Optimal Solution for Fuzzy Assignment Problem using a Measure of Center and its Applications

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***Abstract*—Assignment problem represents a special case of linear programming problem used for allocating resources (mostly workforce) in an optimal way, it is a highly useful tool for operation and project managers for optimizing costs. In this paper a new approach is given to solve the balanced fuzzy assignment problem (FAP). Using this approach, we get an improved solution compared to the solution by conventional methods in a smaller number of iterations. Numerical examples are given to illustrate the procedure.**

Keywords—Balanced fuzzy assignment problem; fuzzy number; Hungarian Method

I. INTRODUCTION

An assignment problem (AP) has many applications in transportation, healthcare, education, and sports. Using assignment problems we minimize time, minimize cost, minimize length path route, and maximize profit. AP is a particular case of transportation problem where the objective is to assign a number of resources to an equal number of activities so as to minimize total cost or maximize total profit of allocation. The term AP first appeared in Votaw and Orden (1952). Hungarian method proposed by Kuhn (1955) is widely used for the solution to APs. AP has been commonly encountered in many problems worldwide, Basirzadeh (2012). Zadeh (1965) has introduced the concept of fuzzy sets to deal with imprecision, vagueness in real life situations. Ahmad and Rabbani (2015) discussed Improved Extreme Point Enumeration Technique for Assignment Problem. Lin and Wen (2004) investigated a fuzzy AP in which the cost depends on the quality of the job. Chen (1985) proposed a fuzzy assignment model that did not consider the differences of individuals and also proved some theorems. Huang and Xu (2005) proposed a solution procedure for the APs with restriction of qualification. Dubois and Fortemps (1999) proposed a flexible AP, which combines with fuzzy theory, multiple criteria decision-making and constraint-directed methodology. Singh and Thakur (2015) solved the fuzzy assignment problem without converting the fuzzy number into a crisp number. Berghman et al. (2014) solved a model for a dock AP. Frimpong and Owusu (2015) use linear programming to clarify the difficulty of over-distribution and under-distribution of the insufficient classroom space.

II. PRELIMINARIES

A. Fuzzy Number

A fuzzy number is a generalization of a regular, real number in the sense that it does not refer to single value but rather to a connected set of values, where each possible value has its own weight between 0 and 1, Dijkman, J.G et. al. (1983). This weight is called a membership function.

A fuzzy number $\underset{\sim}{A}$ is thus a special case of a convex, normalized fuzzy set of the real line R, Michael Hanss (2005), must satisfy the following conditions:

(i) $\mu_{\underset{\sim}{A}}(x_0)$ is a piecewise continuous.

(ii) There exists at least one $x_0 \in R$ with $\mu_{\underset{\sim}{A}}(x_0) = 1$.

(iii) $\underset{\sim}{A}$ must be normal and convex.

B. Defuzzification

Defuzzification is the process of obtaining a single number (crisp value) from the output of the aggregated fuzzy set. It is used to transfer fuzzy inference results into a crisp output. Here, median is used to defuzzify the fuzzy numbers because of its simplicity and accuracy.

C. The Assignment Problem

The assignment problem (balanced) is stated as

$$Minimize\ Z = \sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij}.x_{ij}$$

$$\text{s.t.} \quad \sum_{j=1}^{n} x_{ij} = 1,\ i = 1, 2, \ldots, n$$

$$\sum_{i=1}^{n} x_{ij} = 1,\ \ j = 1, 2, \ldots, n$$

Where, $x_{ij} = \begin{cases} 1 & \text{if } i^{th} \text{ job is assigned to worker } j \\ 0 & \text{otherwise} \end{cases}$

c_{ij} = cost to perform job i by worker j

When the costs $\bar{c}_{ij}$ are fuzzy numbers then the total cost becomes a fuzzy number. We defuzzify the fuzzy cost coefficients into crisp ones by our proposed method.

