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Using Linear Regression Technique to Infer Edge Length Relationship Between Regular Polyhedron and Its Corresponding Dual Polyhedron

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***Abstract*—The combination of a regular polyhedron and its corresponding dual polyhedron has good spatial accumulation characteristics, so many scholars and related manufacturers have been constantly exploring the laws behind them, and had calculated the relevant geometric properties of each regular polyhedron. But there is no relevant data for reference on the combination relationship between regular polyhedron and its corresponding dual polyhedron. Especially when the model is built in parametric form, the dimensional characteristics that have been deduced in the past cannot be used directly, thus hindering the development of related products. Therefore, in this paper, based on the size of the side length required by the parametric modeling method, the optimization and linear regression techniques are used to deduce the side length relationship between each regular polyhedron and its corresponding dual polyhedron, so as to facilitate related products development proceeds.**

Keywords—Linear Regression*; *polyhedron

I. INTRODUCTION

The record of polyhedrons and other mathematical models can first be seen in the ancient Greek era (400 BC). Initially, they only focused on the basic convex regular polyhedron, which is the familiar "Platonic Polyhedron". That is, Tetrahedron, Hexahedron or Cube, Octahedron, Dodecahedron, and Icosahedron. In the last proposition of his book "Elements", Euclid also proved that there are only five types of convex regular polyhedron. Archimede extended his research to more uniform convex polyhedron, a total of 13, later known as "Archimede Polyhedron". Because regular polyhedrons and truncated regular polyhedrons derived from regular polyhedrons have good spatial accumulation properties, they are often used as periodic boundary conditions for molecular simulation calculations. Therefore, the study of polyhedrons has been a part of the study of mathematical structure since two thousand years, and it is also the source of inspiration for many branches of the subject [1].

In terms of architectural engineering, the earliest polyhedral structure is the shape of a stone cave, which shows various symmetrical polyhedrons. The most representative cultural relic is the Egyptian pyramid in 2560 BC. And so far, there have been many unusual polyhedral structures, which have been widely used in buildings all over the world [2]. Several typical polyhedral buildings, such as Toyo Ito Architecture Museum, is a steel-concrete mixed building composed of 4 types of polyhedrons in a crystalline structure mode. It is composed of many independent spaces, and each room is decorated with different materials. Through different display methods, visitors can appreciate the space display of different aspects [3]. Another example is the National Library of Belarus, which is composed of 18 squares and 8 triangles. It was built with reference to the Da Vinci polyhedron and was established on September 15, 1922. A new hall was opened on June 16, 2006. It is one of the largest libraries in the world and the largest library in Eastern Europe [4]. And this thesis intends to use the geometric patterns of these type of regular polyhedrons and their dual polyhedrons to construct the modular creative products.

The analysis of the optimization method can be traced back to the Newton, Lagrange and Cauchy ages. The use of the "Calculus of Variation" method as the basis for the optimization problem was laid by Bernoulli, Euler, Lagrange and Weirstrass. This calculus of variable method is significantly helpful in solving specific problems. For example: static and steady-state analysis of objects and vibration analysis of objects. However, due to the time-consuming process of optimization calculations, progress was slow before the middle of the twentieth century, and many tedious calculations were not solved until the advent of computers. Among them, the development of Linear Programming (LP) is regarded as one of the most important scientific advances of the century.

The optimization algorithm is often used to find the most suitable parameter combination, so that its efficiency and results can meet the requirements of users, and find its suitable algorithm for different types of problems. The fields of application span mathematics, applied sciences, economics, statistics, and even medicine. The general optimization method is to infer the most suitable result through continuous numerical analysis. Therefore, for low-dimensional optimization problems, most of them can have good performance, but for high-dimensional problems with more than 20 or 30 variables, the results are often unsatisfactory. The current analysis optimization methods can be roughly divided into three types: Numerical Method, Enumerative and Random Search. Among them, the numerical method is the most important. It is based on the calculus of mathematics and searches for the best solution in a specific space by seeking extreme values. This is the spirit of the "Hill Climbing Algorithm". That is to say, it has only one or several starting points, and the next iteration value is generated according to the established mathematical model, and the calculation is repeated until the best solution is found. Therefore, it may converge to the local optimal solution, but cannot reach the global optimal solution [5].

