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Construct 3D Models of Regular Polyhedrons and Their Dual Polyhedrons with Optimized Solution Techniques

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***Abstract*—Since ancient Greek philosopher Plato, assigns the classical four elements: fire, air, water, and soil correspond to four regular polyhedrons, and describe the constellation that God uses the dodecahedron to align the entire sky. For thousands of years, people have been constantly exploring the laws behind polyhedron, and with the yearning and curiosity of the order of the universe. Modern makers are dedicated to deconstructing the composition of polyhedron to turn ideas into concrete through self-created tools and body sensations. Because of the regular polyhedrons and truncated regular polyhedrons derived from regular polyhedrons have good spatial accumulation properties, they are often used as periodic boundary conditions for molecular simulation calculations. In addition to introducing how to use parametric design tools and optimization analysis techniques to construct polyhedrons and their corresponding dual polyhedrons, this paper also based on the geometric model of the above-mentioned, a variety of creative products with high-density stacking characteristics will be developed, in order to further illustrate the geometric aesthetics that has continued from ancient times to the present.**

Keywords—Parametric Design*; *polyhedron

I. INTRODUCTION (*Heading 1*)

The record of polyhedrons and other mathematical models can first be seen in the ancient Greek era (400 BC). Initially, they only focused on the basic convex regular polyhedron, which is the familiar "Platonic Polyhedron". That is, Tetrahedron, Hexahedron or Cube, Octahedron, Dodecahedron, and Icosahedron. In the last proposition of his book "Elements", Euclid also proved that there are only five types of convex regular polyhedron. Archimede extended his research to more uniform convex polyhedron, a total of 13, later known as "Archimede Polyhedron". Because regular polyhedrons and truncated regular polyhedrons derived from regular polyhedrons have good spatial accumulation properties, they are often used as periodic boundary conditions for molecular simulation calculations. Therefore, the study of polyhedrons has been a part of the study of mathematical structure since two thousand years, and it is also the source of inspiration for many branches of the subject [1].

In terms of architectural engineering, the earliest polyhedral structure is the shape of a stone cave, which shows various symmetrical polyhedrons. The most representative cultural relic is the Egyptian pyramid in 2560 BC. And so far, there have been many unusual polyhedral structures, which have been widely used in buildings all over the world [2]. Several typical polyhedral buildings, such as Toyo Ito Architecture Museum, is a steel-concrete mixed building composed of 4 types of polyhedrons in a crystalline structure mode. It is composed of many independent spaces, and each room is decorated with different materials. Through different display methods, visitors can appreciate the space display of different aspects [3]. Another example is the National Library of Belarus, which is composed of 18 squares and 8 triangles. It was built with reference to the Da Vinci polyhedron and was established on September 15, 1922. A new hall was opened on June 16, 2006. It is one of the largest libraries in the world and the largest library in Eastern Europe [4]. And this thesis intends to use the geometric patterns of these type of regular polyhedrons and their dual polyhedrons to construct the modular creative products.

The analysis of the optimization method can be traced back to the Newton, Lagrange and Cauchy ages. The use of the "Calculus of Variation" method as the basis for the optimization problem was laid by Bernoulli, Euler, Lagrange and Weirstrass. This calculus of variable method is significantly helpful in solving specific problems. For example: static and steady-state analysis of objects and vibration analysis of objects. However, due to the time-consuming process of optimization calculations, progress was slow before the middle of the twentieth century, and many tedious calculations were not solved until the advent of computers. Among them, the development of Linear Programming (LP) is regarded as one of the most important scientific advances of the century.

The optimization algorithm is often used to find the most suitable parameter combination, so that its efficiency and results can meet the requirements of users, and find its suitable algorithm for different types of problems. The fields of application span mathematics, applied sciences, economics, statistics, and even medicine. The general optimization method is to infer the most suitable result through continuous numerical analysis. Therefore, for low-dimensional optimization problems, most of them can have good performance, but for high-dimensional problems with more than 20 or 30 variables, the results are often unsatisfactory. The current analysis optimization methods can be roughly divided into three types: Numerical Method, Enumerative and Random Search. Among them, the numerical method is the most important. It is based on the calculus of mathematics and searches for the best solution in a specific space by seeking extreme values. This is the spirit of the "Hill Climbing Algorithm". That is to say, it has only one or several starting points, and the next iteration value is generated according to the established mathematical model, and the calculation is repeated until the best solution is found. Therefore, it may converge to the local optimal solution, but cannot reach the global optimal solution [5].

