# Adaptive Piecewise Linear Controllers and Applied to High-Order Industry Processes

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***Abstract*—In this paper, an adaptive piecewise linear control scheme is proposed for improving the performance and response time of high-order industry processes. It is a gain stabilized control technique. No large phase lead compensations or pole zero cancellations are needed for performance improvement. Large gain is used for large tracking error to get fast response. Small gain is used between large and small tracking error for good performance. Large gain is used again for small tracking error to cope with disturbance. It gives a command independent response. The proposed control scheme is applied to an altitude control system, three high-order industry process examples and a complicated hydraulic control system those have been compensated by PID controllers. Time responses show that the proposed method gives significant improvements for response time and performance.**

***Keywords—Piecewise linear controller, Adaptive gain, High-order System, Process control***

## I. INTRODUCTIONS

Gain and phase stabilized are two conventional design methods for feedback control systems. They can be analyzed and designed in gain-phase plots to get wanted gain and phase margins or gain and phase crossover frequencies [1,2]. The gain crossover frequency is closely related to the system band-width( or response time). The phase margin is closely related to performance( or peak overshoot). In general, fast response time and good performance can not be obtained simultaneously for some feedback control systems. For example, the altitude control system of the airframe with altitude and altitude rate feedbacks needs large altitude loop gain for fast response time and low altitude loop gain for good robustness. It is conflict with another. A simple and effective way to solve this problem and can provide better results those of linear controllers are generally expected. The proposed method has been applied for some specified servo mechanical control system[3], and now will be applied for very high-order industry processes.

Variable structure control is a switching control method for feedback control systems[4-7]. It gives good performance and robustness for coping with system uncertainty. But is suffered from chattering problem and state measurements. In this paper, a fast response system and a good performance system are selected for switching. An adaptive switching algorithm is used. There is no discontinuous connection between two systems. Therefore, there is no chattering problem. Gain scheduling has been used successfully to control nonlinear systems for many decades and in many different applications, such as autopilots and chemical processes [8-10]. It is consisted of many linear controllers for operating points to cope with large parameter variations. This concept will be expanded for response time and performance. Operating points are replaced by fast response and good performance conditions and interpolation for gain evaluation is replaced by an adaptive switching point. It is determined by the filtered command tracking errors. Nonlinear controllers syntheses using inverse describing function for use with hard nonlinear system has developed for several researchers[11-14]. They are complicated but effective for nonlinear systems. In this paper, a simple three segments piecewise linear controller is proposed. It is easy to analyses and design. Furthermore, it gives an almost reference input independent response.

## II. THE PIECEWISE LINEAR CONTROLLER

Fig.1 shows piecewise linear description of the symmetrical nonlinearity. Piecewise linear segments $y_{i(+)}/y_{i(-)}$ are in the form of

$$y_{1(+)}=k_1x; \tag{1}$$

$$y_{i(+)}=k_ix+\sum_{j=2}^{i}(k_{j-1}-k_j)D_{j-1}; i>1; \tag{2}$$

$$y_{1(-)}=k_1x; \tag{3}$$

$$y_{i(-)}=k_ix-\sum_{j=2}^{i}(k_{j-1}-k_j)D_{j-1}; i>1; \tag{4}$$

Now, the problem is to determine the values of switch points $D_i$ and gains $k_i$ between $D_i$ and $D_{i+1}$ for the wanted responses time and performance. For illustrating purpose, two switching points $+D_1$, $-D_1$ and two gains $k_1$, $k_2$ will be used to illustrate the advantage of the proposed piecewise linear controller;

i.e., three segments are discussed. In this work, switching points $+D_1$ and $-D_1$ are not fixed and will be determined by the absolute value of the command tracking error for feedback control systems.

