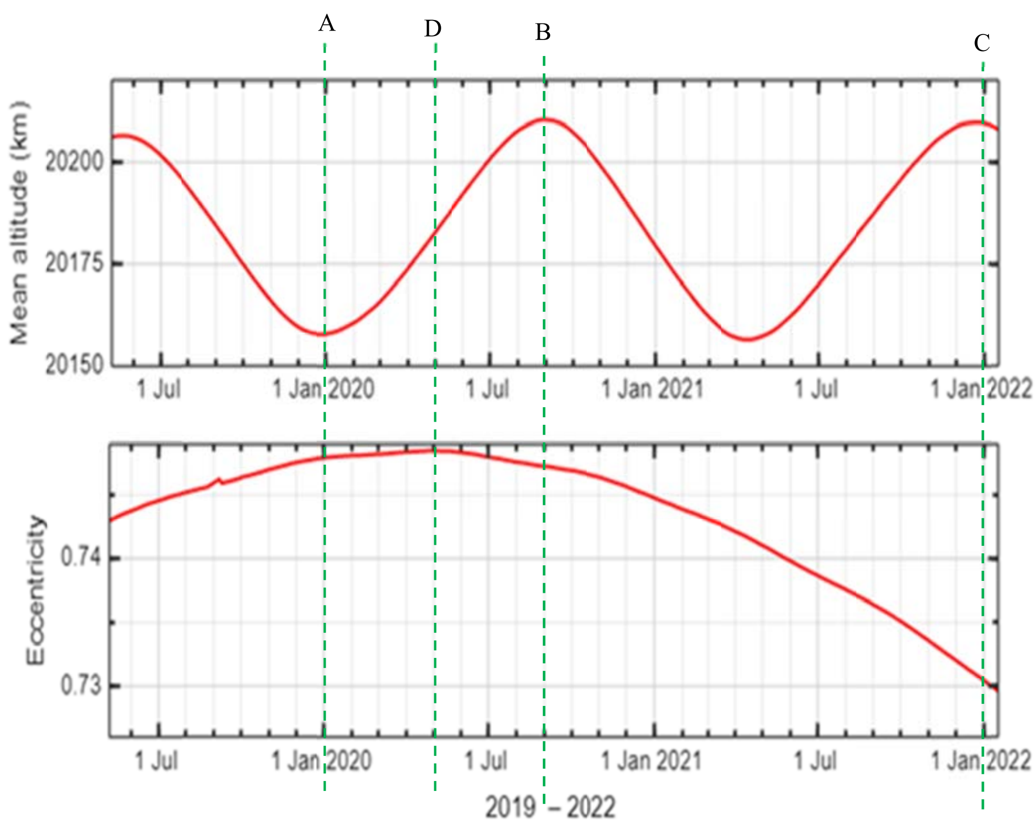

Figure 5 The four points considered the time stamped changes in eccentricity and orbital altitude for the MOLNIYA 1-36 satellite

Table 2 The values of the eccentricity and orbital altitude for the MOLNIYA 1-36 satellite at the four selected points in Figure 5 along with the mean eccentricity and mean orbital attitude.

Point	Eccentricity,	Orbital Altitude
A	0.74800	20160.0
B	0.74700	20216.0
C	0.75000	20184.0
D	0.73100	20214.0
Mean	0.72625	20194.6

4. RESULTS AND DISCUSSION

The results of the visibility time for Molniya 1-36 satellite using the mean orbital eccentricity of 0.72625 and mean orbital altitude of 20194.6 km for minimum elevation angle (ε_{min}) ranging from 0° to 15° are given in Table 3 and Figure 6. The results in table 3 and Figure 6 show that by using the mean orbital eccentricity of 0.72625 and mean orbital altitude of 20194.6 km , the visibility time is highest with a value of 10.989 hours at 0° minimum elevation angle and the visibility time is lowest at a value of 9.157 hours at 15° minimum elevation angle

From the graph plot in Figure 6, the linear relationship between the visibility time of Molniya 1-36 satellite and the ε_{min} is given by trend line analysis as follows;

Visibility time hr)= -0.1221(εmin°) + 10.989 (8)

