Number Of Fuzzy Subgroups Of $Z_2 \times Z_2$, D_8 And S_3 By A New Equivalence Relation

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***Abstract*—Counting the number of the fuzzy subgroups of finite groups has been done by many authors. In early papers, natural equivalence relation is being used to calculate the number of distinct fuzzy subgroups of some finite groups. In this paper; we wish to compute the fuzzy subgroups of some groups by a new equivalence relation ≈ existing in the literature. In fact we will determine the exact number of fuzzy subgroups of $Z_2 \times Z_2$, D_8 and S_3**

Keywords—Equivalence relation, Fuzzy subgroups, Chain of Subgroups, Dihedral group, Symmetric group

Introduction

Without any equivalence relation, the number of fuzzy subgroups of any finite group is infinite. Recently, Tarnauceanu has treated the problem of classifying the fuzzy subgroups of a finite group by a new equivalence relation ≈ introduced in [7].In the present paper we will compute the number of fuzzy subgroups of $Z_2 \times Z_2$, D_8 and S_3.

Preliminaries

Let us denote by $\bar{C}$ the set consisting of all chains of subgroups of G terminated in G. An equivalence relation on $\bar{C}$ can be constructed in the following manner, for two chains

C_1: $H_1 \subset H_2 \subset \ldots \subset H_m = G$

C_2: $K_1 \subset K_2 \subset \ldots \subset Kn = G$

of $\bar{C}$, we put $C_1 \approx C_2$ iff m = n and ∃ f ϵ Aut G such that $f(H_i) = K_i, 1 \leq i \leq n$

First we calculate Fix f_i= $\{H \leq G | f_i(H) = H\}$

In this case the orbit of a chain C ϵ $\bar{C}$ is {f(C)|fϵAut G},while the set of all chains in $\bar{C}$ that are fixed by an automorphism f of G is $Fix_{\bar{C}}(f) = \{C \epsilon \bar{C} | f(C) = C\}$.Now the Burnside's lemma leads to the following theorem.

Theorem 2.1 The number N of all distinct fuzzy subgroups with respect to ≈ of a finite group G is given by

$$N=\frac{1}{|AutG|}\sum_{f\epsilon Aut(G)} |Fix_{\bar{C}}\ (f)|$$

The number of distinct fuzzy subgroups of $Z_2 \times Z_2$

Subgroups of $Z_2 \times Z_2$ are as below

→ $<(0,0)>$
→ $<(0,0),(0,1)>$
→ $<(0,0),(1,0)>$
→ $<(0,0),(1,1)>$
→ $<(0,0),(0,1),(1,0),(1,1)>= Z_2 \times Z_2$

Number of automorphisms of $Z_2 \times Z_2$ is 6.We study all automorphisms one by one

1) $f_1 : Z_2 \times Z_2 \rightarrow Z_2 \times Z_2$

(0, 0) (0, 0) ______

(0, 1) (0, 1) ______

(1, 0) (1, 1) ______

(1, 1) (1, 0) ______

Fix $f_1=\{H \leq Z_2 \times Z_2 | f_1(H)=H\}$

$=\{<(0,0)>,<(0,0),(0,1)>, Z_2 \times Z_2\}$

Lattice of these subgroups are as follows

$Z_2 \times Z_2$

$<(0,0),(0,1)>$ |

$<(0,0)>$ |

Clearly $Fix_{\bar{C}}$ $(f_1)=\{ C\epsilon\bar{C} | f_1(C)=C\}$

$=\{ Z_2 \times Z_2, <(0,0),(0,1)> \subset Z_2 \times Z_2, <(0,0)> \subset Z_2 \times Z_2,$

$<(0,0)> \subset <(0,0),(0,1)> \subset Z_2 \times Z_2\}$

2) $f_2 : Z_2 \times Z_2 \rightarrow Z_2 \times Z_2$

(0, 0) (0, 0) ______

(0, 1) (1, 1) ______

(1, 0) (1, 0) ______

(1, 1) (0,1) ______

Fix $f_2=\{H \leq Z_2 \times Z_2 | f_2(H)=H\}$

$=\{<(0,0)>,<(0,0),(1,0)>, Z_2 \times Z_2\}$

Lattice of these subgroups are as follows

$Z_2\times Z_2$

$<(0,0),(1,0)>$ |

$<(0,0)>$ |

Clearly $Fix_{\bar{C}}(f_2)=\{C\epsilon\bar{C}|f_2(C)=C\}$

$=\{Z_2\times Z_2, <(0,0),(1,0)>\subset Z_2\times Z_2, <(0,0)>\subset Z_2\times Z_2,$