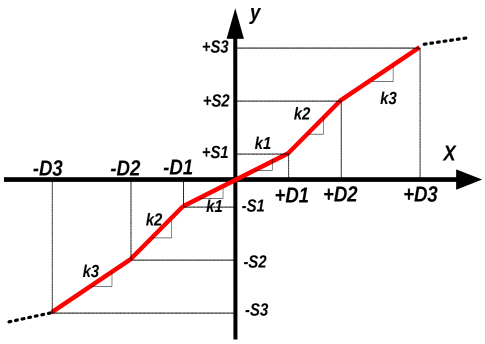


**Fig.1. Piecewise linear description of an adaptive gain.**

Considers a second order numerical example[3] described by

$$G(s) = \frac{1}{s(s+2)} \quad (5)$$

It is closed with a loop gain K. Then the closed-loop transfer function is

$$T(s) = \frac{K}{s^2 + 2s + K} \quad (6)$$

Poles locations and natural frequency( $\omega_n$ ) for two loop gains( $K_1, K_2$ ) are given as:

$K_1 = 0.500; poles: -0.2929, -1.7071$ ;
$K_2 = 10.00; poles: -1.0 \pm j3.0; \omega_n = 3.1623$;

They are an over-damped and an under-damped systems. Time responses are shown in Fig.2 for $K = K_1$ (small-dot-line) and $K = K_2$ (large-dot-line) in which R represents the reference input and C represents the output. The strategy for gain switching is (1) large gain ( $K_2$ ) for large tracking error to get fast response and (2) small gain ( $K_1$ ) for small tracking error (E) to get good performance. It is a variable structure system and can be achieved by selecting a proper switching point $D_1$ of the piecewise linear controller shown in Fig.1. For example, the optimal switching point $D_1$ is selected as 0.525 for R=1 to get both fast response and good performance. Large gain ( $K_2$ ) is used for $|E|> D_1$ and small gain ( $K_1$ ) is used for $|E| \le D_1$ . Step response is shown in Fig.1 (solid-line) also for R=1. It shows that adaptive gain can give a good result for fast response and good performance.

However, it is not true for R is equal to 5, 10 and 50, respectively. Those step responses are shown in Fig.3. Naturally, another switching point $D_1$ for R=5, 10 and 50 can be selected for getting good performance. They are 2.625, 5.250 and 26.250 for R=5, 10 and 50, respectively. They are true for step responses from zeros to 5, 10 and 50 only. Another possible way for the switching point can be dependent on the tracking error (E). A possible switching rule for $D_1$ is found as $D_1 = 0.925|E|$ for good performance. Fig.4 shows time responses for R=1, 5, 10 and 50, respectively. It can be seen that the switching rule gives an input command (R) independent results. However, they are slower than results shown in Figs. 2 and 3.

One possible way to speed up the time response is enlarging the large gain phase in the beginning. A low-pass filter $D(s) = K_n/(T_n s + 1)$ for the absolute tracking error (E) to get $D_1$ is used. Fig.5 shows faster response is get for $K_n = 1.0445$ and $T_n = 1/\omega_n$ . The switching point $D_1$ is shaped for speed up the responses while keeping performance unchanged. Fig.6 shows input independent responses for R=1,5,10 and 50. Note that the natural frequency ( $\omega_n$ ) for $K = K_2$ is used to find $T_n$ . Therefore, it is needed to find $K_n$ only.