The results of the visibility time for Molniya 1-36 satellite using the mean orbital eccentricity and orbital altitude four selected points along with the mean eccentricity and mean orbital attitude for minimum elevation angle (ε_{min}) ranging of 0° are given in Table 4 and Figure 7. The results in Table 3 and Figure 7 show that point B with eccentricity of 0.74700 and orbital altitude of 20216.0 km has the highest visibility time of 666.5381 minutes and highest orbital period of 719.3478 minutes. On the other hand, point A with eccentricity of 0.74800 and orbital altitude of 20160.0 km has the lowest orbital period of 20160.0 minutes and is the third in the visibility time (ranking from highest) with a value of 664.7371 minutes. Furthermore, the mean values with eccentricity of 0.72625 and orbital altitude of 20194.6 km has the lowest visibility time of 659.3155 minutes and is the third in the orbital period (ranking from highest) with a value of 718.4797 minutes. In all, the results obtained from the study show that the orbital elements values do change with time in the course of the orbital motion of the satellites and the changes do affect the instantaneous values of the derived parameters. Also, placing a limit to the minimum elevation angle for the satellite do also reduce the visibility time of the satellite.

Table 3 The results of the visibility time for Molniya 1-36 satellite using the mean orbital eccentricity of 0.72625 and mean orbital altitude of 20194.6 km for minimum elevation angle (ε_{min}) ranging from 0° to 15°

Minimum elevation angle, εmin (°)	Orbital Period, To (min)	Mean Anomaly, M€ radian	Reduction factor (fmin)	Molniya 1-36 visibility time in sec	Molniya 1-36 visibility time in hour
0	718.4797	0.258699	1	39558.93	10.989
2	718.4797	0.258699	0.977778	38679.84	10.744
5	718.4797	0.258699	0.944444	37361.21	10.378
10	718.4797	0.258699	0.888889	35163.49	9.768
15	718.4797	0.258699	0.833333	32965.77	9.157

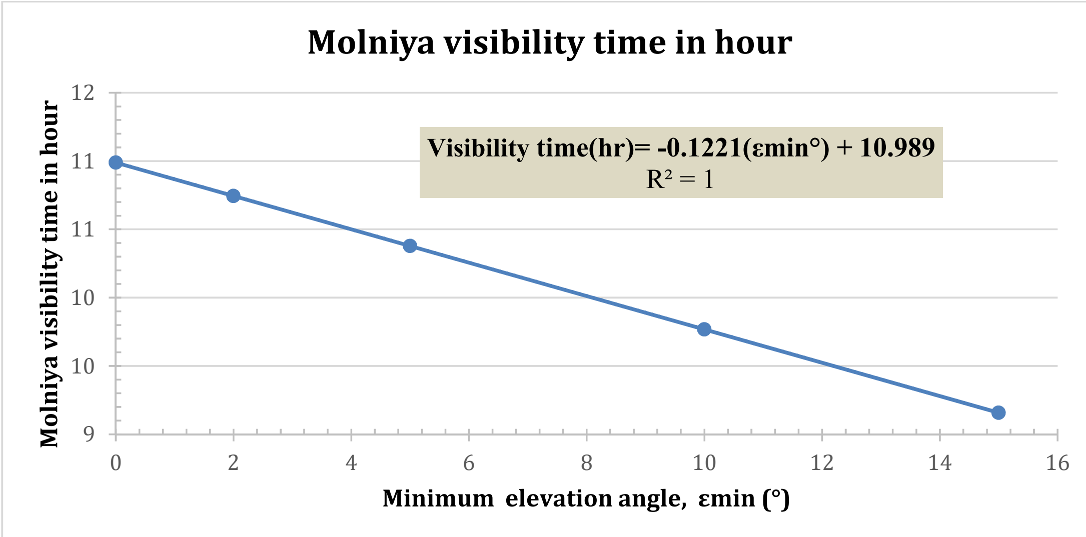

Figure 6 The graph of visibility time for Molniya 1-36 satellite versus minimum elevation angle (ε_{min}) using the mean orbital eccentricity of 0.72625 and mean orbital altitude of 20194.6 km

Table 4 The results of the visibility time for Molniya 1-36 satellite using the mean orbital eccentricity and orbital altitude four selected points along with the mean eccentricity and mean orbital attitude for minimum elevation angle (ε_{min}) ranging of 0°