$<(0,0)>\subset <(0,0),(1,0)>\subset Z_2\times Z_2\}$

Therefore $|Fix_{\bar{C}}(f_2)|=4$

3) $f_3 : Z_2\times Z_2\rightarrow Z_2\times Z_2$

(0,0) (0,0) ____

(0,1) (1,0) ____

(1,0) (0,1) ____

(1,1) (1,1) ____

Fix $f_3=\{H\leq Z_2\times Z_2|f_3(H)=H\}$

$=\{<(0,0)>, <(0,0),(1,1)>, Z_2\times Z_2\}$

Lattice of these subgroups are as follows

$Z_2\times Z_2$

$<(0,0),(1,1)>$ |

$<(0,0)>$ |

Clearly $Fix_{\bar{C}}(f_3)=\{C\epsilon\bar{C}|f_3(C)=C\}$

$=\{Z_2\times Z_2, <(0,0),(1,1)>\subset Z_2\times Z_2, <(0,0)>\subset Z_2\times Z_2,$

$<(0,0)>\subset <(0,0),(1,1)>\subset Z_2\times Z_2\}$

Therefore $|Fix_{\bar{C}}(f_3)|=4$

4) $f_4 : Z_2\times Z_2\rightarrow Z_2\times Z_2$

(0,0) (0,0) ____

(0,1) (1,0) ____

(1,0) (1,1) ____

(1,1) (0,1) ____

Fix $f_4=\{H\leq Z_2\times Z_2|f_4(H)=H\}$

$=\{<(0,0), Z_2\times Z_2\}$

Lattice of these subgroups is as follows

$Z_2\times Z_2$

$<(0,0)>$ |

Clearly $Fix_{\bar{C}}(f_4)=\{C\epsilon\bar{C}|f_4(C)=C\}$

$=\{Z_2\times Z_2, <(0,0)>\subset Z_2\times Z_2\}$

Therefore $|Fix_{\bar{C}}(f_4)|=2$

5) $f_5 : Z_2\times Z_2\rightarrow Z_2\times Z_2$

(0,0) (0,0) ____

(0,1) (1,1) ____

(1,0) (0,1) ____

(1,1) (1,0) ____

Fix $f_5=\{H\leq Z_2\times Z_2|f_5(H)=H\}$

$=\{<(0,0)>, Z_2\times Z_2\}$

Lattice of these subgroups is as follows

$Z_2\times Z_2$

$<(0,0)>$ |

Clearly $Fix_{\bar{C}}(f_5)=\{C\epsilon\bar{C}|f_5(C)=C\}$

$=\{Z_2\times Z_2, <(0,0)>\subset Z_2\times Z_2\}$

Therefore $|Fix_{\bar{C}}(f_5)|=2$

6) $f_6 : Z_2\times Z_2\rightarrow Z_2\times Z_2$

(0,0) (0,0) ____

(0,1) (0,1) ____

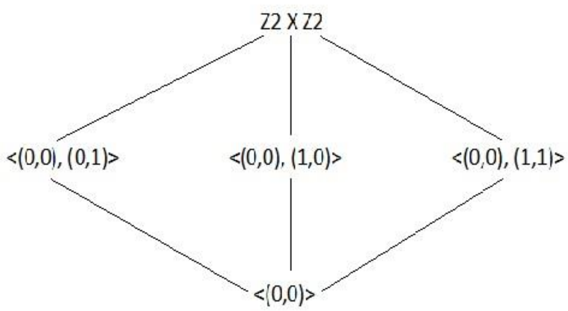
(1,0) (1,0) ____

(1,1) (1,1) ____

Fix $f_6=\{H\leq Z_2\times Z_2|f_6(H)=H\}$

$=\{<(0,0)>, <(0,0),(0,1)>, <(0,0),(1,0)>, <(0,0),(1,1)>, Z_2\times Z_2\}$

Lattice of these subgroups is as follows



Clearly $Fix_{\bar{C}}(f_6)=\{C\epsilon\bar{C}|f_6(C)=C\}$

In this case, members of $Fix_{\bar{C}}(f_6)$ are

$\rightarrow Z_2\times Z_2$

$\rightarrow <(0,0),(0,1)>\subset Z_2\times Z_2$

$\rightarrow <(0,0),(1,0)>\subset Z_2\times Z_2$

$\rightarrow <(0,0),(1,1)>\subset Z_2\times Z_2$

$\rightarrow <(0,0)>\subset Z_2\times Z_2$

$\rightarrow <(0,0)>\subset <(0,0),(0,1)>\subset Z_2\times Z_2$

$\rightarrow <(0,0)>\subset <(0,0),(1,0)>\subset Z_2\times Z_2$

$\rightarrow <(0,0)>\subset <(0,0),(1,1)>\subset Z_2\times Z_2$

Therefore $|Fix_{\bar{C}}\ (f_6)|=8$

So, number N of all distinct fuzzy subgroups of $Z_2 \times Z_2$

$=\frac{1}{6}(3.4+2.2+8)=4$

The number of distinct fuzzy subgroups of S_3

$S_3=\{I,(12),(13),(23),(123),(132)\}$

There are six subgroups of S_3, those are

$\{I\},\{I,(12),\{I,(23)\},\{I,(13)\},\{I,(123),(132)\}$ and S_3 itself.

