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Use of the Modified Model of Induction Machine in the Calculations of Traction Drives

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Abstract - This paper presents a method for calculating an inverter drive system using a modified equivalent diagram of an induction machine. Almost all calculations of an inverter drive system with an induction machine are carried out using the stator flux linkage as a fundamental but rarely used quantity of electromagnetism. The use of this important physical quantity makes it easy to focus attention on the state of the machine's magnetic circuit and its influence on the operation of the entire drive. Basing the analytical description of the induction machine on this physical quantity facilitates the representation of the control characteristics of induction machines and in particular, the easy realization of various mechanical characteristics, including the characteristic called series characteristic. In this paper, the possibilities of the calculation methods used by the authors are demonstrated by means of an example using arbitrary numerical data.

Keywords— Inverter–fed induction machines, traction drives, series-excitation mechanical characteristic, modified equivalent circuit, modified analytical description of induction machine

I. INTRODUCTION

This paper is an extension and continuation of the work presented in [1], [2], [3], [4]. Some formulas and explanations can only be found in these previous articles. All these articles are devoted to the study of the properties of induction machines in traction application.

The introduction of the value of the stator flux linkage into the analytical description of induction machines implies the use of a physical quantity which is the integral of the voltage over time, a quantity which contains more information about the nature and time course of the voltage applied to the machine windings than the voltage itself. The integral of the voltage over time is described by the equation:

$$\int_0^t u(t)dt = \psi(t) - \psi(0) = L \cdot (i(t) - i(0)) \qquad (1)$$

The left-hand side of the equation, containing the integral from the voltage over time, is actually the voltage pulse applied to the magnetic circuit. The first part of the right-hand side is the change of the magnetic flux caused by this impulse, which, according to the second part of the right-hand side, only causes a change of current in a linear magnetic circuit.

From another point of view, the stator flux linkage and electric charge are the fundamental quantities of electromagnetism. Wherever possible these integral quantities should be used instead of the differential quantities which are voltage and current.

An extension of the analytical description of electromagnetic circuits in such a way as to emphasize the importance of physical quantities in their integral form has just been presented using the example of an inverter-powered induction machine in a traction drive system.

II. EXAMPLE OF A TRACTION DRIVE

The equivalent diagram of the modelled drive system is described in detail and shown in Fig. 4 in [2]. The block marked as power inverter represents the induction machine and its place among other components of the drive system. A modified equivalent schematic diagram of the induction machine with further symbols and designations used is shown in Fig.1.

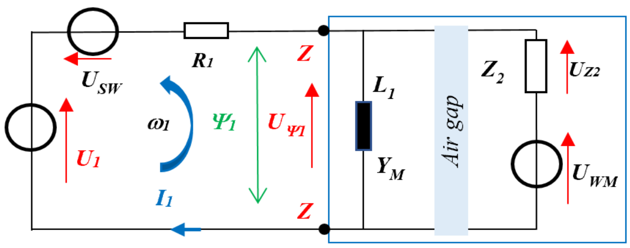

Figure 1. The figure shows a frequency domain, modified equivalent circuit of the induction machine implemented in the algorithm used for calculation of inverter traction drives. The voltage source USW represents the frequency domain symbol of the mechanical switch in the time domain

Nominal data of the machine

U1n = 2040 in [V]	Amplitude of the stator phase voltage
R_1 = 0.055 in [Ω], L_1 = 0.042 in [H]	Resistance and inductance of stator windings and
T_1 =0.76 s, T_2 =0.94 s	Time constants of stator and rotor windings in [s]
σ = 0.082	Coefficient of total leakage of machine
f1n = 50 Hz, f2n = 0.57 Hz	Nominal frequency of stator and rotor in [Hz]
Ψ_1	Stator field flux linkage in [Vs]
Symbol used	
$\omega_i = 2 \cdot \pi \cdot f_i$, $\omega_M = 2 \cdot \pi \cdot f_M$	Angular frequency of current in stator, rotor and mechanical in [1/s]
U_1, I_1, I_2	Complexor of stator voltage in [V]

	stator and rotor current in [A]
U_1, $U_{\Psi 1}$, U_{WM}, U_{Z2}	Complexor of stator voltage, stator field voltage, rotor sink voltage and voltage drop in Z_2 of rotor in [V]

Among the nominal data of the machine there is the stator flux linkage as voltage momentum, which accurately describes the state of the magnetic circuit of the machine. In case of machines powered from inverters this value is, besides the maximum voltage and current values, the most basic information about the machine. This value should be placed on the nameplate of every induction machine suitable for operation with inverter power supply. For accurate calculations, the dependence of the equivalent circuit parameters R_1, L_1, T_1, T_2, σ on the frequency of the supply voltage must be considered.

The relationships between the individual physical quantities which are the parameters of the equivalent circuit diagram from Fig. 1 are presented in the form of the corresponding immittances.

Immittance: Stator voltage to stator current:

$$IM_{U_1 to I_1} = R_1 \cdot \frac{1-\sigma\cdot\omega_1\cdot T_1\cdot\omega_2\cdot T_2+j\cdot(\omega_1\cdot T_1+\omega_2\cdot T_2)}{1+j\cdot\omega_2\cdot T_2} \qquad (2)$$

Immittance: Stator voltage to stator flux linkage:

$$IM_{U_1 to \Psi_1} = \frac{1}{T_1}\cdot\frac{1+j\cdot\omega_2\cdot T_2}{(1+j\cdot\sigma\cdot\omega_2\cdot T_2)} + j\cdot\omega_1 \qquad (3)$$

Immittance: Stator current to stator flux linkage:

$$IM_{I_1 to \Psi_1} = \frac{1}{L_1}\cdot\frac{1+j\cdot\omega_2\cdot T_2}{(1+j\cdot\sigma\cdot\omega_2\cdot T_2)} \qquad (4)$$

Immittance: Flux linkage inducted stator voltage to stator flux linkage:

$$IM_{U_{\Psi_1} to \Psi_1} = j\cdot\omega_1 \qquad (5)$$

Immittance: Flux linkage inducted stator voltage to excitation current trough stator inductance L_1:

$$IM_{\Psi_1 to I_{L_1}} = L_1 \qquad (6)$$

Immittance: Voltage induced in the rotor placed sink to stator flux linkage:

$$IM_{U_{WM} to \Psi_1} = j\cdot\omega_M \qquad (7)$$

Immittance: Voltage drop on the rotor impedance Z_2 to stator flux linkage:

$$IM_{U_{Z_2} to \Psi_1} = j\cdot\omega_2 \qquad (8)$$

Immittance: Current in equivalent rotor circuit to stator flux linkage:

$$IM_{I_2 to \Psi_1} = \frac{j}{L_1}\cdot\frac{(1-\sigma)\cdot\omega_2\cdot T_2}{1+j\cdot\sigma\cdot\omega_2\cdot T_2} \qquad (9)$$

III. CONTROL SIGNALS OF THE INVERTER

In the block diagram of the driving system supplied from the unipolar traction network of symbolic value *3 kV* there is a three-phase traction inverter. In the modelling of modulations taking place in this inverter a special signal controlling conduction states of switchable valves is used. This two states signal encoding the conducting states of the inverter valves (1 = open, 0 = closed) is generated by trigonometric series introduced in [5]. The signal fulfils the conditions allowing to name it generalized function of time and use the theory of distribution for calculation of electric circuits operating in switching modus. The use of the Theory of Distributions allows to bypass many limitations known from the Function Analysis such as: differentiability, fulfillment of Dirichlet's conditions and convergence of Fourier series. The folded form of the switch signal is also used to the description of the transparency and separation states of inverter and have been named in [6] TRANSEP distribution.

