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Analytical Computation Of Non-Rectangular M-Ary Quadrature Amplitude Modulation Bit Error Probability

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***Abstract*— In this paper, analytical computation of non-rectangular M-ary Quadrature Amplitude Modulation (QAM) modulation Bit Error Probability (BER), required minimum bandwidth and bandwidth efficiency was presented. The detailed mathematical expressions for the BER based on Q-function, erf function and erfc function were presented. The BER based on erfc was implemented in Matlab program. Then sample numerical computation was performed with data rate of 1Mps and for a range of modulation index, (M). The results showed that for any given at energy per bit to the spectral noise density, (Eb/No), the BER decreases as modulation index (M) decreases. For instance, at Eb/No =14, BER = 1.3620E-12 for M =2, BER = 3.6843E-06 for M = 4 and BER = 3.1038E-02 for M = 8. The bandwidth efficiency increases as modulation index (M) increases. In all, the results showed that when the higher order and lower order non-rectangular M-ary QAM are compared, the higher order has higher bandwidth efficiency but lower BER for any given Eb/No.**

Keywords*— *Non-Rectangular M-ary QAM, Bit Error Probability, Q-Function, Symbol Error Probability, Complementary Error Function, Required Minimum Bandwidth, Bandwidth Efficiency

1. INTRODUCTION

Quadrature Amplitude Modulation (QAM) is a form of modulation that combines two amplitude-modulated (AM) signals into one channel, in that way it doubles the effective bandwidth [1,2,3,4,5,6,7,8,9]. The QAM modulation scheme operates with two carrier signals where the two signals have the same frequency but differ in phase by one quarter of a cycle (90°) [10,11,12,13,14]. While one of the signal is called the I signal the other signal is called the Q signal. Notably, QAM has been used in digital systems, particularly for wireless communications applications.

In QAM modulation scheme, bits are represented as points on a constellation map which is a graph of the phase and amplitude modulation points [15,16,17,18]. In this paper, the square QAM constellation, which is referred here as non-rectangular QAM, is considered. The square QAM constellations are normally used when the number (n) of bits per symbol, n = log2(M) is even [19,20,21,22,23,24,25,26]. The square QAM has been used widely in different wireless communication standards; among them are satellite communications, digital video broadcast in satellite and terrestrial communications, wireless fidelity (Wi-Fi), as well as in WiMAX, power line Ethernets, and microwave backhaul systems. Particularly, in this paper, the analytical computation of the non-rectangular multi-level QAM modulation bit error probability is presented. The relevant analytical models for the BEP are presented along with some numerical examples.

2. METHODOLOGY

2.1 Non-Rectangular M-ary QAM (MQAM_NRec) Bit Error Probability (BER)

The Non-Rectangular M-ary QAM (MQAM_NRec) Bit Error Probability (BER) denoted as $\mathrm{P_{bMQAM\_NRec}}(Qfn)$ can be expressed in respect of Q-function as follows;

$$\mathrm{P_{bMQAM\_NRec}}(Qfn) = \left(\frac{4}{(\mathrm{Log}_2(M))}\right) Q\left(\sqrt{\left(\frac{3(\mathrm{Log}_2 M)}{M-1}\right)\left(\frac{\varepsilon_b}{N_0}\right)}\right) \quad (1)$$

If the complementary error function (erfc) is used, the BER denoted as $\mathrm{P_{bMQAM\_NRec}}(erfc)$ can be expressed as;

$$\mathrm{P_{bMQAM_{NRec}}}(erfc) = \left(\frac{2}{(\mathrm{Log}_2(M))}\right) erfc\left(\sqrt{\left(\frac{3(\mathrm{Log}_2 M)}{2(M-1)}\right)\left(\frac{\varepsilon_b}{N_0}\right)}\right) \quad (2)$$

Correspondingly, if error function (erfc) is used, the BER denoted as $\mathrm{P_{bMQAM\_NRec}}(erf)$ can be expressed as;

$$\mathrm{P_{bMQAM_{NRec}}}(erf) = \left(\frac{2}{(\mathrm{Log}_2(M))}\right)\left(1 - erf\left(\sqrt{\left(\frac{3(\mathrm{Log}_2 M)}{2(M-1)}\right)\left(\frac{\varepsilon_b}{N_0}\right)}\right)\right) \quad (3)$$

Alternatively, the BER expression based on erfc can be generalized as follows;

$$\mathrm{P_{bMQAM\_NRec}}(erfc) = (A) erfc\left(\sqrt{(B)\left(\frac{\varepsilon_b}{N_0}\right)}\right) \quad (4)$$

Where

$$\mathrm{A} = \frac{2}{(\mathrm{Log}_2(M))} \quad (5)$$

$$\mathrm{B} = \frac{3(\mathrm{Log}_2 M)}{2(M-1)} \quad (6)$$

2.2 Non-Rectangular M-ary QAM (MQAM_NRec) Symbol Error Probability (SER)

The Non-Rectangular M-ary QAM (MQAM_NRec) Symbol Error Probability (SER), denoted as