$|Aut(S_3)|=3!=2\times3=6$

Therefore number of automorphism of S_3 is 6.

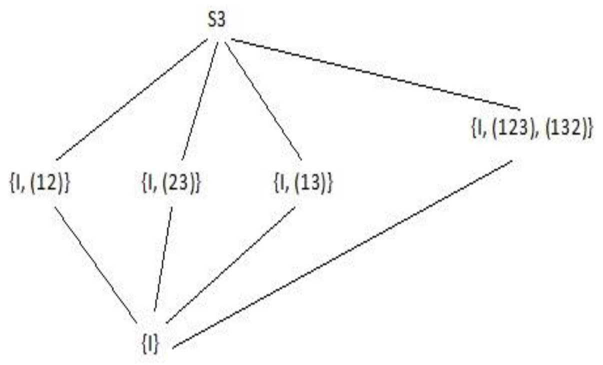
Now, we will study all automorphism one by one.

1) : I ——— I
(12) ——— (12)
(13) ——— (13)
(23) ——— (23)
(123) ——— (123)
(132) ——— (132)

$Fix(f_1)=\{H\leq S_3|f_1(H)=H\}$

$=\{\{I\},\{I,(12)\},\{I,(23)\},\{I,(13)\},\{I,(123),(132)\},S_3\}$

Lattice of these subgroups is as follows.



$Fix_{\bar{C}}\ (f_1) = \{ C\epsilon\bar{C}|f_1(C) = C\}$
Clearly members of $Fix_{\bar{C}}\ (f_1)$ are
$\rightarrow S_3$

$\rightarrow\{I,(12)\}\subset S_3$

$\rightarrow\{I,(23)\}\subset S_3$

$\rightarrow\{I,(13)\}\subset S_3$

$\rightarrow\{I,(123),(132)\}\subset S_3$

$\rightarrow\{I\}\subset S_3$

$\rightarrow\{I\}\subset_{\{I,(12)\}}\subset S_3$

$\rightarrow\{I\}\subset_{\{I,(23)\}}\subset S_3$

$\rightarrow\{I\}\subset_{\{I,(13)\}}\subset S_3$

$\rightarrow\{I\}\subset_{\{I,(123),(132)\}}\subset S_3$

Therefore $|Fix_{\bar{C}}\ (f_1)|=10$

2)

f_2: I ——— I
(12) ——— (13)
(13) ——— (23)
(23) ——— (12)
(123) ——— (123)
(132) ——— (132)

$Fix(f_2)=\{H\leq S_3|f_2(H)=H\}$

$=\{\{I\},\{I,(123),(132)\},S_3\}$

Lattice of these subgroups is as follows

S_3
|
$\{I,(123),(132)\}$
|
$\{I\}$

$Fix_{\bar{C}}\ (f_2)=\{ C\epsilon\bar{C}|f_2(C)=C\}$
Clearly members of $Fix_{\bar{C}}\ (f_2)$ are
$\rightarrow S_3$

$\rightarrow_{\{I,(123),(132)\}}\subset S_3$

$\rightarrow\{I\}\subset S_3$

$\rightarrow\{I\}\subset_{\{I,(123),(132)\}}\subset S_3$

Therefore $|Fix_{\bar{C}}\ (f_2)|=4$

3)

f_3: I ——— I
(12) ——— (23)
(13) ——— (12)
(23) ——— (13)
(123) ——— (123)
(132) ——— (132)

$Fix(f_3)=\{H\leq S_3|f_3(H)=H\}$

$=\{\{I\},\{I,(123),(132)\},S_3\}$

Lattice of these subgroups is as follows

S_3
|
$\{I,(123),(132)\}$
|
$\{I\}$

$Fix_{\bar{C}}\ (f_3)=\{ C\epsilon\bar{C}|f_3(C)=C\}$
Clearly members of $Fix_{\bar{C}}\ (f_3)$ are
$\rightarrow S_3$

$\rightarrow_{\{I,(123),(132)\}}\subset S_3$

$\rightarrow\{I\}\subset S_3$

$\rightarrow\{I\}\subset_{\{I,(123),(132)\}}\subset S_3$

Therefore $|Fix_{\bar{C}}\ (f_3)|=4$

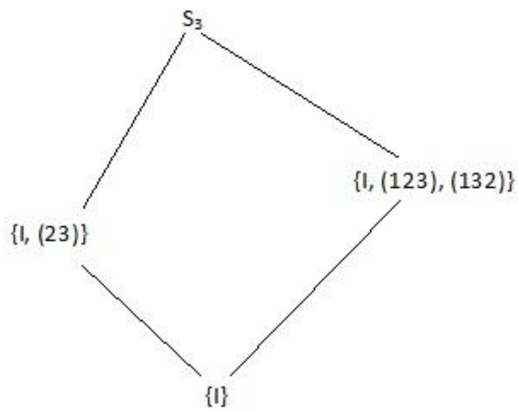
4)

f_4: I ——— I
(12) —— (13)
(13) —— (12)
(23) —— (23)
(123) —— (132)
(132) —— (123)