Compared with numerical methods, the heuristic algorithm of artificial intelligence technology is to imitate different natural phenomena respectively, using the concept of random search method to randomly select many starting points, and search for the best solution at the same time. Each individual in the ethnic group has a search point. After the evolution of generations, each search point approaches the best solution direction [6]. In this paper, it is only necessary to deduce the relative length of various polyhedrons to dual polyhedrons through optimized analysis. Therefore, the "Hill Climbing Algorithm" is the most efficient way.

The regression analysis method is to use statistical methods to fit the data, by analyzing the characteristics of each variable, and approximating the variables according to the weighting direction or the part with the smallest error. In various physical and statistical problems, some data are obtained by observing or experimenting on the relevant quantities for many times. They are scattered and not only inconvenient to deal with, but usually cannot accurately and fully reflect their inherent laws. In order to obtain the inherent law between the data or use the current data to predict the expected data, it is necessary to use a continuous curve to approximately describe or compare the functional relationship between the coordinates represented by the discrete point groups on the plane. Regression analysis can be roughly divided into Linear Regression and Nonlinear Regression. Linear regression [7], also known as linear fitting method, is a regression analysis that uses a least squares function called linear regression equation to model the relationship between one or more independent variables and dependent numbers. This function is a linear combination of one or more model parameters called regression coefficients. If there is only one independent variable, it is called simple linear regression, and if there is more than one independent variable, it is called multivariable linear regression. While linear regression models are often fitted by least squares approximation, they may also be fitted by other methods, such as least absolute error regression. The least squares approximation can also be used to fit those nonlinear models. Therefore, although "least squares" and "linear regression" are closely related, they cannot be equated.

Parametric design is a design method that defines parameter rules and related processes as the design basis. Unlike traditional designers who are accustomed to directly determine the form and shape of design through experience based on specific design conditions, the design process of parametric design is not directly dealing with "shape", but the "logic" behind the design. The design method that controls the various parameter factors that affect the design and the correlation between each parameter to produce, evaluate, and adjust the geometric form of the design plan in real time. Parametric digital modeling through the acquisition and analysis of digital data, combined with computer-aided manufacturing tools such as RP or CNC, can quickly view the design, greatly improving the efficiency and feasibility of non-traditional products design. This digital continuum produced by the integrated application of model software to manufacturing hardware has gradually driven the comprehensive innovation of contemporary digital design concepts to production methods [8]. In this paper, "Creo Parametric" modeling software is used as the construction software for all kinds of polyhedrons. At the same time, this software is also used as a tool for subsequent relative dual polyhedrons construction.

II. Geometric structure of regular polyhedron and its dual polyhedron

Regular polyhedron refers to the various faces of a convex polyhedron, which are composed of the same regular polygon. Therefore, based on the original definition of regular polyhedrons, four other equivalent properties can be re-derived:

(1) Each vertex of a regular polyhedron is connected with the same number of edges.
(2) Each vertex of a regular polyhedron connects the same number of faces.
(3) Each vertex of the regular polyhedron is located on the same sphere.
(4) The angle between each face of a regular polyhedron and the face is equal.

There are only five types of regular polyhedrons that can meet the above conditions: regular tetrahedron, regular hexahedron, regular octahedron, regular dodecahedron, and regular icosahedron; that is, the so-called Platonic polyhedron. The vertex of the corresponding dual polyhedron is the correspondence face of the original polyhedron, and the face of the dual polyhedron is the correspondence vertex of the original polyhedron. In addition, the edge defined by adjacent vertices can correspond to two adjacent faces, and the intersection line of these faces also defines an edge line of the dual polyhedron. Table 1 is a summary table of the dual polyhedrons corresponding to the Platonic polyhedrons.

TABLE 1 The Dual Polyhedron Corresponding to The Platonic Polyhedron

Platonic polyhedron	dual polyhedron
regular tetrahedron	regular tetrahedron
regular hexahedron	regular octahedron
regular octahedron	regular hexahedron
regular dodecahedron	regular icosahedron
regular icosahedron	regular dodecahedron

III. APPLICATION OF OPTIMAL ANALYSIS TECHNIQUES

In the field of optimization research, many algorithms have been proposed, such as hill climbing algorithm, simulated annealing method, genetic algorithm, tabu search method, ant colony algorithm, particle swarm algorithm and so on. If the number of particles is used to distinguish, the above algorithms can be divided into two types: "single-particle type" and "multi-particle type". Among them, "hill-climbing algorithm", "simulated annealing method", and "taboo search method" belong to the "single-particle algorithm", while "genetic algorithm", "ant colony algorithm", "particle swarm algorithm", " The bee colony algorithm" are the multi-particle algorithm. Although most of the current academic research focuses on multi-particle algorithms, it is difficult to analyze the quality of these algorithms, which is caused by the inherent complexity of multi-particle systems. In this article, we only need to deduce the relative length of various polyhedrons to the side polyhedron through optimization analysis. Therefore, a well-known, simple and fast basic algorithm-"hill climbing algorithm" is adopted. As the optimized calculation tool in this article.