Point	Eccentricity, e	Orbital Altitude, h (km)	Orbital Period, To (min)	Molniya visibility time, tv (min)	Normalised Orbital Period, To (min)	Normalised visibility time, tv (min)
A	0.74800	20160.0	717.0768	664.7371	99.80	100.82
B	0.74700	20216.0	719.3478	666.5381	100.12	101.10
C	0.75000	20184.0	718.0498	666.2448	99.94	101.05
D	0.73100	20214.0	719.2666	661.5273	100.11	100.34
Mean	0.72625	20194.6	718.4797	659.3155	100.00	100.00

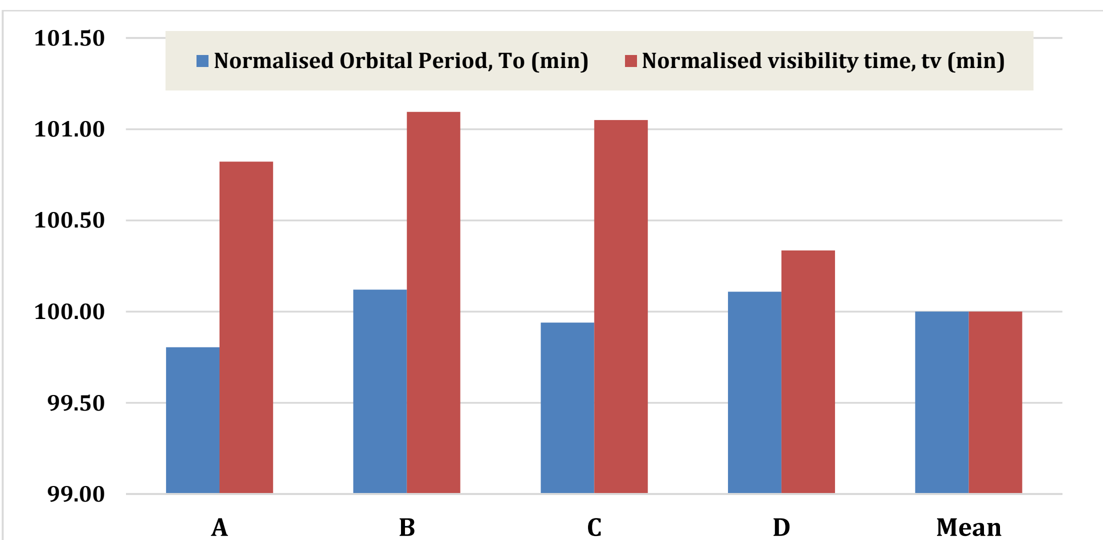

Figure 7 The bar chart of the normalised orbital period and normalised visibility time time for Molniya 1-36 satellite using the mean orbital eccentricity and orbital altitude four selected points along with the mean eccentricity and mean orbital attitude for minimum elevation angle (ε_{min}) ranging of 0°

5. CONCLUSION

The algorithm for a program for computing the visibility time for orbits that has high eccentricity and is elliptical in shape is presented. In order to demonstrate the applicability of the algorithm, some sample numerical examples were presented using Molniya 1-36 satellite as the case study. The algorithm enables the computation of the visibility time of the sample satellite when there are different values of minimum elevation angles. Also, the computation was conducted for different instantaneous values of the orbital eccentricity and orbital altitudes.