Then the fuzzy objective function is:

$$Minimize\ \bar{Z} = \sum_{i=1}^{n}\sum_{j=1}^{n} \bar{c}_{ij}.x_{ij}$$

III. PROPOSED APPROACH TO REDUCE THE FUZZY ASSIGNMENT PROBLEM

A. When the Number of Values of Fuzzy Numbers are Even

In this case, first we will find the median of the series of fuzzy numbers as

$$\text{Median of } (x_1, x_2, ..., x_n) = \text{Average of} \left[\left(\frac{n}{2}\right)^{th} item + \left(\frac{n}{2}+1\right)^{th} item \right] \quad (1)$$

Where $R(x_1, x_2, ..., x_n) = Median\ of\ the\ series$

e.g., (i) (1, 6, 11, 16, 21, 26) have conversion as Median = 13.5

(ii) (1, 3, 5, 7, 9, 11, 13, 15) have conversion as Median = 8 etc.

B. When the Number of Values of Fuzzy Numbers are Odd

In this case, first we will find the median of the series of fuzzy numbers as

$$\text{Median of } (x_1, x_2, ..., x_n) = \left(\frac{n}{2}+1\right)^{th} item \quad (2)$$

Where $R(x_1, x_2, ..., x_n) = Median\ of\ the\ series$

e.g., (3, 5, 6, 7, 10) have conversion as Median = 6

IV. PROPOSED METHOD

The method consists of the following steps:

(1) Determine the fuzzy cost table from the given problem.

(2) Reduced the fuzzy assignment table using the formula given in Eq. (1) or (2) according to even or odd type of problem.

(3) Apply the Hungarian method to find the optimum assignment. Calculate fuzzy optimal solution by using $\sum_{i=1}^{n}\sum_{j=1}^{n} \bar{c}_{ij}.x_{ij}$.

V. NUMERICAL EXAMPLES

A. For the number of values of fuzzy numbers as even

Consider the problem of assigning five operators *O*1, *O*2, *O*3, *O*4, *O*5 to five machines *M*1, *M*2, *M*3, *M*4, *M*5. The assignment costs are given the following table:

	M1	M2	M3	M4	M5
O1	(2, 3, 5, 7, 9, 10)	(4, 5, 7, 9, 11, 12)	(5, 6, 8, 10, 12, 13)	(3, 4, 6, 8, 10, 11)	(9, 10, 12, 14, 16, 17)
O2	(5, 6, 8, 10, 12, 13)	(1, 2, 4, 6, 8, 9)	(4, 5, 7, 9, 11, 12)	(7, 8, 10, 12, 14, 15)	(3, 4, 6, 8, 10, 11)
O3	(6, 7, 9, 11, 13, 14)	(3, 4, 6, 8, 10, 11)	(2, 3, 5, 7, 9, 10)	(4, 5, 7, 9, 11, 12)	(5, 6, 8, 10, 12, 13)
O4	(7, 8, 10, 12, 14, 15)	(10, 11, 13, 15, 17, 18)	(5, 6, 8, 10, 12, 13)	(9, 10, 12, 14, 16, 17)	(8, 9, 11, 13, 15, 16)
O5	(5, 6, 8, 10, 12, 13)	(3, 4, 6, 8, 10, 11)	(7, 8, 10, 12, 14, 15)	(6, 7, 9, 11, 13, 14)	(9, 10, 12, 14, 16, 17)

Assign the operator to a different machine so that the total cost of assignment is minimized.

Solution: First, we have to reduce the given fuzzy assignment table using Eq.(1)

Now, we have hexagonal fuzzy number as (median) c_{ij} value.