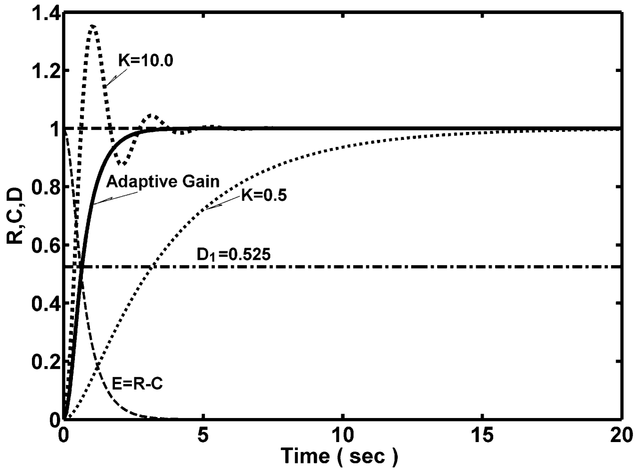


**Fig.2. Time responses for K=0.5, 10 and Adaptive gain of the illustrating example.**

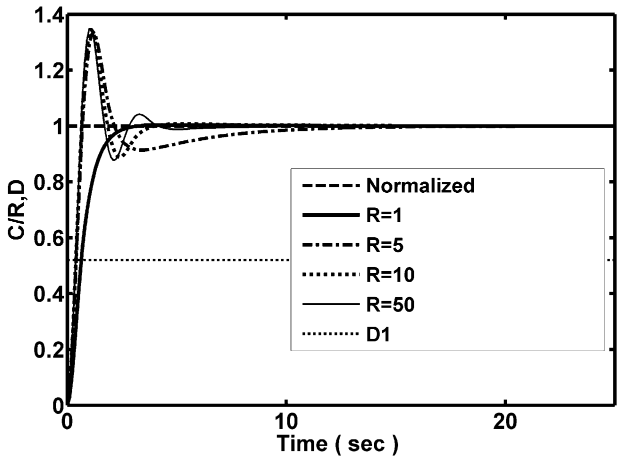


**Fig.3 Time responses for R=1,5,10,50 with $D_1$=0.525 of the illustrating example.**

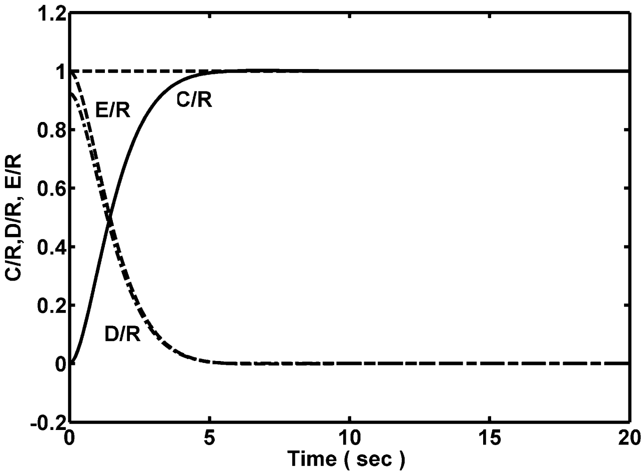


**Fig.4 Time responses for R=1,5,10,50 with $D_1 = 0.925|E|$ of the illustrating example.**

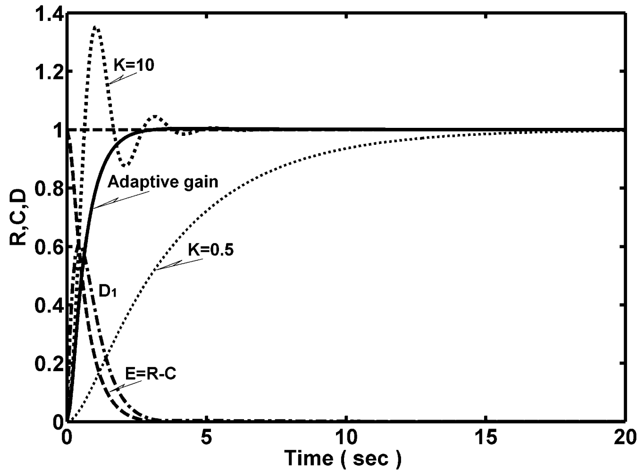


**Fig.5 Time responses for R=1 with $D(s) = K_n/(T_n s+1)$ of the illustrating example.**

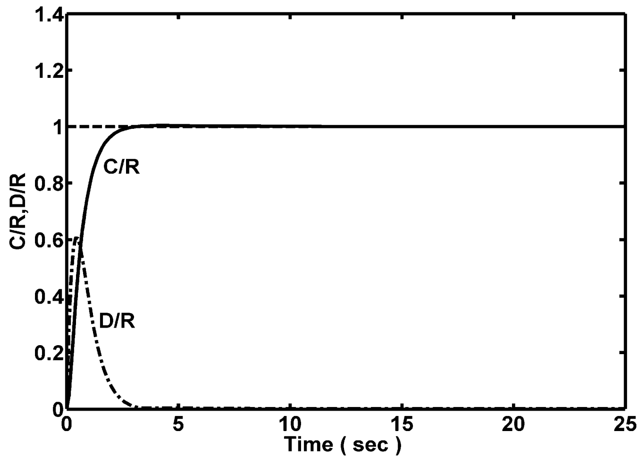


**Fig.6 Time responses for R=1,5,10 and 50 of the illustrating example.**

The design procedures for the proposed method using the adaptive piecewise linear controller are

***Step 1:*** Selecting two loop gains for fast response and good performance, respectively. In general, high loop gain ( $K = K_2$ ) for fast responses and low gain ( $K = K_1$ ) for good performance. The rise time of the system with high gain meets the design specification. The peak overshoot of the system with low gain meet the design specification.