The generator of the pulse series controlling the conduction state of a single-phase inverter is presented in the form of two equations. The first equation defines the relative value of the fundamental harmonic amplitude, phase and frequency in the phase voltage of the induction machine. The second equation, sums the number *"n"* harmonics of the control signal into a series of 0/1 level signal called control distribution in [6]. In order to shorten the length of the all equation, the exponential form of the notation and the function sinus cardinalis is used. The parameters of this control distribution are:

1. *m1* - relative amplitude of fundamental harmonic
2. *f1* - frequency of fundamental harmonic of pulse series
3. f_T – switching frequency of controllers
4. $\phi 1$, $\phi 2$, $\phi 3$ - phase angles of voltages with amplitude *m1*
5. $\varepsilon\iota$ = *0 Rad* - initial phase of impulse sequence
6. *n* - number of harmonics summed up

Three control pulse generators are required for the three-phase inverter. The signals controlling these phase inverters are obtained by changing the value of index "*i*" in equation (10) from *"R"* to *"S"* and to *"T"*.

$$\begin{cases} m_{SIi}(t) = 0.5 + 0.5\cdot m1\cdot cos(2\cdot\pi\cdot f1\cdot t-\phi i) \\ tsd_{SIi}(t) = m_{SIi}(t)\left\{1+2\sum_n Re\left[sinc(n\cdot\pi\cdot m_{SIi}(t))\cdot e^{jn\cdot\left(2\pi\cdot f_T\cdot t-\varepsilon i\right)}\right]\right\} \end{cases} \qquad (10)$$

In the calculations presented, the possibility of summarizing of these three phase control distributions into a complex form of the rotating distribution vector of the inverter according to the formula (11) was used:

$$Vtsd_{MI}(t) = \frac{2}{3}\sum_i tsd_{SIi}(t)\cdot e^{j\frac{(i-1)\cdot 2\cdot\pi}{3}} \qquad (11)$$

The analytically generated rotating vector of the bridge inverter distribution for *m1 = 0.81* and $\psi 1$ *= 0.64 Rad*, *fT = 810 Hz*, $\varepsilon 1$ *= 0 Rad*, *n = 500* in a *20 ms* time window with Fourier transform resolution *fr = 50 Hz* is shown in Figure 3d.

IV. CALCULATION OF THE MACHINE SUPPLY

In a voltage inverter, two waveform changes occur simultaneously, which are described by separate equations: voltage and current modulation. These two processes are described in time domain in equation (12) containing rotating vectors of voltage, current and distribution of the inverter. The first inversion produces three pulsating phase voltages from a unipolar DC circuit voltage. The second inversion describes the current modulation and changes the three phase currents into a single pulsating bidirectional DC link current. The calculations written in vector form in (12) are usually performed for each phase separately.

$$\begin{cases} \overrightarrow{Vu_1(t)} = u_{DC}(t) \cdot \overrightarrow{Vtsd_{3PH}(t)} \\ i_{DC}(t) = \frac{3}{2} \cdot Re\left[\overrightarrow{V\iota_1(t)} \cdot \overrightarrow{Vtsd_{3PH}{}^*(t)}\right] \end{cases} \quad (12)$$

The phase voltages calculated in the time domain were transformed into the frequency domain using the Fourier Transform. In the calculation of the machine phase currents the previously introduced immittance was used.

$$I1_i = \frac{U1_i}{IM_{U_1toI_1}} \quad (13)$$

The stator currents were used to calculate the voltage drops across the stator winding resistance of R_1.

$$UR1_i = R1 \cdot I1_i \quad (14)$$

The next quantity to be calculated is the magnetic flux linkage of the stator winding. In the modified equivalent diagram of an induction machine this quantity plays an important role:

$$\Psi1_i = \frac{U1_i}{IM_{U_1to\Psi_1}} \quad (15)$$

The voltage induced in the machine windings by the stator flux linkage can be counted in many ways. In this paper, the method shown in equation (16) is used:

$$U\Psi1_i = IM_{U_{\Psi_1}to\Psi_1} \cdot \Psi1_i \quad (16)$$

The current flowing through inductance L_1 caused by the direct action of voltage $U_{\Psi 1}$ is the magnetizing current.

$$IL1_i = \frac{\Psi1_i}{IM_{\Psi_1toI_{L1}}} \quad (17)$$

The rotor current is shown as a function of the stator flux linkage:

$$I2_i = IM_{I_2to\Psi_1} \cdot \Psi1_i \quad (18)$$

On the impedance Z_2 located in the rotor circuit there is a voltage loss equal to:

$$UZ2_i = IM_{U_{WZ}to\Psi_1} \cdot \Psi1_i \quad (19)$$

The energy entering the rotor is absorbed by the energy sink. The sink draws the electrical power and converts it into the mechanical power at the machine shaft. The complex amplitude of the voltage at the terminals of this sink has the value:

$$UWM_i = IM_{U_{WM}to\Psi_1} \cdot \Psi1_i \quad (20)$$

V. POWER FLOW THROUGH THE SYSTEM

For power flow description, the results of all calculations performed in the frequency domain were inverse transformed into the time domain. All three-phase quantities are represented in the form of rotating vectors to simplify the notation of operations.

The power consumed by the induction machine is described by following equation:

$$p_1(t) = \frac{3}{2} \cdot Re\left\{\overrightarrow{Vu_1(t)} \cdot \overrightarrow{V\iota_1{}^*(t)}\right\} \quad (21)$$

The power absorbed by the sink located in the rotor is expressed by the product of the rotating voltage vector at the sink and the conjugate value of the rotating current vector in the rotor circuit. If all unaccounted losses are neglected, this power is equal to the mechanical power at the machine shaft:

$$p_{WM}(t) = \frac{3}{2} \cdot Re\left\{\overrightarrow{Vu_{WM}(t)} \cdot \overrightarrow{V\iota_1{}^*(t)}\right\} \quad (22)$$

It remains to calculate the torque of the induction machine. The value of the torque is described by the first equation in the equation system (23) of the induction machine as an electrical into mechanical energy converter. The second equation describes the value of the voltage induced in the machine windings in response to excitation by the rotating magnetic flux linkage vector of the stator.

$$\begin{cases} m_M(t) = \frac{3}{2} \cdot p \cdot Re\left[j \cdot \overrightarrow{V\psi_1(t)} \cdot \overrightarrow{V\iota_1{}^*(t)}\right] \\ \overrightarrow{Vu_{\Psi 1}(t)} = (\omega_2 + p \cdot \Omega_M) \cdot j \cdot \overrightarrow{V\psi_1(t)} \end{cases} \quad (23)$$

The value of the torque on the shaft of an induction machine depends additionally on the number of pole pairs "p" of the stator winding. This number acts similarly to a mechanical gearbox with a ratio equal to "p". The number of the pole pairs increases the value of the developed torque and reduces the angular speed of the motor shaft.

VI. DC LINK LOAD CURRENT OF THE INVERTER

The three-phase currents flowing through the stator windings of an induction machine are demodulated into bidirectional DC circuit current in the process described in the second equation in the equations system (12).

VII. RESULT OF MODELLING FOR f1 = 30 HZ

Due to the volume of the performed calculations, this article is limited to modelling only the inverter-converter part of the whole drive system, surrounded by the blue line in Figure 2. The operation of this part of the system was modelled for a stator frequency of *30 Hz* and an inverter switching frequency of *fT = 810 Hz*. This higher switching frequency was chosen due to the development trends in inverter technology. Some abbreviated results of calculations for the frequency of *50 Hz* and power supply with a full *180°* block of the inverter output voltage are also shown to compare the results of the applied calculation method with the method most frequently quoted in technical literature for inverter feeding of induction machines.