REFERENCES

1. Kos, A., Milutinović, V., & Umek, A. (2019). Challenges in wireless communication for connected sensors and wearable devices used in sport biofeedback applications. *Future generation computer systems*, *92*, 582-592.
2. Simeon, Ozuomba. (2016) "Comparative Analysis Of Rain Attenuation In Satellite Communication Link For Different Polarization Options." *Journal of Multidisciplinary Engineering Science and Technology (JMEST) Vol. 3 Issue 6, June - 2016*
3. Morocho-Cayamcela, M. E., Lee, H., & Lim, W. (2019). Machine learning for 5G/B5G mobile and wireless communications: Potential, limitations, and future directions. *IEEE Access*, *7*, 137184-137206.
4. Johnson, Enyenihi Henry, Simeon Ozuomba, and Ifiok Okon Asuquo. (2019). Determination of Wireless Communication Links Optimal Transmission Range Using Improved Bisection Algorithm. *Universal Journal of Communications and Network, 7(1), 9-20.*
5. Simeon, Ozuomba (2014) "Fixed Point Iteration Computation Of Nominal Mean Motion And Semi Major Axis Of Artificial Satellite Orbiting An Oblate Earth." *Journal of Multidisciplinary Engineering Science and Technology (JMEST) Vol. 1 Issue 4, November – 2014*
6. Arefin, M. T., Ali, M. H., & Haque, A. F. (2017). Wireless body area network: An overview and various applications. *Journal of Computer and Communications*, *5*(7), 53-64.
7. Meister, T., Ellinger, F., Bartha, J. W., Berroth, M., Burghartz, J., Claus, M., ... & Zhang, Z. (2017, November). Program FFlexCom—High frequency flexible bendable electronics for wireless communication systems. In *2017 IEEE International Conference on Microwaves, Antennas, Communications and Electronic Systems (COMCAS)* (pp. 1-6). IEEE.
8. Simeon, Ozuomba. (2016) "Comparative Analysis Of Rain Attenuation In Satellite Communication Link For Different Polarization Options." *Journal of Multidisciplinary Engineering Science and Technology (JMEST) Vol. 3 Issue 6, June – 2016*
9. Mahendran, R. (2016, May). Integrated LiFi (Light Fidelity) for smart communication through illumination. In *2016 International Conference on Advanced Communication Control and Computing Technologies (ICACCCT)* (pp. 53-56). IEEE.
10. Simeon, Ozuomba. (2017). "Determination Of The Clear Sky Composite Carrier To Noise Ratio For Ku-Band Digital Video Satellite Link" *Science and Technology Publishing (SCI & TECH) Vol. 1 Issue 7, July – 2017*
11. Chi, N., Zhou, Y., Wei, Y., & Hu, F. (2020). Visible light communication in 6G: Advances, challenges, and prospects. *IEEE Vehicular Technology Magazine*, *15*(4), 93-102.
12. Kalu, C., Ozuomba, Simeon. & Udofia, K. (2015). Web-based map mashup application for participatory wireless network signal strength mapping and customer support services. *European Journal of Engineering and Technology, 3 (8)*, 30-43.
13. Samuel, Wali, Simeon Ozuomba, and Philip M. Asuquo (2019). EVALUATION OF WIRELESS SENSOR NETWORK CLUSTER HEAD SELECTION FOR DIFFERENT PROPAGATION ENVIRONMENTS BASED ON LEE PATH LOSS MODEL AND K-MEANS ALGORITHM. EVALUATION, 3(11). *Science and Technology Publishing (SCI & TECH) Vol. 3 Issue 11, November - 2019*
14. Zeinab, K. A. M., & Elmustafa, S. A. A. (2017). Internet of things applications, challenges and related future technologies. *World Scientific News*, *67*(2), 126-148.
15. Govindraj, V., Sathiyanarayanan, M., & Abubakar, B. (2017, August). Customary homes to smart homes using Internet of Things (IoT) and mobile application. In *2017 International Conference On Smart Technologies For Smart Nation (SmartTechCon)* (pp. 1059-1063). IEEE.
16. Samuel, W., Ozuomba, Simeon, & Constance, K. (2019). SELF-ORGANIZING MAP (SOM) CLUSTERING OF 868 MHZ WIRELESS SENSOR NETWORK NODES BASED ON EGLI PATHLOSS MODEL COMPUTED RECEIVED SIGNAL STRENGTH. *Journal of Multidisciplinary Engineering Science and Technology (JMEST) Vol. 6 Issue 12, December - 2019*
17. Njoku, Felix A., Ozuomba Simeon, and Fina Otosi Faithpraise (2019). Development Of Fuzzy Inference System (FIS) For Detection Of Outliers In Data Streams Of Wireless Sensor Networks. *International Multilingual Journal of Science and Technology (IMJST) Vol. 4 Issue 10, October - 2019*
18. Yaqoob, I., Hashem, I. A. T., Mehmood, Y., Gani, A., Mokhtar, S., & Guizani, S. (2017). Enabling communication technologies for smart cities. *IEEE Communications Magazine*, *55*(1), 112-120.