***Step 2:*** determining parameters of low-pass filter $D(s) = K_n/(T_n s+1)$ to find the optimal switching point $D_1$. The natural frequency ( $\omega_n$ ) for the high gain system ( $K = K_2$ ) is used to find $T_n$. The natural frequency ( $\omega_n$ ) is close related to the rise time. Another parameter $K_n$ can be found by the optimization method using performance index formulated by integration of the absolute error (IAE) and integration of the square error(ISE) or online parameterized method[15,16]. The iteration rule for finding $K_n$ is formulated as

$$G_n(kT+T) = G_n(kT) \times \{\alpha[Mpc/Mps]^j + (1-\alpha)\}; \quad (7)$$

$$K_n = G_n(kT+T) \quad (8)$$

where Mps is the specification of the Peak point; Mpc is the peak point found using $K_n = G_n(kT)$ ; T is simulation period of one step response; and k is the k$^{th}$ step responses.

The proposed control scheme will be applied to an altitude control system, four numerical examples and those have been compensated by PID controllers.

## III. NUMERICAL EXAMPLES

***Example 1***: Consider an altitude control system shown in Fig.7. This is a control system example that fast responses and good performance are hard to be get simultaneously.

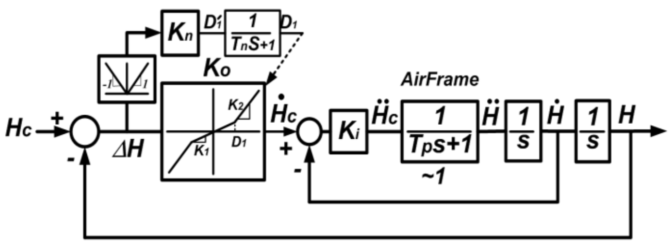


**Fig.7. An altitude control loop.**

The closed-loop transfer function is

$$T(s) = \frac{K_o K_i}{T_p s^3 + s^2 + K_i s + K_o K_i} \quad (9)$$

The characteristic equation of *T(s)* is

$$\Delta(s) = 0.05 s^3 + s^2 + s + K_o \quad (10)$$

for $T_p = 0.05$ and $K_i = 1$. Pole locations and natural frequency( $\omega_n$ ) of dominate poles for low gain( $K_o = K_1$ ) and high gain( $K_o = K_2$ ) are given as:

$$K_1 = 0.048; poles: -0.050, -1.0023, -18.947;$$
$$K_2 = 1.200; poles: -19.0146, -0.4927 \pm j1.0097 (\omega_n = 1.1235)$$
;

The on-line rule for finding $K_n$ is formulated as

$$G_n(kT+T) = G_n(kT) \times \{0.9[Mpc/Mps]^3 + 0.1\}; \quad (11)$$

$$K_n = G_n(kT+T) \quad (12)$$

with $G_n(0) = 0.5$, $T = 20 seconds$ and $Mps = 1.001$. The found $G_n(kT)$ are

$$G_n(0) = 0.5; G_n(T) = 0.5605; G_n(2T) = 05789;$$
$$G_n(3T) = 0.582; G_n(4T) = 0.582; ...$$

$G_n(kT)$ is converged to be 0.582 within three period simulations. The found switching point is in the form of

$$D_1 = 0.582 |\Delta H| / (0.8901s + 1); \quad (13)$$

The step response is shown in Fig.8. It can be seen that the piecewise-linear controller gives an input independent and good performance result.

The PID controllers are used widely in industry processes[16-23] The proposed control scheme can be used to improve the response time and performance of the designed system. The control configuration is shown in Fig.9. Another three numerical examples will be proposed to illustrate the advantage of the proposed method.