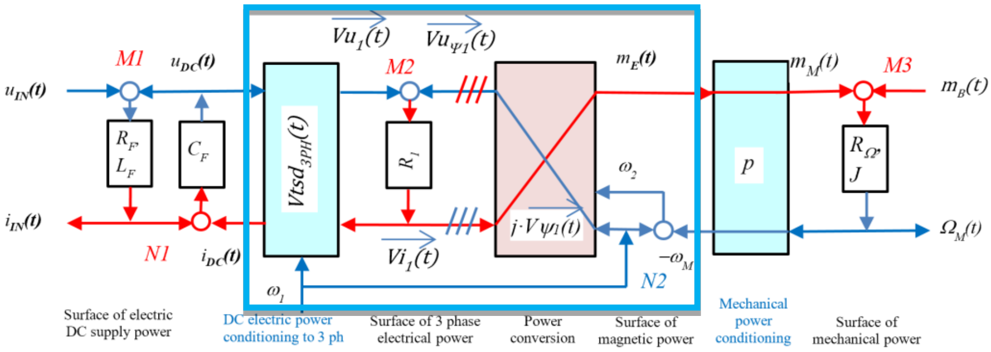

Figure 2. The figure presents a part of DC fed traction drive calculated with using of modified analytical description of induction machine. Input quantities of the blue block are: DC link voltage $u_{DC}(t)$ and the angular speed of the stator magnetic field $\omega_M(t)$. Output quantities are: torque $m_M(t)$ and DC link current $i_{DC}(t)$

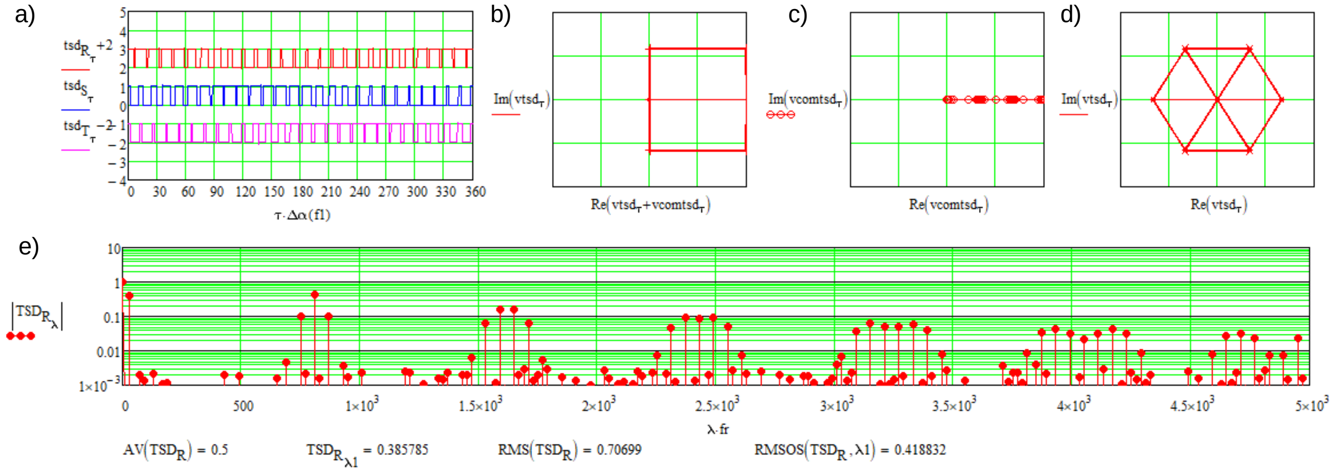

Figure 3. The figure presents in the time and frequency domain the signals controlling the internal state of the three-phase inverter.
a) Instantaneous values of the signals controlling the switching state of the three-phase inverter.
b) Rotating vector of the control distribution in the form containing the common mode components,
c) The common mode part of the rotating vector of the control distribution,
d) Rotating vector of the distribution describing generating the phase voltages,
e) Spectrum of the signal controlling the transparency state of the controller in the first phase of the three-phase inverter (dimensionless signal)

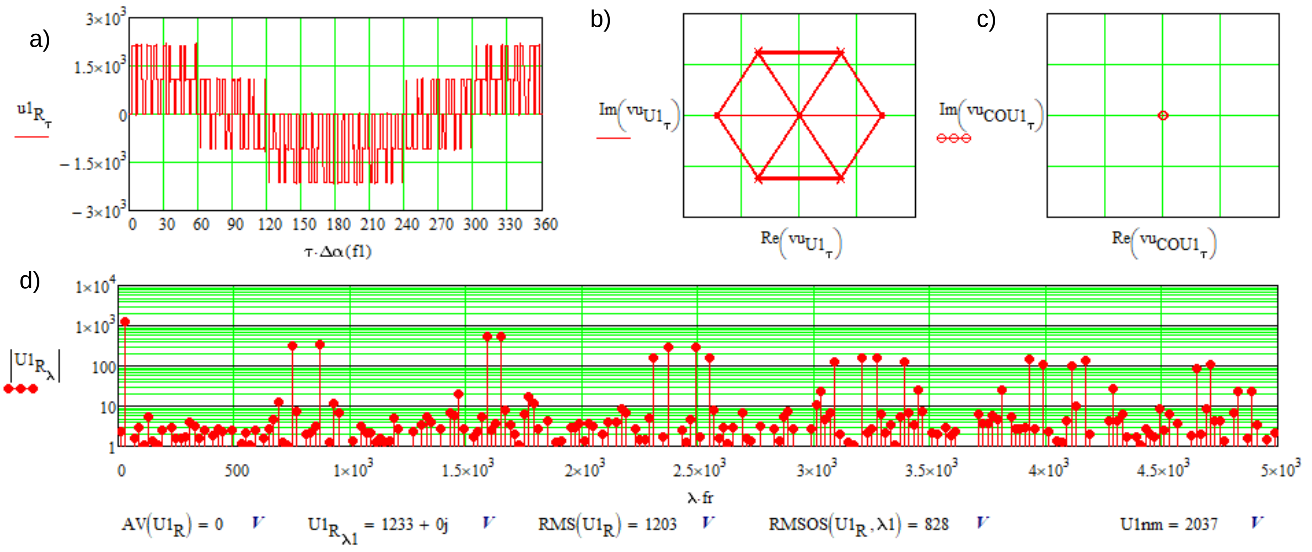

Figure 4. The figure presents in the time and frequency domain the voltages on the stator phase windings of an induction machine.
a) The instantaneous value of the voltage on the phase winding R,
b) The rotating vector of the phase voltages,
c) An image showing that there are no common mode harmonics in the phase voltages,
d) The amplitude spectrum of the phase voltage. Harmonic amplitudes in Volts

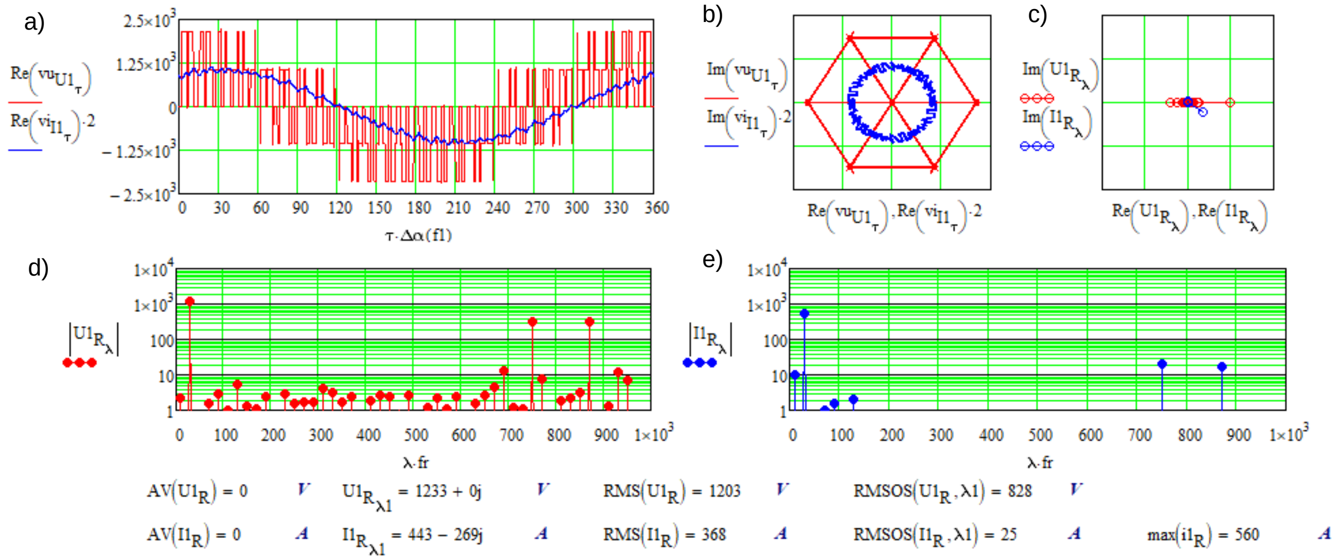

Figure 5. The figure presents in the time and frequency domain the stator voltages and currents of a machine.
a) Instantaneous values of voltage and current in winding R,
b) Rotating vectors of voltage and current,
c) Complexor of voltage and current in the phase R of an induction machine.
d) Amplitude spectrum of phase voltage,
e) Amplitude spectrum of phase current