A. Hill-Climbing Algorithm，HC

The so-called "hill-climbing algorithm" is a simple regional search algorithm in the single-particle algorithm. Since its process is quite similar to the continuous upward movement of humans when climbing a mountain, it is called a hill-climbing algorithm. The hill-climbing algorithm can be said to be a heuristic method. The search strategy is to constantly find the best solution around, and then continue to move on to the best solution until it can no longer be improved. The implementation is quite easy and the execution speed is very fast. Therefore, it is often used as a benchmark for comparison of various optimization algorithms. And when the problem to be asked has multiple parameters, we can increase or decrease the value of a certain parameter by one unit in turn in the process of gradually obtaining the optimal solution through the hill climbing method. For example, the solution of a certain problem needs to use three integer-type arguments x1, x2, x3. At the beginning, set these three arguments to (2, 2, -2), increase/decrease x1 by 1, and get Two solutions (1,2, -2), (3, 2,-2); increase/decrease x2 by 1, and get two solutions (2,3, -2), (2,1, -2); Increase/decrease x3 by 1, and get two solutions (2,2,-1), (2,2,-3), so you get a solution set: (2,2,-2), (1, 2 ,-2), (3, 2,-2), (2,3,-2), (2,1,-2), (2,2,-1), (2,2,-3). Find the optimal solution from the above solution set, and then construct a solution set for this optimal solution according to the above method, and then find the optimal solution. In this way, the "hill-climbing" operation process ends until the optimal solution of the previous time and the optimal solution of the next time are the same. However, after this algorithm falls into the best solution in the area, it cannot jump out, that is, it cannot find a better solution. This is because the hill climbing algorithm only finds neighboring points for comparison, and does not allow walking in the worse direction, which makes climbing hills. In the more complex nonlinear programming problems, the algorithm is easy to fall into a poor area, and it is difficult to find the best solution in the whole domain.

Therefore, some experts have devised a jumping strategy that increases with the number of failures. This method is called Hill-Climbing with Jumping strategies (HCJ). The jumping mechanism makes it easier for the mountain climbing algorithm to leave the valley and find more good solution quickly. The difference between HCJ and the traditional hill-climbing algorithm is that the jumping steps of HCJ will increase with the number of failures. Therefore, when the particle is at the bottom of the valley, it will cause continuous jumping failure. At this time, HCJ will randomly increase the jumping range. By increasing the range, HCJ will have the opportunity to jump out of the current valley and move to a lower valley, allowing HCJ to find a better solution. In practice, in order to overcome the failure caused by the border area, then randomly select individuals from the range to make adjustments, and the size of the adjustment range is determined by random steps. When this neighbor selection method is used in the hill climbing algorithm, the individual can adjust in two directions, and there is a half chance that it will adjust in the correct direction. Therefore, the hill-climbing algorithm can be expressed by the following function:

```
Algorithm Hill-Climbing(pi)
    p = pi;
     while not isEnd( )
      pn = move(p);
      if pn.energy() <= p.energy()
      p = pn;
End Algorithm
```

The entire execution flow of the hill-climbing algorithm is shown in Figure 1.

B. Tetrahedron's Dual Polyhedron Optimization Execution Result

As mentioned earlier, this article intends to use the hill-climbing algorithm to deduce the relative length of the sides of the various polyhedrons. Here for the regular tetrahedron; suppose the side length of the regular tetrahedron is 100, and the initial side length of the dual regular tetrahedron is 50, The side length of the dual regular tetrahedron is the "independent variable", and the distance between the end point of the dual regular tetrahedron and the plane of the regular tetrahedron "Dist = 0" is the objective function, the initial assembled state is shown in Figure 2. After the optimization calculation, when the side length of the dual regular tetrahedron is 33.3338, the objective function requirement can be met. The convergence process is shown in Figure 3, and the result is shown in Figure 4.