Compared with numerical methods, the heuristic algorithm of artificial intelligence technology is to imitate different natural phenomena respectively, using the concept of random search method to randomly select many starting points, and search for the best solution at the same time. Each individual in the ethnic group has a search point. After the evolution of generations, each search point approaches the best solution direction [6]. In this paper, it is only necessary to deduce the relative length of various polyhedrons to dual polyhedrons through optimized analysis. Therefore, the "Hill Climbing Algorithm" is the most efficient way.

Parametric design is a design method that defines parameter rules and related processes as the design basis. Unlike traditional designers who are accustomed to directly determine the form and shape of design through experience based on specific design conditions, the design process of parametric design is not directly dealing with "shape", but the "logic" behind the design. The design method that controls the various parameter factors that affect the design and the correlation between each parameter to produce, evaluate, and adjust the geometric form of the design plan in real time. Parametric digital modeling through the acquisition and analysis of digital data, combined with computer-aided manufacturing tools such as RP or CNC, can quickly view the design, greatly improving the efficiency and feasibility of non-traditional products design. This digital continuum produced by the integrated application of model software to manufacturing hardware has gradually driven the comprehensive innovation of contemporary digital design concepts to production methods [7]. In this paper, "Creo Parametric" modeling software is used as the construction software for all kinds of polyhedrons. At the same time, this software is also used as a tool for subsequent creative product creation.

II. Geometric structure of regular polyhedron and its dual polyhedron

Regular polyhedron refers to the various faces of a convex polyhedron, which are composed of the same regular polygon. Therefore, based on the original definition of regular polyhedrons, four other equivalent properties can be re-derived:

(1) Each vertex of a regular polyhedron is connected with the same number of edges.
(2) Each vertex of a regular polyhedron connects the same number of faces.
(3) Each vertex of the regular polyhedron is located on the same sphere.
(4) The angle between each face of a regular polyhedron and the face is equal.

There are only five types of regular polyhedrons that can meet the above conditions: regular tetrahedron, regular hexahedron, regular octahedron, regular dodecahedron, and regular icosahedron; that is, the so-called Platonic polyhedron. The vertex of the corresponding dual polyhedron is the correspondence face of the original polyhedron, and the face of the dual polyhedron is the correspondence vertex of the original polyhedron. In addition, the edge defined by adjacent vertices can correspond to two adjacent faces, and the intersection line of these faces also defines an edge line of the dual polyhedron. Table 1 is a summary table of the dual polyhedrons corresponding to the Platonic polyhedrons.

TABLE 1 THE DUAL POLYHEDRON CORRESPONDING TO THE PLATONIC POLYHEDRON

Platonic polyhedron	dual polyhedron
regular tetrahedron	regular tetrahedron
regular hexahedron	regular octahedron
regular octahedron	regular hexahedron
regular dodecahedron	regular icosahedron
regular icosahedron	regular dodecahedron

III. Application of optimal analysis techniques

In the field of optimization research, many algorithms have been proposed, such as hill climbing algorithm, simulated annealing method, genetic algorithm, tabu search method, ant colony algorithm, particle swarm algorithm and so on. If the number of particles is used to distinguish, the above algorithms can be divided into two types: "single-particle type" and "multi-particle type". Among them, "hill-climbing algorithm", "simulated annealing method", and "taboo search method" belong to the "single-particle algorithm", while "genetic algorithm", "ant colony algorithm", "particle swarm algorithm", " The bee colony algorithm" are the multi-particle algorithm. Although most of the current academic research focuses on multi-particle algorithms, it is difficult to analyze the quality of these algorithms, which is caused by the inherent complexity of multi-particle systems. In this article, we only need to deduce the relative length of various polyhedrons to the side polyhedron through optimization analysis. Therefore, a well-known, simple and fast basic algorithm-"hill climbing algorithm" is adopted. As the optimized calculation tool in this article.