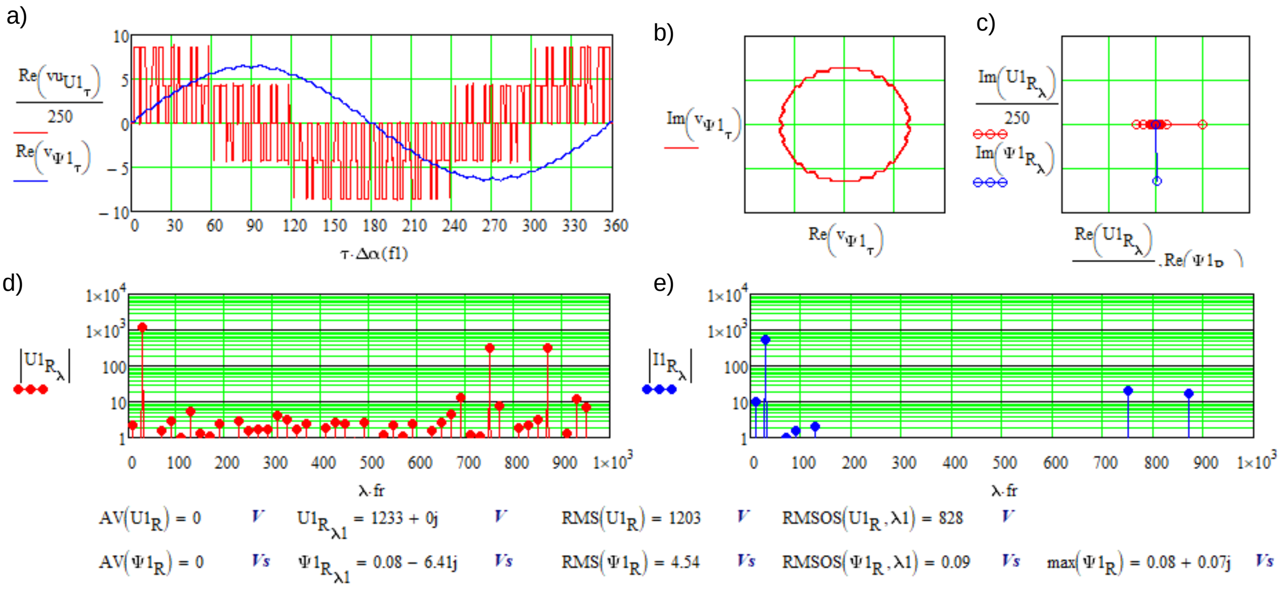

Figure 6. The figure presents in the time and frequency domain the instantaneous value of the phase voltage and the stator winding flux linkage of the machine,
a) Instantaneous values of the stator terminal voltage and the stator windings flux linkage phase R,
b) Rotating vector of the stator flux linkage,
c) Complexor of the voltage and flux linkage in phase R of the induction machine.
d) Amplitude spectrum of phase voltage,
e) Amplitude spectrum of the stator flux linkage

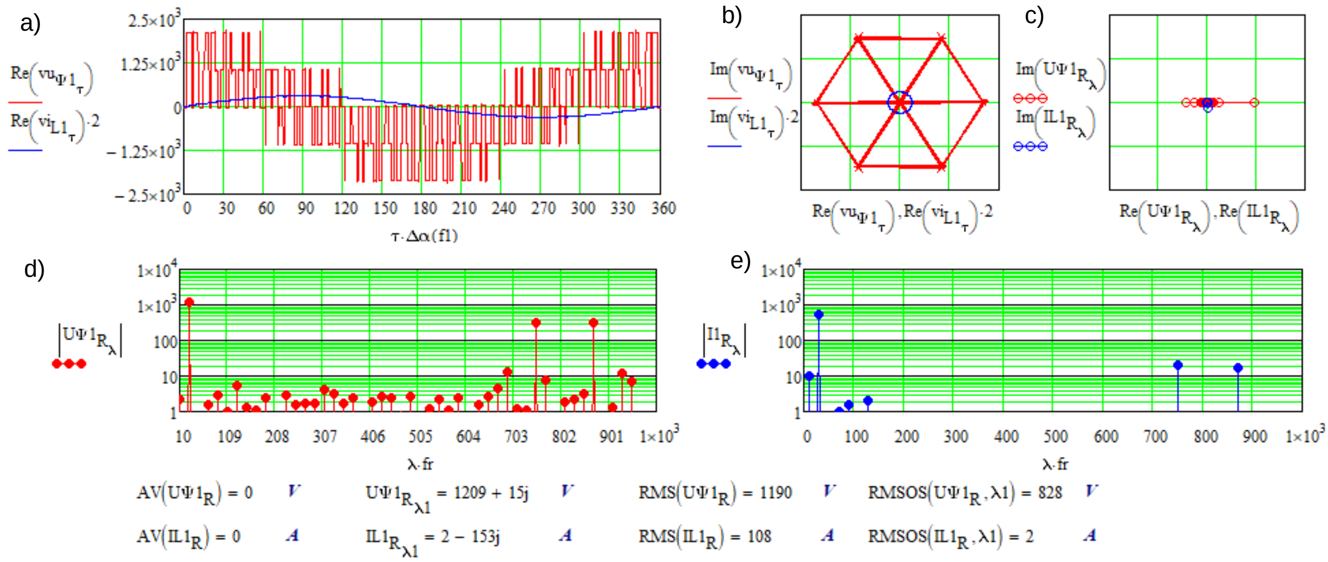

Figure 7. The figure presents in the time and frequency domain the voltage induced by the stator flux linkage and the current magnetizing the magnetic circuit of an induction machine.
a) instantaneous value of voltage acting on the inductance and instantaneous value of current flowing through inductance L_1,
b) rotating vectors of voltage and current through L1,
c) Complexor of voltage and current on the excitation branch,
d) voltage amplitude spectrum,
e) current amplitude spectrum