19. Zhao, X., Askari, H., & Chen, J. (2021). Nanogenerators for smart cities in the era of 5G and Internet of Things. *Joule*, *5*(6), 1391-1431.
20. Simeon, Ozuomba. (2020). "APPLICATION OF KMEANS CLUSTERING ALGORITHM FOR SELECTION OF RELAY NODES IN WIRELESS SENSOR NETWORK." *International Multilingual Journal of Science and Technology (IMJST) Vol. 5 Issue 6, June – 2020*
21. Priyanka, E. B., & Thangavel, S. (2020). Influence of Internet of Things (IoT) In Association of Data Mining Towards the Development Smart Cities-A Review Analysis. *Journal of Engineering Science & Technology Review*, *13*(4).
22. Ahmed, E., Yaqoob, I., Gani, A., Imran, M., & Guizani, M. (2016). Internet-of-things-based smart environments: state of the art, taxonomy, and open research challenges. *IEEE Wireless Communications*, *23*(5), 10-16.
23. Simeon, Ozuomba. (2020). "Analysis Of Effective Transmission Range Based On Hata Model For Wireless Sensor Networks In The C-Band And Ku-Band." *Journal of Multidisciplinary Engineering Science and Technology (JMEST) Vol. 7 Issue 12, December – 2020*
24. Rosengren, A. J., Skoulidou, D. K., Tsiganis, K., & Voyatzis, G. (2019). Dynamical cartography of Earth satellite orbits. *Advances in Space Research*, *63*(1), 443-460.
25. Grewal, M. S., Andrews, A. P., & Bartone, C. G. (2020). *Global navigation satellite systems, inertial navigation, and integration*. John Wiley & Sons.
26. Maral, G., Bousquet, M., & Sun, Z. (2020). *Satellite communications systems: systems, techniques and technology*. John Wiley & Sons.
27. Sharma, S. K., Chatzinotas, S., & Ottersten, B. (2016). In-line interference mitigation techniques for spectral coexistence of GEO and NGEO satellites. *International Journal of Satellite Communications and Networking*, *34*(1), 11-39.
28. Azzari, G., & Lobell, D. B. (2017). Landsat-based classification in the cloud: An opportunity for a paradigm shift in land cover monitoring. *Remote Sensing of Environment*, *202*, 64-74.
29. Nassar, R., McLinden, C., Sioris, C. E., McElroy, C. T., Mendonca, J., Tamminen, J., ... & Wunch, D. (2019). The atmospheric imaging mission for northern regions: AIM-North. *Canadian journal of remote sensing*, *45*(3-4), 423-442.
30. Blake, J. A., Chote, P., Pollacco, D., Feline, W., Privett, G., Ash, A., ... & Watson, C. (2021). DebrisWatch I: A survey of faint geosynchronous debris. *Advances in Space Research*, *67*(1), 360-370.
31. Oltrogge, D. L., Alfano, S., Law, C., Cacioni, A., & Kelso, T. S. (2018). A comprehensive assessment of collision likelihood in Geosynchronous Earth Orbit. *Acta Astronautica*, *147*, 316-345.
32. Reid, T. G., Walter, T., Enge, P. K., & Sakai, T. (2016). Orbital representations for the next generation of satellite-based augmentation systems. *GPS solutions*, *20*(4), 737-750.
33. Liu, F., Lu, S., & Sun, Y. (2019). *Guidance and Control Technology of Spacecraft on Elliptical Orbit*. Berlin: Springer.
34. Truchly, P., & Cervienka, J. (2016, November). Simple visualization tool for analysis of satellite orbits and constellations. In *2016 International Conference on Emerging eLearning Technologies and Applications (ICETA)* (pp. 359-364). IEEE.
35. Alessi, E. M., Buzzoni, A., Daquin, J., Carbognani, A., & Tommei, G. (2021). Dynamical properties of the Molniya satellite constellation: Long-term evolution of orbital eccentricity. *Acta Astronautica*, *179*, 659-669.
36. Capderou, M. (2002). *Satellites: orbites et missions*. Springer Science & Business Media.
37. Chapter 5 Orbit and Ground Track of a Satellite available at : http://fgg-web.fgg.uni-lj.si/~/mkuhar/Pouk/SG/Seminar/Vrste_tirnic_um_Zemljinih_sat/Orbit_and_Ground_Track_of_a_Satellite-Capderou2005.pdf
38. Morgan-Jones, I., & Loskot, P. (2019). Regional Coverage Analysis of LEO Satellites with Kepler Orbits. *arXiv preprint arXiv:1910.10704*.