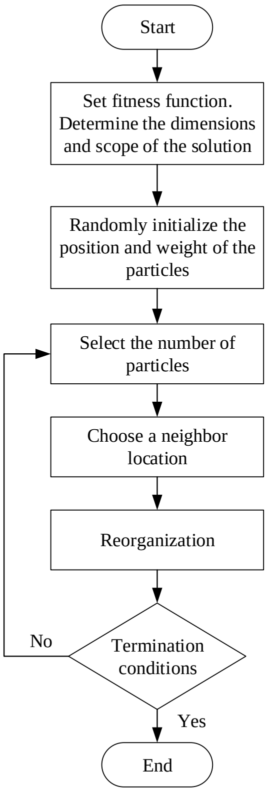

Figure 1 The entire execution flow of the hill-climbing algorithm

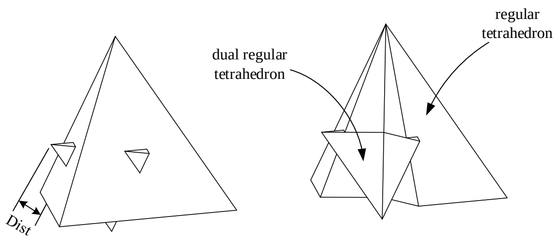

Figure 2 The initial assembled state

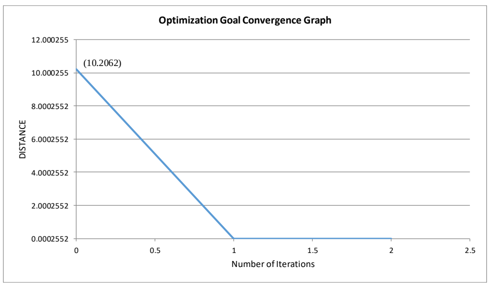

Figure 3 The convergence process

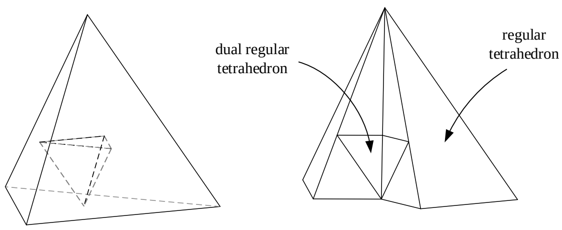

Figure 4 The result assembled state

IV. APPLICATION OF REGRESSION ANALYSIS

Regression analysis is the general term for statistical techniques that use one variable or a set of variables to predict another variable. The predicted variable is called the dependent variable, and the variable used for prediction is called the independent variable. Regression analysis that predicts dependent variable based on only one independent variable is called "simple regression", and if there are two or more independent variables, it is called "multiple regression". The purpose of regression analysis is to find the most appropriate mathematical equation to express the relationship between the independent variable and the dependent variable. This equation is called the regression equation. If it is assumed that the functional relationship between the independent variable and the dependent variable is linear, it is called linear regression, otherwise it is called nonlinear regression. In this paper, the hill-climbing algorithm is used to deduce the side length values of various polyhedrons and their dual polyhedrons of different sizes. At the same time, regression analysis techniques are used to deduce the corresponding relations to facilitate the construction of dual polyhedrons of different sizes.

Simple linear regression model is a statistical method to explore the relationship between an independent variable and another dependent variable. The relationship between the independent variable and the dependent variable can be divided into three types: positive relationship, negative relationship and no relationship. The relationship between the independent variable and the dependent variable can be divided into two types: linear and non-linear. If the independent variable X is assumed to be X offset value, for example, the dependent variable Y is the side length. The relationship between the two is a straight line relationship, then the regression equation can be expressed as:

$$y_i = b_0 + b_1 \times x_i$$

Where i = 1,……,n

y_i = dependent variable y i-th variable
x_i = dependent variable x i-th variable
b_0 = intercept of regression model
b_1 = regression coefficient or slope
n = number of independent variables

If we use the corresponding values of the independent variables x_i and y_i in the sample data, and use the independent variable x_i, intercept b_0 and slope b_1 to calculate the estimated value $\hat{y}_i$ of the dependent variable y_i, let the sum of squares (Sum Square Error, SSE) that the difference between the dependent variable y_i and its estimated value $\hat{y}_i$ is minimum. Which is a characteristic of the least square method.

$$\text{Min SSE} = min \sum_{i=1}^{n} (y_i - \hat{y}_i)^2 = min \sum_{i=1}^{n} (y_i - b_0 - b_1 \times x_i)^2$$