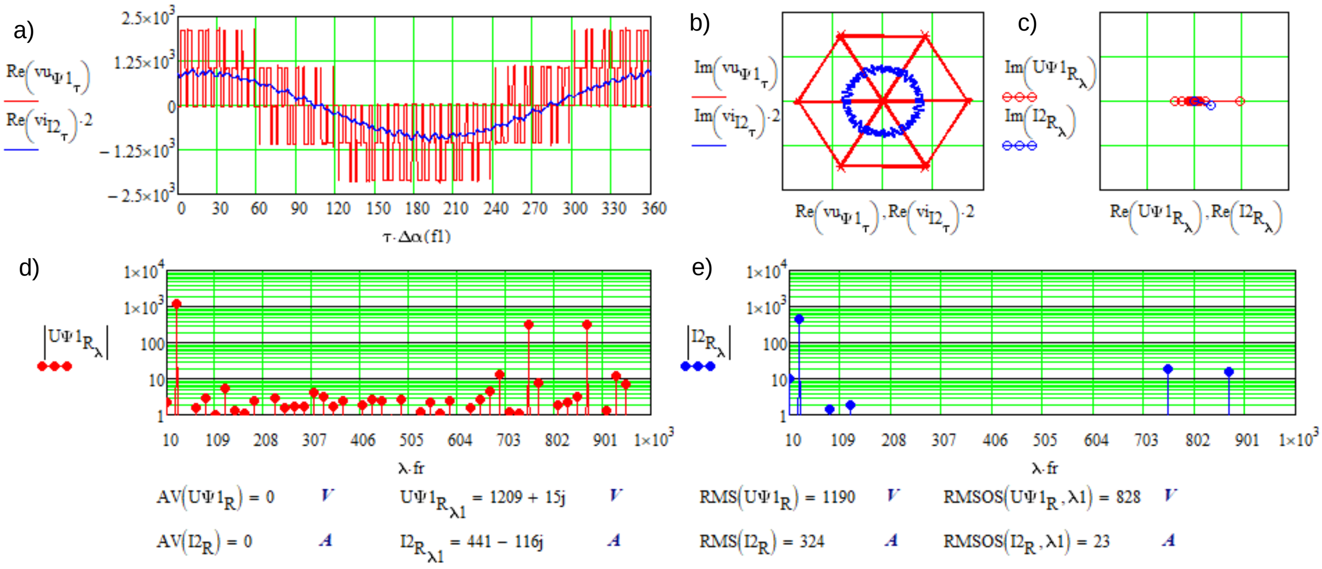

Figure 8. The figure presents in time and in the frequency domain the voltage induced by the stator flux linkage and current of the equivalent rotor circuit of an inverter - fed induction machine.
a) instantaneous values of inducted voltage and current flowing through the rotor circuit,
b) rotating vectors of flux inducted voltage and current through the rotor circuit,
c) complexor of the voltage and rotor current,
d) amplitude spectrum of the voltage induced by stator flux linkage,
e) amplitude spectrum of rotor current

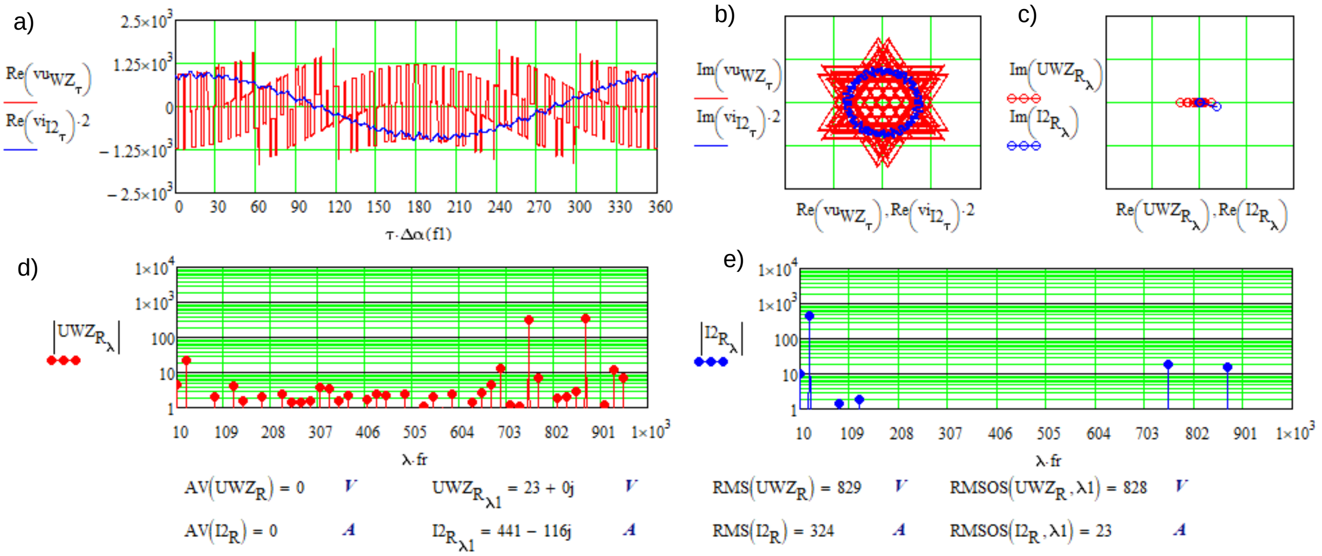

Figure 9. The figure presents in the time domain and the frequency domain the voltage loss on the equivalent rotor impedance of an inverter-powered induction machine,
a) instantaneous value of the voltage loss on the impedance of the rotor circuit and the current flowing in the rotor,
b) rotating vectors of the voltage on the impedance and the current in the rotor circuit,
c) complexor of this voltage and current,
d) amplitude spectrum of the voltage on the impedance,
e) amplitude spectrum of the rotor current

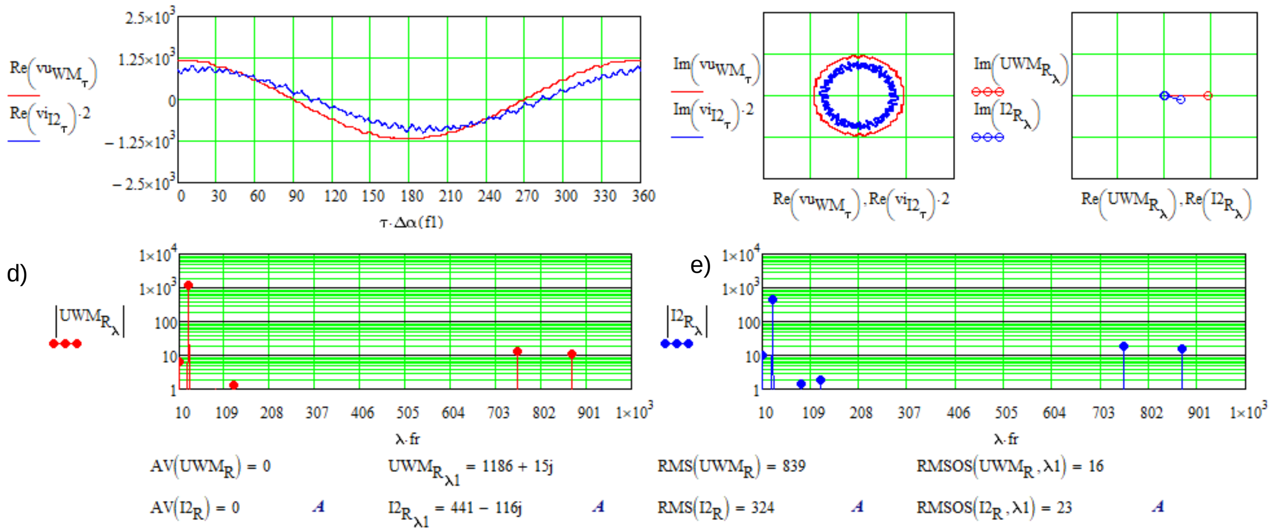

Figure 10. The figure presents in the time and frequency domain the voltage on the rotor voltage sink and the rotor current of the inverter powered induction machine,
a) instantaneous value of the voltage on the sink and the current flowing in the rotor,
b) rotating vectors of the voltage at the sink and the current in the rotor circuit,
c) complexor of this voltage and current,
d) amplitude spectrum of the voltage on the sink,
e) amplitude spectrum of the rotor current