A. Hill-Climbing Algorithm, HC

The so-called "hill-climbing algorithm" is a simple regional search algorithm in the single-particle algorithm. Since its process is quite similar to the continuous upward movement of humans when climbing a mountain, it is called a hill-climbing algorithm. The hill-climbing algorithm can be said to be a heuristic method. The search strategy is to constantly find the best solution around, and then continue to move on to the best solution until it can no longer be improved. The implementation is quite easy and the execution speed is very fast. Therefore, it is often used as a benchmark for comparison of various optimization algorithms. And when the problem to be asked has multiple parameters, we can increase or decrease the value of a certain parameter by one unit in turn in the process of gradually obtaining the optimal solution through the hill climbing method. For example, the solution of a certain problem needs to use three integer-type arguments x1, x2, x3. At the beginning, set these three arguments to (2, 2, -2), increase/decrease x1 by 1, and get Two solutions (1,2, -2), (3, 2,-2); increase/decrease x2 by 1, and get two solutions (2,3, -2), (2,1, -2); Increase/decrease x3 by 1, and get two solutions (2,2,-1), (2,2,-3), so you get a solution set: (2,2,-2), (1, 2 ,-2), (3, 2,-2), (2,3,-2), (2,1,-2), (2,2,-1), (2,2,-3). Find the optimal solution from the above solution set, and then construct a solution set for this optimal solution according to the above method, and then find the optimal solution. In this way, the "hill-climbing" operation process ends until the optimal solution of the previous time and the optimal solution of the next time are the same. However, after this algorithm falls into the best solution in the area, it cannot jump out, that is, it cannot find a better solution. This is because the hill climbing algorithm only finds neighboring points for comparison, and does not allow walking in the worse direction, which makes climbing hills. In the more complex nonlinear programming problems, the algorithm is easy to fall into a poor area, and it is difficult to find the best solution in the whole domain.

Therefore, some experts have devised a jumping strategy that increases with the number of failures. This method is called Hill-Climbing with Jumping strategics (HCJ). The jumping mechanism makes it easier for the mountain climbing algorithm to leave the valley and find more good solution quickly. The difference between HCJ and the traditional hill-climbing algorithm is that the jumping steps of HCJ will increase with the number of failures. Therefore, when the particle is at the bottom of the valley, it will cause continuous jumping failure. At this time, HCJ will randomly increase the jumping range. By increasing the range, HCJ will have the opportunity to jump out of the current valley and move to a lower valley, allowing HCJ to find a better solution. In practice, in order to overcome the failure caused by the border area, then randomly select individuals from the range to make adjustments, and the size of the adjustment range is determined by random steps. When this neighbor selection method is used in the hill climbing algorithm, the individual can adjust in two directions, and there is a half chance that it will adjust in the correct direction. Therefore, the hill-climbing algorithm can be expressed by the following function:

```
Algorithm Hill-Climbing(pi)
    p = pi;
      while not isEnd( )
        pn = move(p);
        if pn.energy() <= p.energy()
        p = pn;
End Algorithm
```

The entire execution flow of the hill-climbing algorithm is shown in Figure 6.

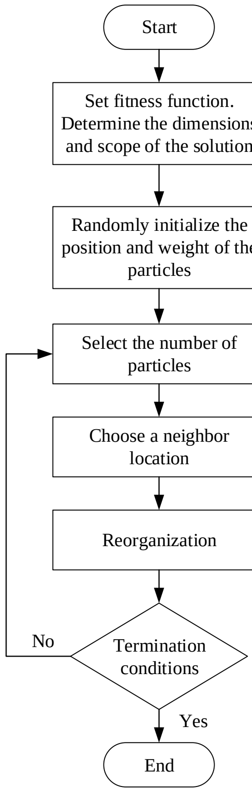


Figure 1 The entire execution flow of the hill-climbing algorithm

B. Tetrahedron 's dual polyhedron optimization execution result

As mentioned earlier, this article intends to use the hill-climbing algorithm to deduce the relative length of the sides of the various polyhedrons. Here for the regular tetrahedron; suppose the side length of the regular tetrahedron is 100, and the initial side length of the dual regular tetrahedron is 50, The side length of the dual regular tetrahedron is the "independent variable", and the distance between the end point of the dual regular tetrahedron and the plane of the regular tetrahedron "Dist = 0" is the objective function, the initial assembled state is shown in Figure 2. After the optimization calculation, when the side length of the dual regular tetrahedron is 33.3338, the objective function requirement can be met. The convergence process is shown in Figure 3, and the result is shown in Figure 4.