Therefore, the regression equation of the simple linear regression model can be expressed as;

$$y = b_0 + b_1 \times x$$

The regression coefficient (b_1) and intercept (b_0) of the regression equation can be obtained by using the differential equation as shown below;

$$b_1 = \frac{\sum_{i=1}^{n}(x_i \times y_i) - \frac{\sum_{i=1}^{n} x_i \times \sum_{i=1}^{n} y_i}{n}}{\sum_{i=1}^{n} x_i^2 - \frac{(\sum_{i=1}^{n} x_i)^2}{n}}$$

$$b_0 = \bar{y} - b_1 \times \bar{x}$$

Where $\bar{y}$ is the sample mean of the dependent variable, $\bar{x}$ is the sample mean of the independent variable.

For example;

Table 2 Side Length Values of Regular Tetrahedron and Its Dual Polyhedron

Item	Tetrahedron side length	Dual tetrahedron side length
1	100	33.3338
2	90	30.0004
3	80	26.6668
4	70	23.3334
5	60	20.0001
6	50	16.6668

Then;

$$b_1 = \frac{11833.44 - \frac{450 \times 150.0013}{6}}{35500 - \frac{202500}{6}} = \frac{583.3425}{1750} = 0.3333$$

$$b_0 = \frac{150.0013}{6} - 0.3333 \times \frac{450}{6} = 0.002717$$

So the regression equation for this example is:

$$y = b_0 + b_1 \times x = 0.002717 + 0.3333 \times x$$

V. RESULTS AND DISCUSSION

In this paper, the hill-climbing algorithm is used to deduce the side length values of various and different size polyhedrons relative to their dual polyhedrons. At the same time, regression analysis techniques are used to deduce the corresponding relational expressions to facilitate the construction of dual polyhedrons of different sizes. The result as follows:

A. *Regular Tetrahedron and Its Dual Polyhedron*

Through the optimized execution results, the side length values of the regular tetrahedron and its dual polyhedron (regular tetrahedron) are shown in Table 2.

So the regression equation is;

$$y = b_0 + b_1 \times x = 0.002717 + 0.3333 \times x \qquad (1)$$

Figure 5 is the side length regression graph of regular tetrahedron and its dual polyhedron (regular tetrahedron).

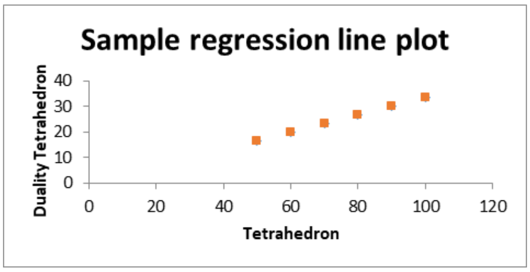

Figure 5 The side length regression graph of regular tetrahedron and its dual polyhedron

B. *Regular Hexahedron and Its Dual Polyhedron*

Through the optimized execution results, the side length values of the regular hexahedron and its dual polyhedron (regular octahedron) are shown in Table 3.

Table 3 Side Length Values of Regular Hexahedron and Its Dual Polyhedron

Item	Hexahedron side length	Dual octahedron side length
1	100	70.7110
2	90	63.6401
3	80	56.5687
4	70	49.4977
5	60	42.4266
6	50	35.3555

So the regression equation is;

$$y = b_0 + b_1 \times x = -0.00009 + 0.707111 \times x \qquad (2)$$

Figure 6 is the side length regression graph of the regular hexahedron and its dual polyhedron (regular octahedron).

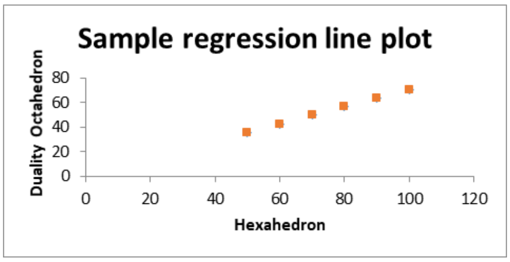

Figure 6 The side length regression graph of regular hexahedron and its dual polyhedron

C. *Regular Octahedron and Its Dual Polyhedron*

Through the optimized execution results, the side length values of the regular octahedron and its dual polyhedron (regular hexahedron) are shown in Table 4.