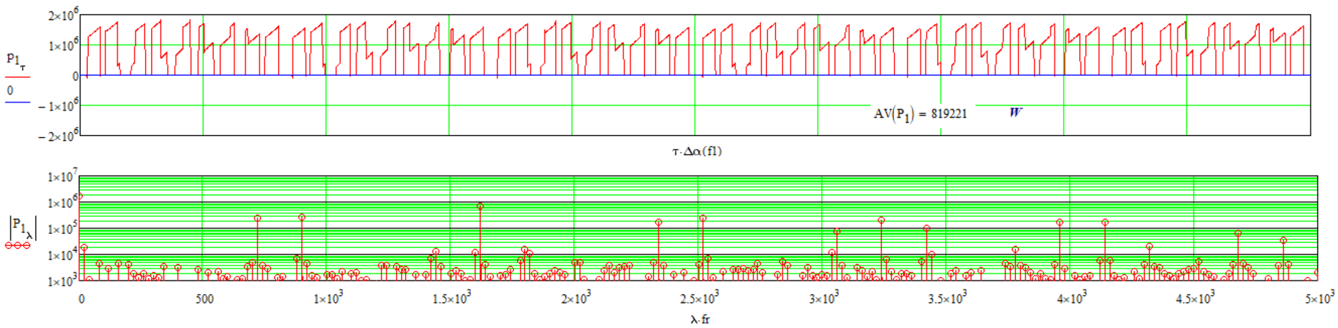

Figure 11. The figure presents the instantaneous value and the amplitude spectrum of the power at the stator terminals of an induction machine

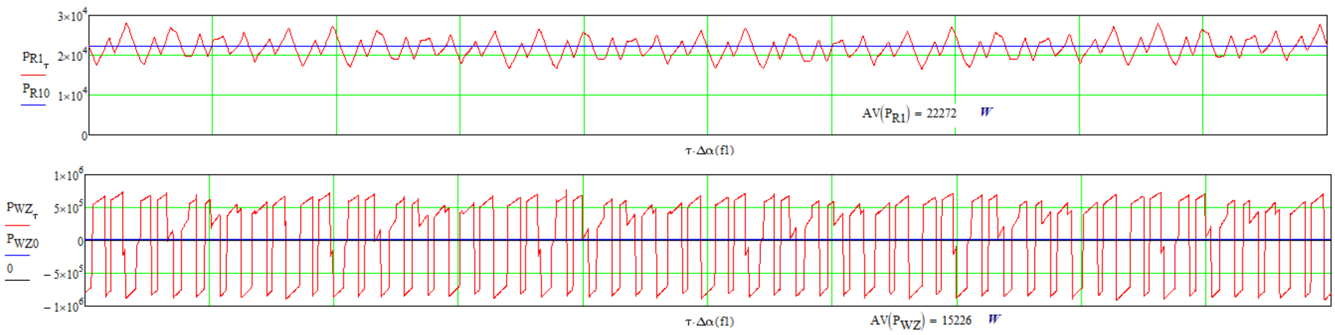

Figure 12. The figure presents the instantaneous power at the stator winding resistance and the instantaneous power at the equivalent rotor impedance Z_2, a) instantaneous power at $p_{R1}(t)$ (red), active power at this resistance (blue), b) instantaneous power and active power $p_{WZ}(t)$ at the equivalent rotor impedance Z_2

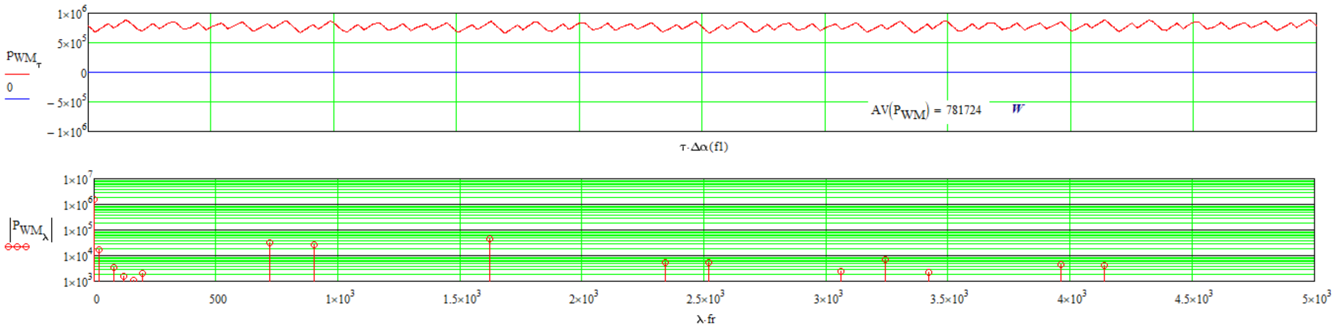

Figure 13. The figure presents the instantaneous value and the amplitude spectrum of the power at the terminals of the power channel in the rotor circuit of an induction machine. After neglecting all unaccounted losses, this power is equal to the mechanical power at the machine shaft

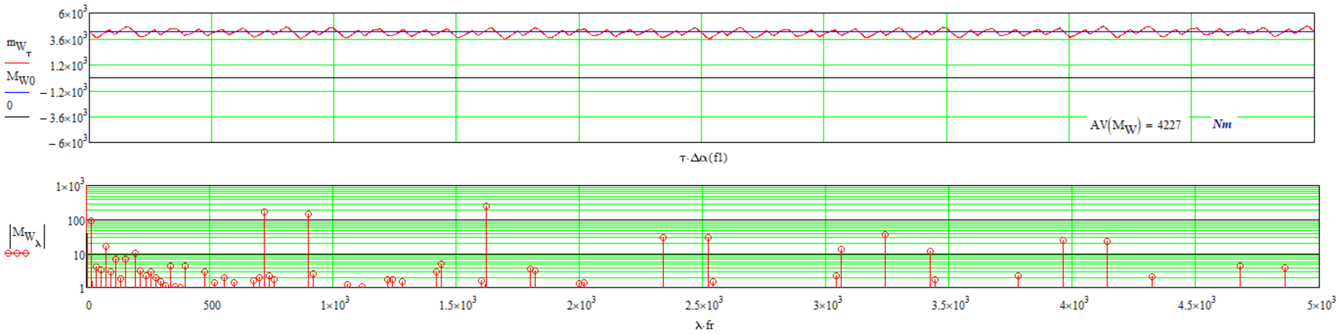

Figure 14. The figure presents the instantaneous value and the amplitude spectrum of the torque on the rotor and shaft of an induction machine

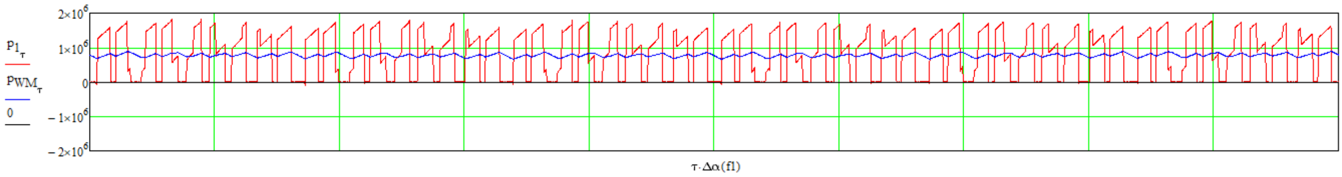

Figure 15. The figure presents a comparison of the instantaneous values of the power at the stator terminals and at the shaft of an induction machine

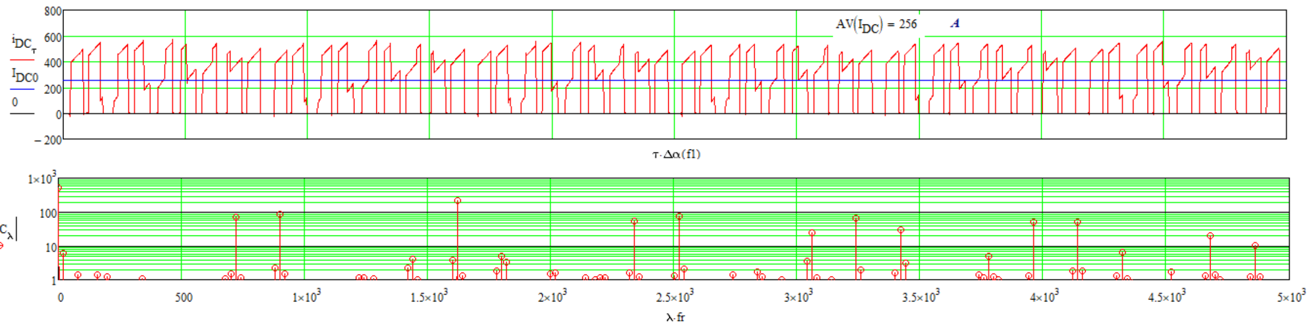

Figure 16. The figure presents the instantaneous value and the amplitude spectrum of the current drawn from the DC circuit by the three-phase inverter supplying the induction machine

VIII. MODELLING RESULTS FOR f1=50 HZ WITH FULL DRIVE CONTROLLABILITY IN THE WEAKENED FIELD RANGE

Operation with an increased average value of the DC link voltage up to 3600 V and avoidance of inverter pulsations with 180° voltage blocks length allows full controllability of the drive system within the entire stator frequency range.