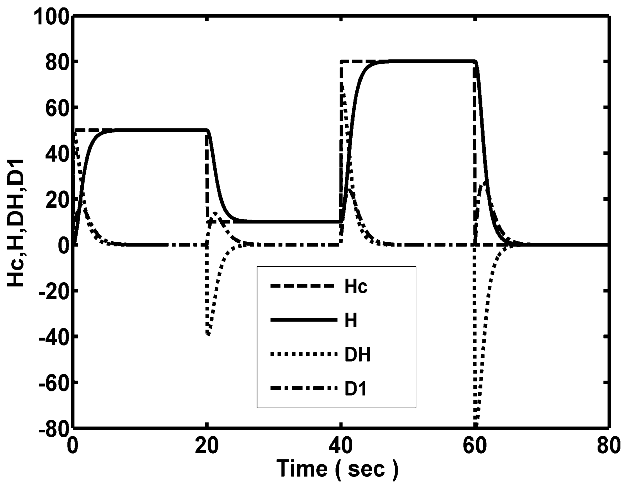


**Fig.8. Step responses of the controlled altitude control system.**

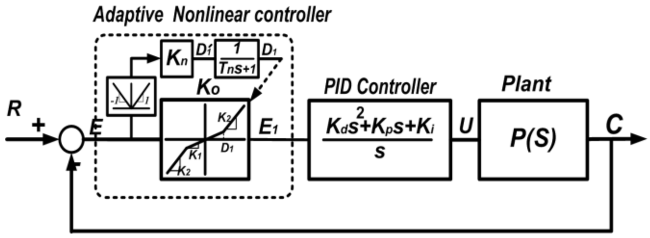


**Fig.9. Control configuration of the industry process.**

***Example 2:*** Consider a stable plant has the transfer function[17]

$$G(s) = \frac{e^{-s}}{(s+1)^2} \quad (14)$$

It is a second order dynamic plus a pure time delay (SOPDT). In this example, a PID controller with parameters

$$K_p = 1.1953; K_i = 0.5942; K_d = 0.7338;$$

is designed first. And then low gain $K_1 = 0.50$ is selected and high gain $K_2 = 2.587$ is selected for the system is just in the sustaining oscillating condition. The oscillation frequency is $\omega_n = 1.5708 rad/s$. Time responses using low gain ( $K_1 = 0.50$ ) and high gain ( $K_2 = 2.587$ ) are shown in Fig.10. They show an over-damped and zero-damped systems. Now, applying the proposed control scheme to the system using

$$K_1 = 0.5000; K_2 = 2.587; K_n = 1.1385; T_n = 0.6366;$$

The $K_n$ is found by following on-line computing rule :

$$G_n(kT+T) = G_n(kT) \times \{0.9[Mpc/Mps]^2 + 0.1\}; \quad (15)$$

$$K_n = G_n(kT+T) \quad (16)$$

with $G_n(0) = 0.5$, $T = 25 seconds$ and $Mps = 1.001$. The found $G_n(kT)$ are

$$G_n(0) = 0.5; G_n(T) = 1.0794; G_n(2T) = 1.1365;$$
$$G_n(3T) = 1.1385; G_n(4T) = 1.1385; ...$$

$G_n(kT)$ is converged to be 1.1385 within three period simulations.

The time response is shown in Fig.10 also. It can be seen that the proposed method can give fast response and good performance simultaneously. It is the combination of over-damped and zero-damped systems with $D_1$. Zero-damped system is used for fast responses and over-damped system is used for good performance. Naturally, it is input command(R) independent also.

Simulation results of the proposed method and four other methods are presented for comparisons. They are Ziegler-Nichols method[18,19] for finding PI and PID compensators, Tan et al[20,21] for finding PID compensator and Majhi[17] for finding PI compensator. Parameters of five found compensators are given below:

(1)*Proposed Method*:

$K_p = 1.1953; K_i = 0.5942; K_d = 0.7338;$

$K_1 = 0.5000; K_2 = 2.587; K_n = 1.1385; T_n = 0.6366;$

(2)*ZN(PI)* : $K_p = 1.240$ and $K_i = 0.251$.

(3)*ZN(PID)* : $K_p = 1.6367, K_i = 0.4187 \; and \, K_d = 0.5972$.

(4)*Tan's(PID)*:

$K_p = 0.620, K_i = 0.5636 \; and \; K_d = 0.1705$.

(5)*Majhi's(PI)*: $K_p = 0.864 \; and \, K_i = 0.3653$.

Time responses are shown in Fig.11. Integral of the Square Error(*ISE*), and Integral of the Absolute Error (*IAE*) are given in Table 1. From Table 1 and Fig.11, one can see that the proposed method gives faster response and better performance than those of other methods presented.