Table 4 Side Length Values of Regular Hexahedron and Its Dual Polyhedron

Item	Octahedron side length	Dual hexahedron side length
1	100	47.1406
2	90	42.4267
3	80	37.7125
4	70	32.9984
5	60	28.2844
6	50	23.5703

So the regression equation is;

$$y = b_0 + b_1 \times x = -0.000052 + 0.471407 \times x \quad (3)$$

Figure 7 is the side length regression graph of the regular octahedron and its dual polyhedron (regular hexahedron).

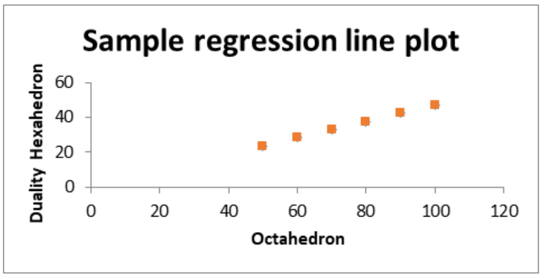

Figure 7 The side length regression graph of the regular octahedron and its dual polyhedron

D. Dodecahedron and Its Dual Polyhedron

Through the optimized execution result, the side length values of the regular dodecahedron and its dual polyhedron (regular icosahedron) are shown in Table 5.

Table 5 Side Length Values of Regular Dodecahedron and Its Dual Polyhedron

Item	Dodecahedron side length	Dual icosahedron side length
1	100	117.0822
2	90	105.3740
3	80	93.6659
4	70	81.9577
5	60	70.2495
6	50	58.5413

So the regression equation is;

$$y = b_0 + b_1 \times x = 0.000438 + 1.170818 \times x \quad (4)$$

Figure 8 is the side length regression graph of the regular dodecahedron and its dual polyhedron (icosahedron).

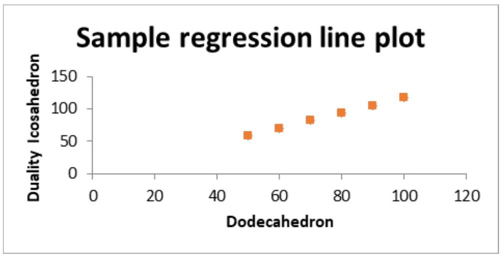

Figure 8 The side length regression graph of the regular dodecahedron and its dual polyhedron

E. Icosahedron and Its Dual Polyhedron

Through the optimized execution results, the side length values of the regular icosahedron and its dual polyhedron (regular dodecahedron) are shown in Table 6.

Table 6 Side Length Values of Regular Icosahedron and Its Dual Polyhedron

Item	Icosahedron side length	Dual dodecahedron side length
1	100	53.9346
2	90	48.5413
3	80	43.1477
4	70	37.7543
5	60	32.3608
6	50	26.9674

So the regression equation is;

$$y = b_0 + b_1 \times x = 0.00011 + 0.539345 \times x \quad (5)$$

Figure 9 is the side length regression graph of the regular icosahedron and its dual polyhedron (dodecahedron).

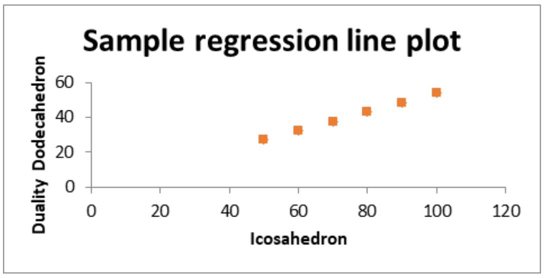

Figure 9 The side length regression graph of the regular icosahedron and its dual polyhedron (dodecahedron)

VI. CONCLUSION

The regular polyhedron matches its corresponding dual polyhedron and has good spatial accumulation properties, so it is often used as a periodic boundary condition for molecular simulation calculations. Therefore, the study of polyhedrons has been a part of the study of mathematical structure since two thousand years, and it is also the source of inspiration for many branches of the subject. had calculated the relevant geometric properties of each regular polyhedron. But there is no relevant data for reference on the combination relationship between regular polyhedron and its corresponding dual polyhedron. Especially when the model is built in parametric form, the dimensional characteristics that have been deduced in the past cannot be used directly, thus hindering the development of related products. Therefore, in this paper, based on the size of the side length required by the parametric modeling method, we use the hill-climbing algorithm to infer the side lengths of various and different sizes of polyhedrons. At the same time, we use regression analysis techniques to derive the corresponding relations to facilitate the construction of different sizes of dual polyhedrons, so as to facilitate related products development proceeds.

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