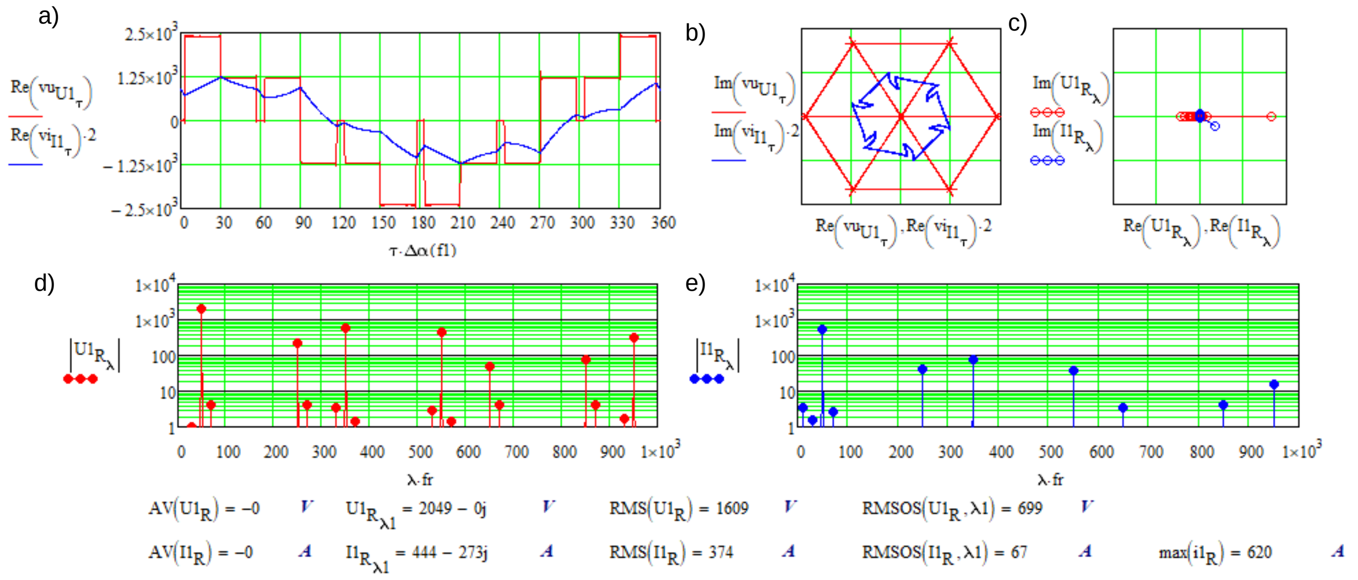

Figure 17. The figure presents the instantaneous values, rotating vectors, complexor diagrams and amplitude spectra of the phase voltages and currents of a machine operating at stator frequencies higher than 50 Hz with 3-voltage pulses at invertor terminals

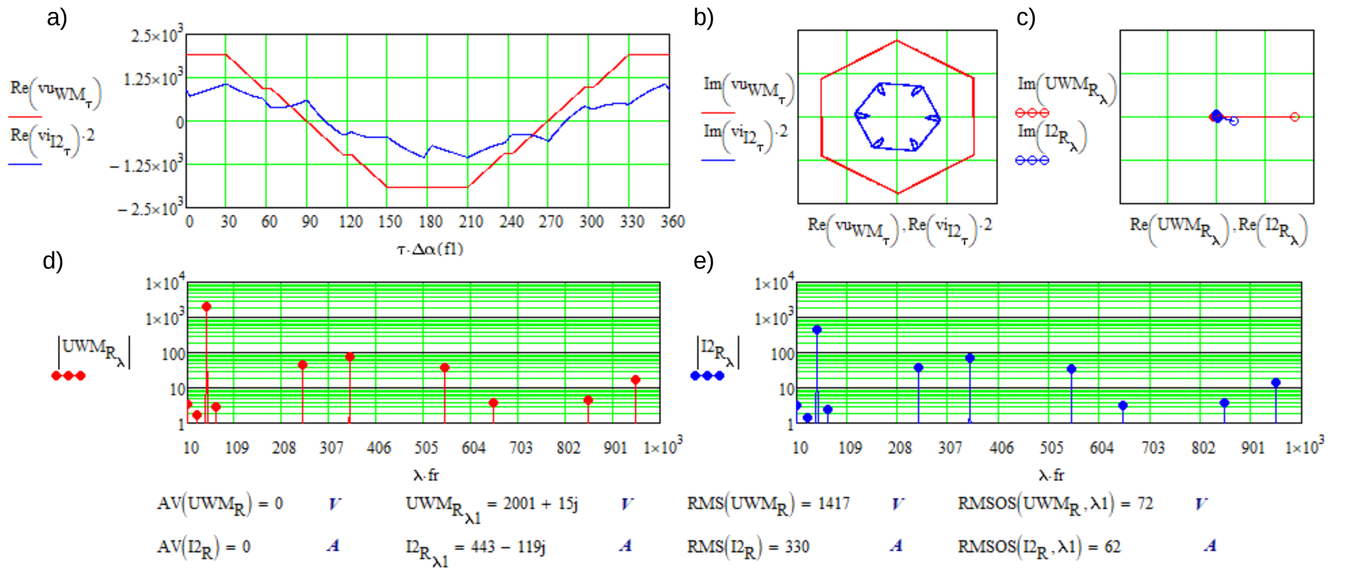

Figure 18. The figure presents instantaneous values, rotating vectors, complexor diagrams and amplitude spectra of the rotor sink voltage and rotor phase currents of a machine supplied at stator frequencies higher than 50 Hz with 3 voltage pulses at the inverter output terminals

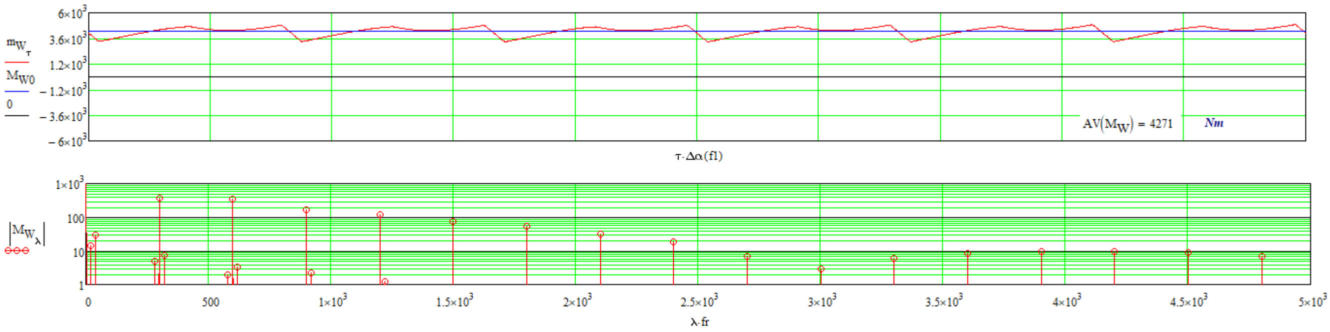

Figure 19. The figure presents the instantaneous value and the amplitude spectrum of the torque on the rotor and shaft of an induction machine supplied with 3 voltage pulses at the output terminals of inverter

IX. MODELLING RESULTS FOR f1=50 HZ WITHOUT INCREASE OF THE DC CIRCUIT VOLTAGE

Operation without increased average value of the DC link voltage with the intention to use the full rated value of machine phase voltage means that pulsation with one 180° voltage block is necessary. Operation with a full block of even slightly varying voltage in the DC circuit leads to a loss of full controllability of the drive system in the weakened field range.