**Table 1. The ISE and IAE of five methods for Example 2.**

| | Proposed | ZN(PI) | ZN(PID) | Tan's(PID) | Majhi's(PI) |
|---|---|---|---|---|---|
| ISE | 1.347 | 2.268 | 1.770 | 2.247 | 2.465 |
| IAE | 1.599 | 4.011 | 2.876 | 3.073 | 4.066 |

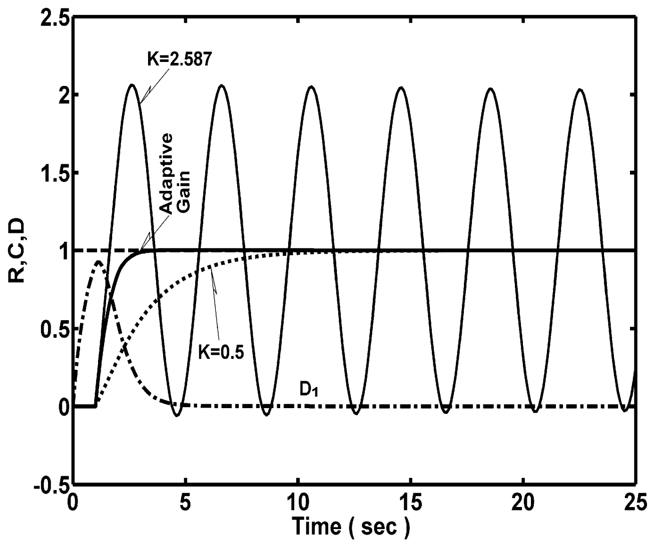


**Fig.10. Step responses for constant gains ( K=0.5 & 2.587) and adaptive gain with $D_1$ of Example 2.**

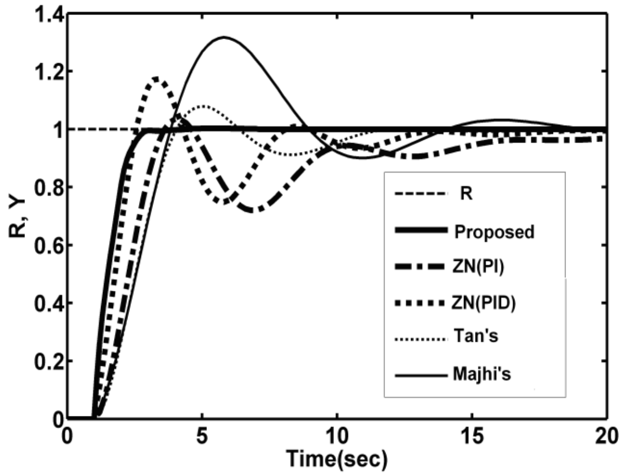


**Fig.11. Comparisons with other methods for Example 2.**

***Example 3:*** Consider the sixth-order plant[17]

$$G(s) = \frac{1}{(s+1)^6} \qquad (17)$$

The optimized PID controller with parameters

$K_p = 1.1800; K_i = 0.3380; K_d = 2.9181;$

Is used for comparison purpose. The performance index $50IAE + ISE$ is used. The dominate pole for high gain $K_2 = 1.400$ are $-1.017 \pm j0.7805$. The natural frequency is $\omega_n = 0.7871 rad/s$. It is an under-damped system. The low gain $K_1 = 0.05$ is selected. It is an over-damped system. Parameters for the proposed control scheme are

$K_1 = 0.0500; K_2 = 1.4000; K_n = 0.4828; T_d = 1.2705;$

Fig.12 shows time responses using optimization method and the proposed method for R=1,5,10 and 50. It can be seen that the proposed method gives better result than that of the optimization method.
Simulation results of the proposed and four other methods are presented for comparisons. They are Ziegler-Nichols rule[18,19] for finding PI and PID compensators, Ho et al.[22] for finding PID compensator and Majhi[17] for finding PI compensator. Parameters of five found compensators are given below:

*(1)The proposed method:*

$K_p = 1.1800; K_i = 0.3380; K_d = 2.9181;$

$K_1 = 0.0500; K_2 = 1.4000; K_n = 0.4828; T_d = 1.2705;$

*(2)ZN(PI)* : $K_p = 1.079 \; and \, K_i = 0.110$.

*(3)ZN(PID)* : $K_p = 1.4248, K_i = 0.1838 \; and \; K_d = 1.360$.