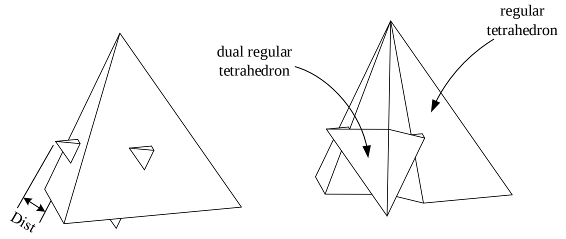


Figure 2 The initial assembled state

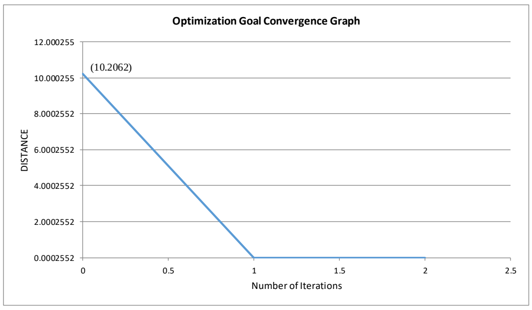


Figure 3 The convergence process

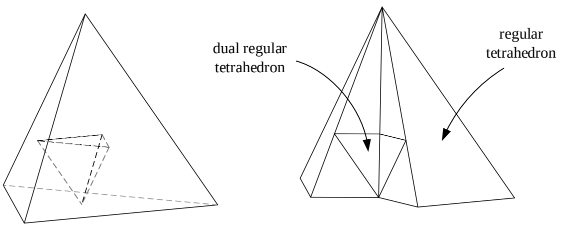


Figure 4 The result assembled state

C. Hexahedron 's dual polyhedron optimization execution result

For the regular hexahedron; suppose the side length of the regular hexahedron is 100, the initial side length of the dual regular octahedron is 85, the side length of the dual regular octahedron is the "independent variable", and the distance between the end point of the dual regular octahedron and the plane of the regular hexahedron "Dist = 0" is the objective function, and its initial combined state is shown in Figure 5. After the optimization calculation, when the side length of the dual octahedron is 70.7110, the objective function requirement can be met. The convergence process is shown in Figure 6, and the result is shown in Figure 7.