$P_{sMQAM\_NRec}(Qfn)$ can be expressed in respect of Q-function as follows;

$$P_{sMQAM\_NRec}(Qfn) = 4\left(Q\left(\sqrt{\left(\frac{3}{M-1}\right)\left(\frac{\varepsilon_s}{N_0}\right)}\right)\right) \quad (7)$$

If the complementary error function (erfc) is used, the SER denoted as $P_{sMQAM\_NRec}(erfc)$ can be expressed as;

$$P_{sMQAM\_NRec}(erfc) = 2\left(erfc\left(\sqrt{\left(\frac{3}{2(M-1)}\right)\left(\frac{\varepsilon_s}{N_0}\right)}\right)\right) \quad (8)$$

Correspondingly, if error function (erf) is used, the SER denoted as $P_{sMQAM\_NRec}(erf)$ can be expressed as;

$$P_{sMQAM\_NRec}(\mathrm{erf}) = 2\left(1 - erf\left(\sqrt{\left(\frac{3}{2(M-1)}\right)\left(\frac{\varepsilon_s}{N_0}\right)}\right)\right) \quad (9)$$

2.3 The Required Minimum Bandwidth and Bandwidth Efficiency of the Non-Rectangular M-ary QAM (MQAM_NRec)

When the data rate, R_b is given, then, the required minimum bandwidth, W (Hz) for the non-rectangular M-ary QAM (MQAM_NRec) is given as;

$$W = \frac{\text{Data Rate}}{\text{symbol rate}} = \frac{R_b}{\log_2(M)} \quad (10)$$

Where M is the modulation index. Also,

$$\text{Symbol rate or Baud Rate} = \log_2(M) \quad (11)$$

Furthermore, Bandwidth Efficiency, η is given as

$$\eta = \frac{R_b}{W} = \frac{R_b}{\left(\frac{R_b}{\log_2(M)}\right)} = \log_2(M) \quad (12)$$

3. RESULTS AND DISCUSSION

The Bit Error Probability (BER), the required minimum bandwidth and the bandwidth efficiency of non-rectangular M-ary QAM are computed using Matlab program. The computation was performed with data rate of 1Mps. In Table 1 and Figure 1 are the results of the computation of BER versus Eb/No for different modulation index. The results showed that for any given Eb/No, the BER decreases as modulation index (M) decreases. For instance, at Eb/No =14 , BER = 1.3620E-12 for M =2, BER = 3.6843E-06 for M = 4 and BER = 3.1038E-02 for M = 8. However, the results in Table 2 show that the bandwidth efficiency increases as modulation index (M) increases. Essentially, when the higher order and lower order non-rectangular M-ary QAM are compared, the higher order has higher bandwidth efficiency but lower BER for any given Eb/No.

Table 1 The result of the computation of BER versus Eb/No for different modulation index for the non-rectangular M-ary QAM

Signal Levels or Modulation index, M	4	16	64	256	1024
K bits/symbol	2	4	6	8	10
Eb/No(dB)	**MQAM_NRec BER For M=4**	**MQAM_NRec BER For M=16**	**MQAM_NRec BER For M=64**	**MQAM_NRec BER For M=256**	**MQAM_NRec BER For M=1024**
0	1.5730E-01	1.8555E-01	1.9766E-01	1.8975E-01	1.7281E-01
2	7.5012E-02	1.3008E-01	1.6700E-01	1.7483E-01	1.6586E-01
4	2.5002E-02	7.8158E-02	1.3230E-01	1.5670E-01	1.5722E-01
6	4.7766E-03	3.7162E-02	9.5397E-02	1.3511E-01	1.4652E-01
8	3.8182E-04	1.2330E-02	5.9794E-02	1.1023E-01	1.3342E-01
10	7.7442E-06	2.3389E-03	3.0323E-02	8.2994E-02	1.1763E-01
12	1.8012E-08	1.8488E-04	1.1113E-02	5.5490E-02	9.9080E-02
14	1.3620E-12	3.6843E-06	2.4617E-03	3.1038E-02	7.8149E-02
16	0.0000E+00	8.3336E-09	2.4820E-04	1.3226E-02	5.5984E-02
18	0.0000E+00	6.0296E-13	7.2585E-06	3.7036E-03	3.4749E-02
20	0.0000E+00	0.0000E+00	3.0102E-08	5.3899E-04	1.7362E-02
22	0.0000E+00	0.0000E+00	5.6851E-12	2.8092E-05	6.2187E-03
24	0.0000E+00	0.0000E+00	0.0000E+00	2.9018E-07	1.3292E-03