*(4)Majhi's(PI):* $K_p = 0.7736 \; and \, K_i = 0.1547$.

*(5)Ho's(PID)* : $K(s) = 1.3(1 + 0.189/s + 1.3s/(0.13s+1))$.

Time responses are shown in Fig.13. Integral of the Square Error(ISE), and Integral of the Absolute Error (IAE) are given in Table 2. From Table 2 and Fig.13, one can see that the proposed method gives faster response and better performance than those of other methods.

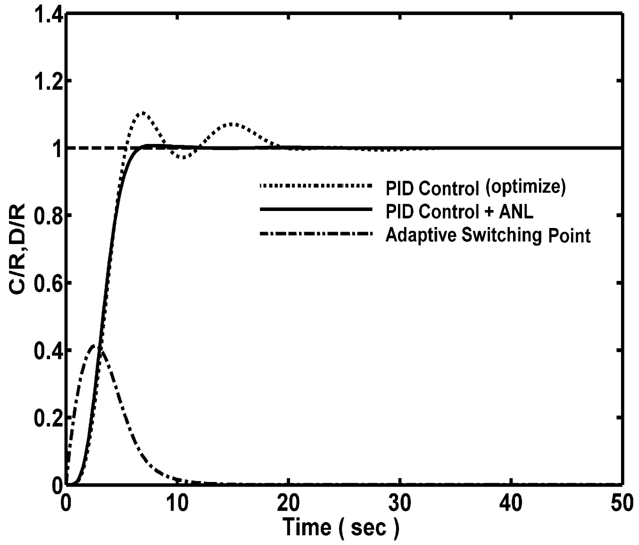


**Fig.12. Normalized responses of C/R and D/R of Example 3.**

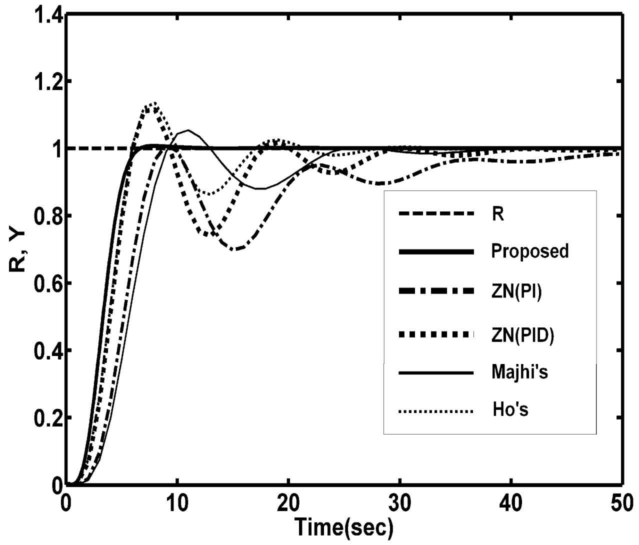


**Fig.13. Comparisons with other methods for Example 3.**

**Table 2. The ISE and IAE of five methods for Example 3.**

| | Proposed | ZN(PI) | ZN(PID) | Ho's(PID) | Majhi's(PI) |
|---|---|---|---|---|---|
| ISE | 2.748 | 5.335 | 4.023 | 5.215 | 3.740 |
| IAE | 3.477 | 9.279 | 6.492 | 7.219 | 5.425 |

***Example 4:*** Consider the very high order plant[17]

$$G(s)=\frac{1}{(s+1)^{20}} \quad (19)$$

The optimized PID controller with parameters

$$K_p=0.666; K_i=0.06015; K_d=4.4119;$$

Is used for comparison purpose. The performance index $50IAE+ISE$ is used. The dominate pole for high gain $K_2=2.000$ are $+0.0126\pm j0.19337$ $(\omega_n=0.19378)$. It is an unstable system. The low gain $K_1=0.200$ is selected. It is an over-damped system. Parameters for the proposed control scheme are

$$K_1=0.20; K_2=2.000; K_n=0.6816; T_d=5.1605;$$

Fig.14 shows time responses using optimization method and the proposed method for R=1,5,10 and 50. It can be seen that the proposed method gives better result than that of the optimization method. Note that the compensated system is a combination of an over-damped and an unstable system. That is the proposed control scheme can stabilized the system and gives fast response and good performance.
Simulation results of the proposed and four other methods are presented for comparisons. They are Ziegler-Nichols method for finding PI and PID compensators[18,19], Zhuang et al[23] for finding PI compensator and Majhi[17] for finding PI compensator. Parameters of five found compensators are given below:

*(1)The proposed method*

$K_p=0.666; K_i=0.06015; K_d=4.4119;$

$K_1=0.20; K_2=2.000; K_n=0.6816; T_d=5.1605;$

*(2)ZN(PI)* : $K_p=0.585$ *and* $K_i=0.0305$.