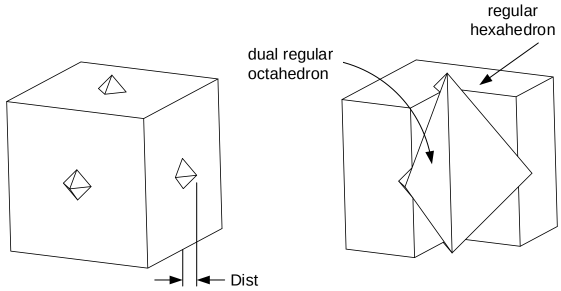


Figure 5 The initial assembled state

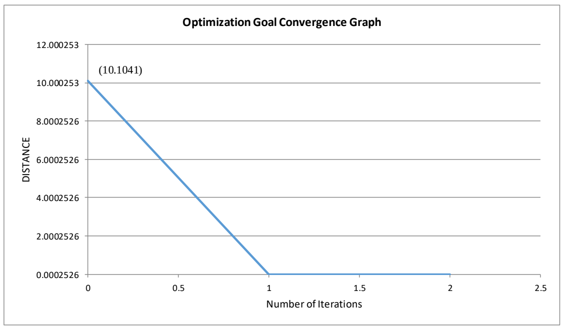


Figure 6 The convergence process

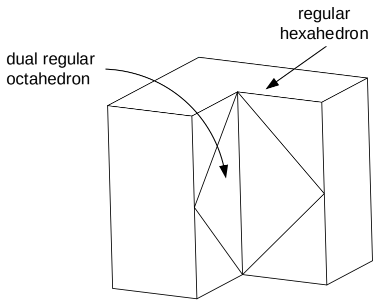


Figure 7 The result assembled state

D. Octahedron 's dual polyhedron optimization execution result

For a regular octahedron; suppose the side length of the regular octahedron is 100, the initial side length of the dual regular hexahedron is 55, the side length of the dual regular hexahedron is the "independent variable", and the distance between the end point of the dual regular hexahedron and the plane of the regular octahedron "Dist = 0" is the objective function is the objective function, and its initial combined state is shown in Figure 8. After the optimization calculation, when the side length of the dual regular hexahedron is 47.1406, the objective function requirement can be met. The convergence process is shown in Figure 9 and the result is shown in Figure 10.

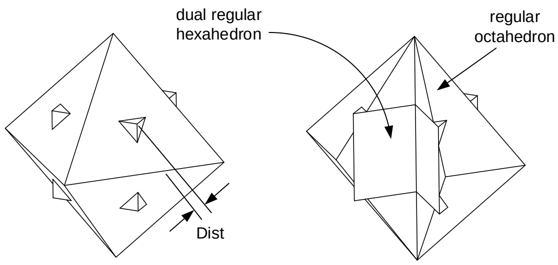


Figure 8 The initial assembled state

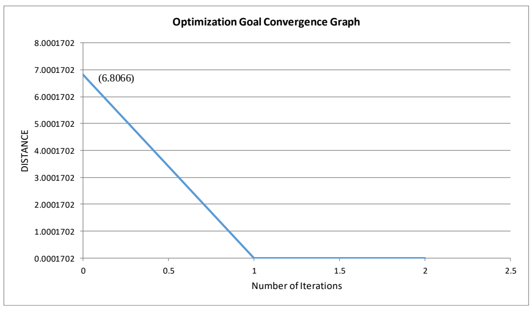


Figure 9 The convergence process

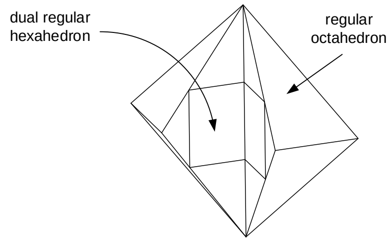


Figure 10 The result assembled state

E. Dodecahedron's dual polyhedron optimization execution result

For the regular dodecahedron; suppose the side length of the regular dodecahedron is 100, the initial side length of the dual regular icosahedron is 125, the side length of the dual regular icosahedron is the "independent variable", the distance between the end point of the dual regular icosahedron and the plane of the regular dodecahedron "Dist = 0" is the objective function, and its initial combined state is shown in Figure 11. After the hill-climbing algorithm is optimized, when the side length of the dual icosahedron is 117.0822, the objective function requirement can be met. The convergence process is shown in Figure 12, and the result is shown in Figure 13.