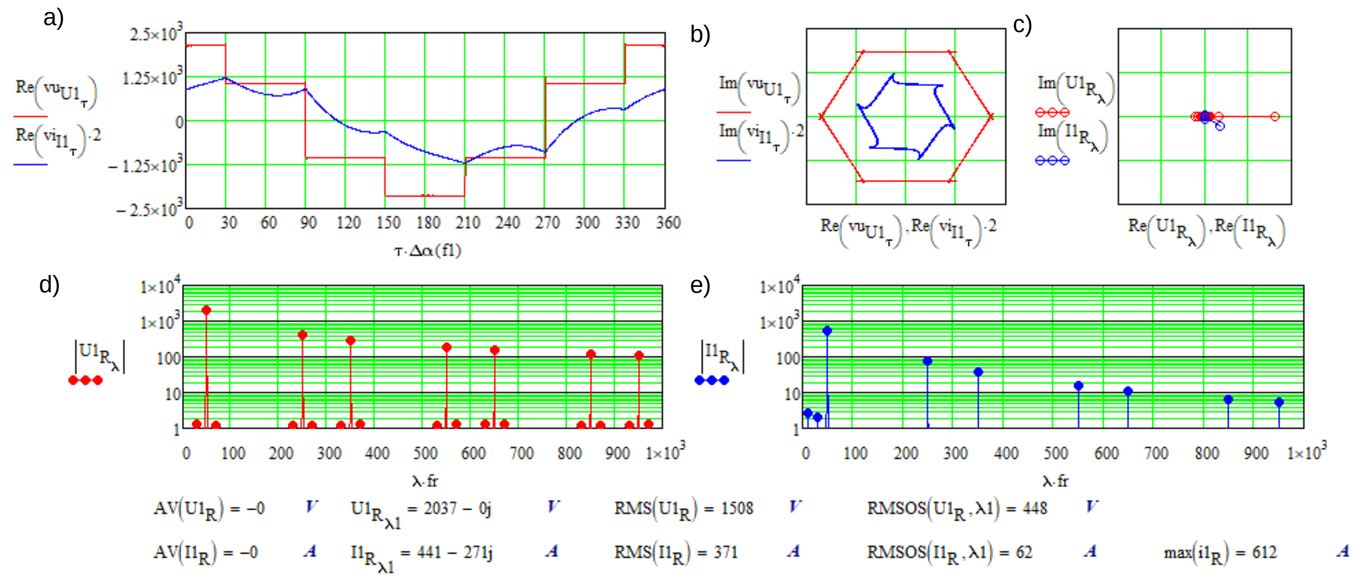

Figure 20. The figure presents the instantaneous values, rotating vectors, complexor diagrams and amplitude spectra of the phase voltages and currents of a machine operating at stator frequencies higher than 50 Hz with single 180° long voltage pulses at invertor terminals

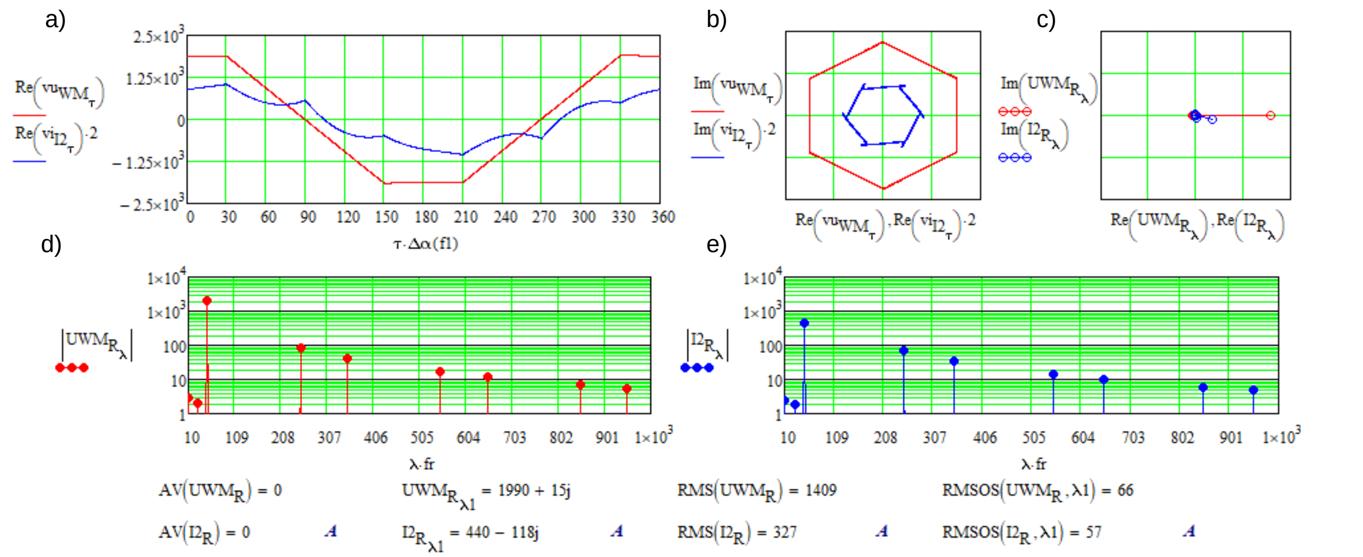

Figure 21. The figure presents instantaneous values, rotating vectors, complexor diagrams and amplitude spectra of the rotor sink voltage and rotor phase currents of a machine supplied at stator frequencies higher than 50 Hz with single voltage pulse at the inverter output terminals. Pulsation with 180° voltage pulses causes loss of full control capability of the drive system in the range of weakened field

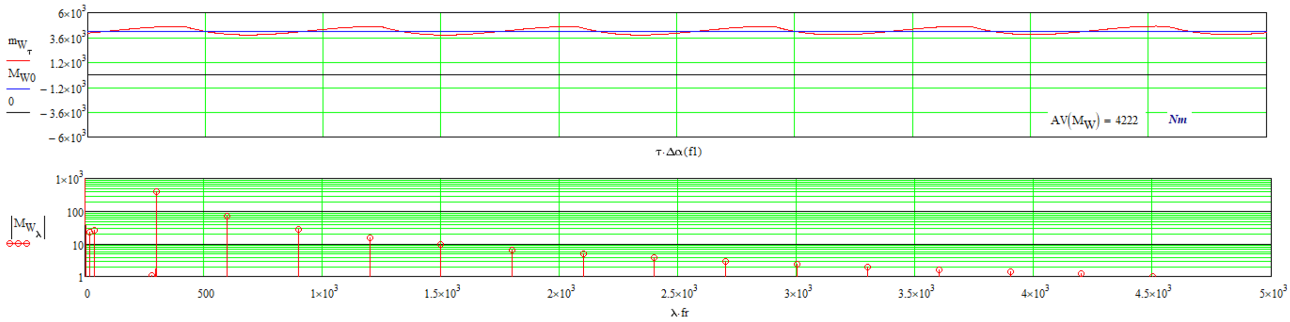

Figure 22. The figure presents the instantaneous value and the amplitude spectrum of the torque on the rotor and shaft of an induction machine supplied with single voltage pulses at the output terminals of inverter

X. CONCLUSION

The inverter drive calculation method used in this papier allows accurate dimensioning of the elements of electrical part of the drive. This method uses the advantages of the fast Fourier Transform to perform part of the calculation in the time domain and part in the frequency domain.

For the control the transparency and separation states of inverters valves a generalized function of time in the form of a trigonometric series, called the inverter TRANSEP distribution, is used.

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