*(3)ZN(PID) :* $K_p=0.77256, K_i=0.05088$ *and* $K_D=4.9135$.

*(4)Zhuang's(PI):* $K_p=0.473$ *and* $K_i=0.058$.

*(5)Majhi's(PI) :* $K_p=0.5097$ *and* $K_i=0.0443$.

Integral of the Square Error(ISE) and Integral of the Absolute Error (IAE) are given in Table 3. Time responses are shown in Fig.14. From Table 3 and Fig.14, one can see that the proposed method gives faster responses time and better performance than those of other methods.

**Table 3. The ISE and IAE of five methods for Example 4.**

| | Proposed | ZN(PI) | ZN(PID) | Zhuang's(PID) | Majhi's(PI) |
|---|---|---|---|---|---|
| ISE | 13.954 | 21.227 | 16.216 | 20.191 | 21.814 |
| IAE | 15.801 | 32.708 | 22.971 | 26.830 | 32.913 |

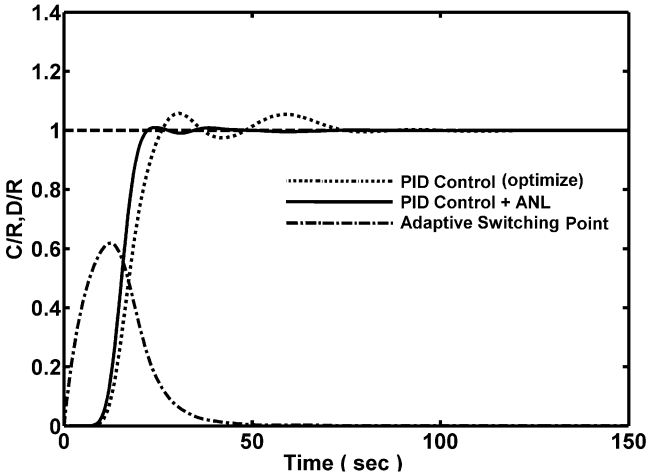


**Fig.14. normalized responses of C/R and D/R of Example 4.**

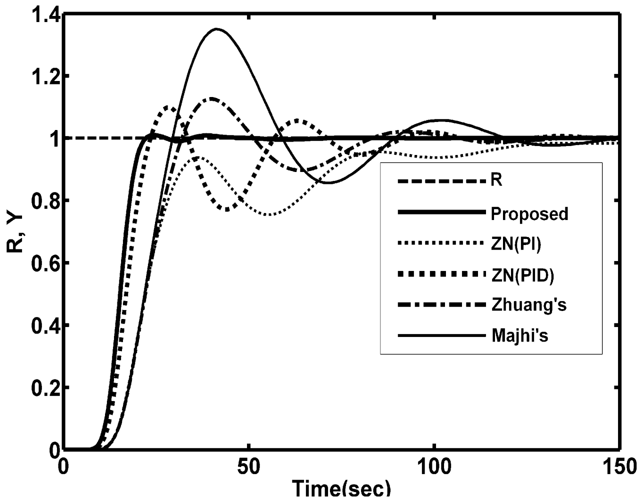


**Fig.15. Comparisons with other methods for Example 4.**

## IV. CONCLUSIONS

The proposed adaptive piecewise linear controller has been shown that provides controlled systems are reference input independent and both good performance and fast response were obtained simultaneously. Three segments piecewise linear controller provided a switching algorithm for low gain and high systems; i.e., low gain for performance and high gain for response time. The switching points were dependent on the command tracking errors. There are zero-damped, under-damped and unstable systems used in Example 2, 3 and 4 individually to get fast responses in large tracking phases.

Four numerical examples were designed and comparesons were made with six famous on-line computing and control methods. They have illustrated better performance and fast response of the proposed method than those of other mentioned methods

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