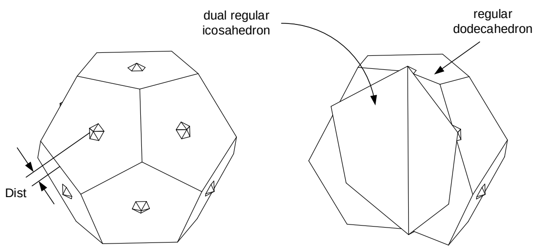


Figure 11 The initial assembled state

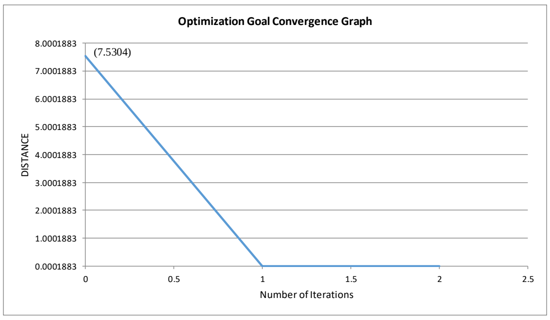


Figure 12 The convergence process

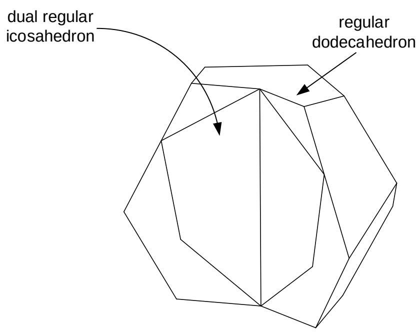


Figure 13 The result assembled state

F. Icosahedron 's dual polyhedron optimization execution result

For the regular icosahedron; suppose the side length of the regular icosahedron is 100, the initial side length of the dual regular dodecahedron is 58, the side length of the dual regular dodecahedron is the "independent variable", t and the distance between the end point of the dual regular dodecahedron and the plane of the regular icosahedron "Dist = 0" is the objective function, and its initial combined state is shown in Figure 14. After the optimization of the hill climbing algorithm, when the side length of the dual regular dodecahedron is 53.9346, the objective function requirement can be met. The convergence process is shown in Figure 15, and the result is shown in Figure 16.

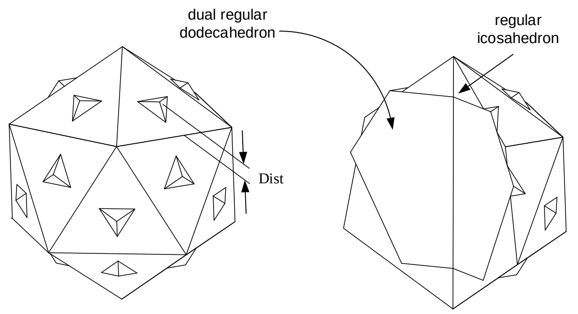


Figure 14 The initial assembled state

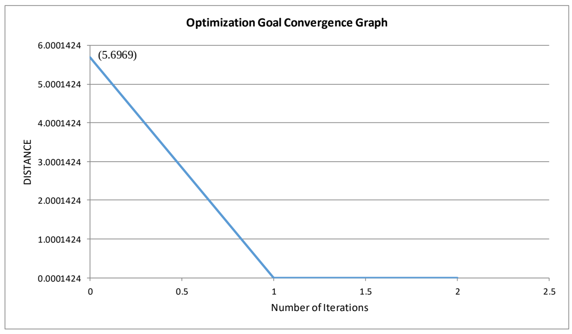


Figure 15 The convergence process

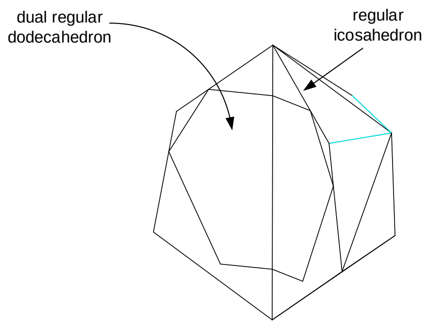


Figure 16 The result assembled state

IV. CREATIVE PRODUCT DEVELOPMENT

Based on the polyhedron and its dual polyhedron mentioned above, the geometric of their 3D models are characterized by high-density stacking. Therefore, this paper uses these geometric characteristics to develop creative products with geometric aesthetics, hoping to further interpret the geometric aesthetics that have continued from ancient times to the present.

The following takes the regular hexagon and its dual polyhedron (regular octagon) as an example to illustrate the development steps of the rotary table storage box.

Step 1: Construct a regular hexahedron, as shown in Figure 17.

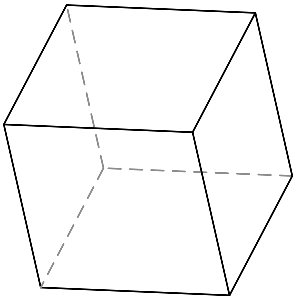


Figure 17 Regular hexahedron

Step 2: Construct the corresponding dual polyhedron (octahedron), as shown in Figure 18.

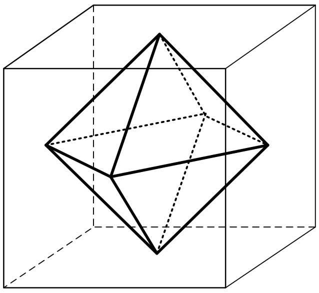


Figure 18 Regular octahedron

Step 3: Through the operation of the surface, respectively extend the surfaces of the dual polyhedron to disassemble the parts, as shown in Figure 19.