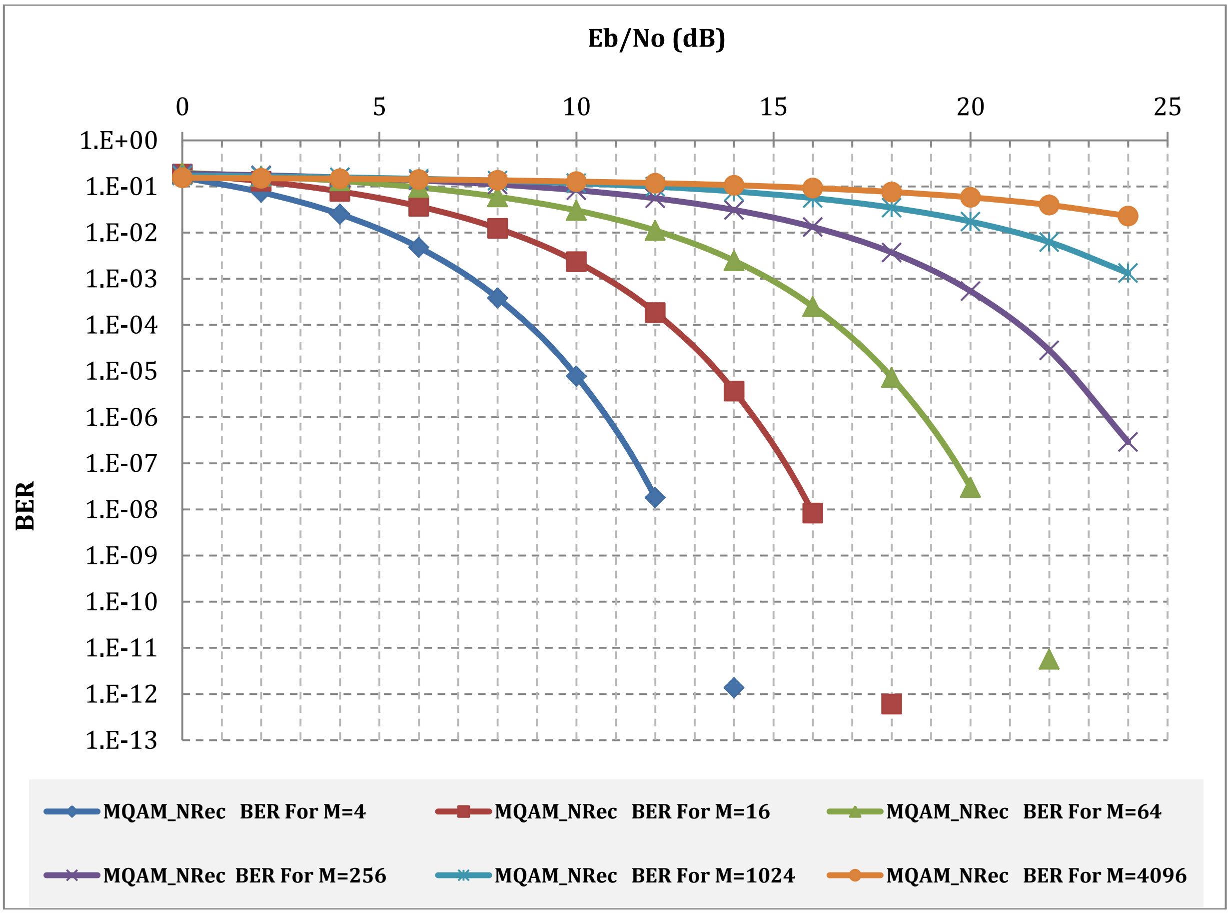

Figure 1 BER versus Eb/No (dB) for the non-rectangular M-ary QAM (MQAM_NRec)

Table 2 The result of the computation of required minimum bandwidth and bandwidth efficiency for the non-rectangular M-ary QAM

Signal Levels or Modulation index, M	Bits Per Symbol = $\log_2(M)$	Data Rate , Rb (bps) [Given , 1 Mbps]	Minimum Bandwidth , W (Hz) = Data Rate/ $\log_2(M)$	Baud Rate , Rs (sps) = Data Rate/ $\log_2(M)$	Bandwidth Efficiency = $\log_2(M)$
4	2	1.E+06	5.E+05	5.E+05	2.0
16	4	1.E+06	3.E+05	3.E+05	4.0
64	6	1.E+06	2.E+05	2.E+05	6.0
256	8	1.E+06	1.E+05	1.E+05	8.0
1024	10	1.E+06	1.E+05	1.E+05	10.0
4096	12	1.E+06	8.E+04	8.E+04	12.0
16384	14	1.E+06	7.E+04	7.E+04	14.0
32768	15	1.E+06	7.E+04	7.E+04	15.0

4. CONCLUSION

Analysis of the Bit Error Probability (BER), the required minimum bandwidth and the bandwidth efficiency of non-rectangular multilevel Quadrature Amplitude Modulation (M-ary QAM) modulation scheme is studied. The BER model based on Q-function, erf function and complementary erfc function are presented along with the corresponding symbol error probability models. The models erfc function-based model was implemented in Matlab program and some computations were performed for various modulation index. In all, the results showed that when the higher order and lower order non-rectangular M-ary QAM are compared, the higher order has higher bandwidth efficiency but lower BER for any given Energy per Bit (Eb) to the Spectral Noise Density (No).

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