	M1	M2	M3	M4	M5
O1	6	8	9	7	13
O2	9	5	8	11	7
O3	10	7	6	8	9
O4	11	14	9	13	12
O5	9	7	11	10	13

After applying the Hungarian method to find the optimum assignment, the optimum assignment is as follows:

Operators	Machines	Costs
O1	*M1*	(2, 3, 5, 7, 9, 10)
O2	*M5*	(3, 4, 6, 8, 10, 11)
O3	*M4*	(4, 5, 7, 9, 11, 12)
O4	*M3*	(5, 6, 8, 10, 12, 13)
O5	*M2*	(3, 4, 6, 8, 10, 11)

The fuzzy optimal cost is

(2, 3, 5, 7, 9, 10) + (3, 4, 6, 8, 10, 11) + (4, 5, 7, 9, 11, 12) + (5, 6, 8, 10, 12, 13) + (3, 4, 6, 8, 10, 11) = (17, 22, 32, 42, 52, 57)

R(17, 22, 32, 42, 52, 57) = 37 units

B. For the number of values of fuzzy numbers as odd

Suppose five tutors T1, T2, T3, T4 and T5 are to be assigned to teach five different subject of a course S1, S2, S3, S4 and S5. The preparation time (in hours) for different topics varies from tutor to tutor and which is given in the following table. Each tutor allowed only one subject at a time. Calculate an assignment of timetable so as to minimize the total course preparation time for all subjects of a course.

	S1	S2	S3	S4	S5
P1	(3, 4, 5, 6, 7)	(10, 11, 12, 13, 14)	(3, 4, 5, 6, 7)	(8, 9, 10, 11, 12)	(3, 3, 3, 3, 3)
P2	(7, 8, 9, 10, 11)	(9, 10, 11, 12, 13)	(8, 9, 10, 11, 12)	(7, 8, 9, 10, 11)	(3, 3, 3, 3, 3)
P3	(5, 6, 7, 8, 9)	(7, 8, 9, 10, 11)	(6, 7, 8, 9, 10)	(4, 5, 6, 7, 8)	(3, 3, 3, 3, 3)
P4	(5, 6, 7, 8, 9)	(3, 4, 5, 6, 7)	(8, 9, 10, 11, 12)	(4, 5, 6, 7, 8)	(3, 3, 3, 3, 3)
P5	(6, 7, 8, 9, 10)	(4, 5, 6, 7, 8)	(6, 7, 8, 9, 10)	(10, 11, 12, 13, 14)	(3, 3, 3, 3, 3)

Solution:

First, we have to reduce the given fuzzy assignment table using Eq.(2)

Now, we have pentagonal fuzzy number as (median) c_{ij} value.

	S1	S2	S3	S4	S5
P1	5	12	5	10	3
P2	9	11	10	9	3
P3	7	9	8	6	3
P4	7	5	10	6	3
P5	8	6	8	12	3

After applying the Hungarian method to find the optimum assignment, the optimum assignment is as follows:

Professor	Subject	Time taken (in hours)
P1	S1	(3, 4, 5, 6, 7)
P2	S5	(3, 3, 3, 3, 3)
P3	S4	(4, 5, 6, 7, 8)
P4	S2	(3, 4, 5, 6, 7)
P5	S3	(6, 7, 8, 9, 10)

The fuzzy optimal time is

(3, 4, 5, 6, 7) + (3, 3, 3, 3, 3) + (4, 5, 6, 7, 8) + (3, 4, 5, 6, 7) + (6, 7, 8, 9, 10) = (19, 23, 27, 31, 35)

R(19, 23, 27, 31, 35) = 27 hours

VI. The Advantage of the Proposed Method

(1) The proposed method works well for maximization as well as minimization FAP.

(2) The best possible solution can be obtained via this method quickly and with a less computational efforts.

(3) In this method, we solve FAP by converting into a crisp value.

(4) Octagonal, decagonal, hendecagonal, etc., can be solve simply by proposed method.

VII. Conclusion

We considered balanced FAP and enhanced an existing algorithm to solve it more powerfully, resulting in the proposed algorithm. We solved two numerical examples one is for cost minimizing and second, is for time minimizing balanced fuzzy assignment problem to obtain fuzzy optimal solution by using proposed method. For these problems, we convert the hexagonal and pentagonal fuzzy number into the median number respectively, due to this the lengthy calculation is minimized. However, the proposed algorithm is simpler, uncomplicated to understand and having a smaller number of iteration than fuzzy Hungarian method.

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