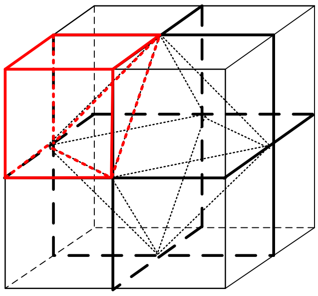


Figure 19 Extend the surfaces of the dual polyhedron to disassemble the parts

Step 4: Perform structural disassembly operations on the upper, middle and bottom of each part, as shown in Figure 20.

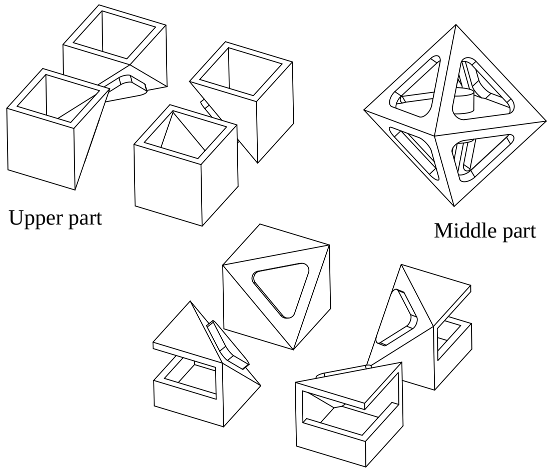


Figure 20 The upper, middle and bottom of each part

Step 5: Complete the combination of the upper, middle and bottom of each part structure, as shown in Figure 21.

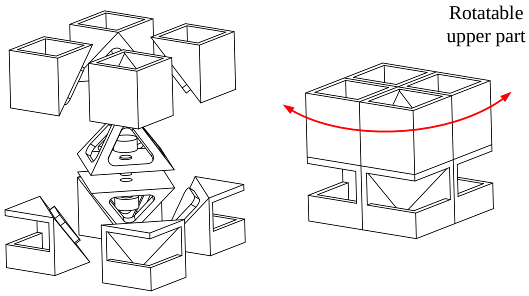
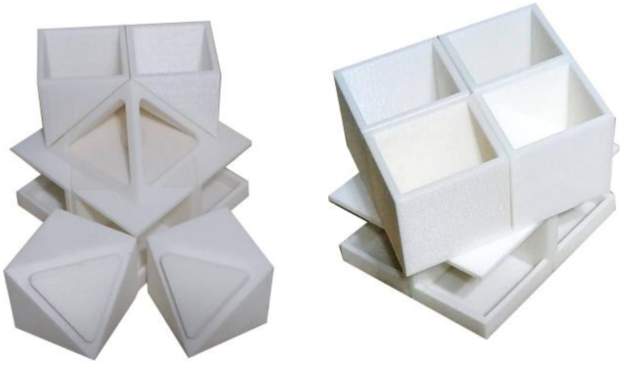


Figure 21 The rotary table storage box

V. CONCLUSION

The regular polyhedron matches its corresponding dual polyhedron and has good spatial accumulation properties, so it is often used as a periodic boundary condition for molecular simulation calculations. Therefore, the study of polyhedrons has been a part of the study of mathematical structure since two thousand years, and it is also the source of inspiration for many branches of the subject. In this paper, we use the hill-climbing algorithm to infer the side lengths of various and different sizes of polyhedrons. At the same time, we based on the above-mentioned polyhedron and its dual polyhedron, the geometric of their 3D models are characterized by high-density stacking, developing a rotary storage box composed of a regular hexagon and its dual polyhedron (regular octagon) with geometric aesthetics. And print out other physical products in 3D, hoping to further illustrate the geometric aesthetics that has continued from ancient times to the present. Figure 22 is the printed product of the rotary storage box composed of a regular hexahedron and its dual polyhedrons (regular octahedron). Figure 23 is the printed product of the rotary storage box composed of a regular dodecahedron and its dual polyhedron (a regular icosahedron).



Separation status Combination status

Figure 22 The printed product of the rotary storage box composed of a regular hexahedron and its dual polyhedrons

Separation status Combination status

Figure 23 The printed product of the rotary storage box composed of a regular dodecahedron and its dual polyhedron

ACKNOWLEDGMENT

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