$Fix(f_4)=\{H\leq S_3|f_4(H)=H\}$

$=\{\{I\},\{I,(23)\},\{I,(123),(132)\},S_3\}$

Lattice of these subgroups is as follows



$Fix_{\bar{C}}\ (f_4)=\{\ C\epsilon\bar{C}|f_4(C)=C\}$
Clearly members of $Fix_{\bar{C}}\ (f_4)$ are
$\rightarrow S_3$

$\rightarrow \{I,(23)\} \subset S_3$

$\rightarrow \{I,(123),(132)\} \subset S_3$

$\rightarrow \{I\} \subset S_3$

$\rightarrow \{I\} \subset \{I,(23)\} \subset S_3$

$\rightarrow \{I\} \subset \{I,(123),(132)\} \subset S_3$

Therefore $|Fix_{\bar{C}}\ (f_4)|=6$

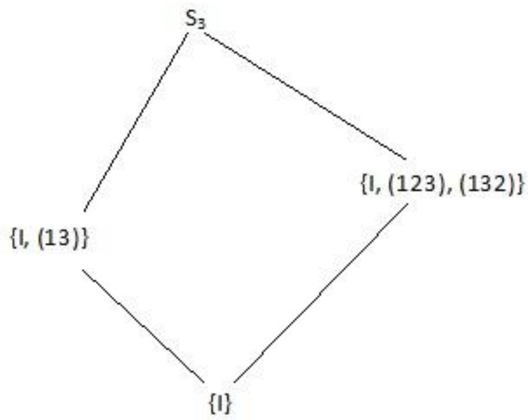
5)

f_5: I ——— I
(12) —— (23)
(13) —— (13)
(23) —— (12)
(123) —— (132)
(132) —— (123)

$Fix(f_5)=\{H\leq S_3|f_5(H)=H\}$

$=\{\{I\},\{I,(13)\},\{I,(123),(132),S_3\}$

Lattice of these subgroups is as follows



$Fix_{\bar{C}}\ (f_5)=\{\ C\epsilon\bar{C}|f_5(C)=C\}$
Clearly members of $Fix_{\bar{C}}\ (f_5)$ are
$\rightarrow S_3$

$\rightarrow \{I,(13)\} \subset S_3$

$\rightarrow \{I,(123),(132)\} \subset S_3$

$\rightarrow \{I\} \subset S_3$

$\rightarrow \{I\} \subset \{I,(13)\} \subset S_3$

$\rightarrow \{I\} \subset \{I,(123),(132)\} \subset S_3$

Therefore $|Fix_{\bar{C}}\ (f_5)|=6$

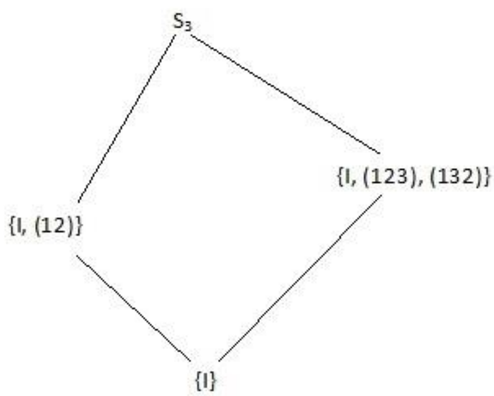
6)

f_6: I ——— I
(12) —— (12)
(13) —— (23)
(23) —— (13)
(123) —— (132)
(132) —— (123)

$Fix(f_6)=\{H\leq S_3|f_6(H)=H\}$

$=\{\{I\},\{I,(12)\},\{I,(123),(132),S_3\}$

Lattice of these subgroups is as follows



$Fix_{\bar{C}}\ (f_6)=\{\ C\epsilon\bar{C}|f_6(C)=C\}$
Clearly members of $Fix_{\bar{C}}\ (f_6)$ are

$\rightarrow S_3$

$\rightarrow_{\{I,(12)\}} \subset S_3$

$\rightarrow_{\{I,(123),(132)\}} \subset S_3$

$\rightarrow \{I\} \subset S_3$

$\rightarrow \{I\} \subset_{\{I,(12)\}} \subset S_3$

$\rightarrow \{I\} \subset_{\{I,(123),(132)\}} \subset S_3$

Therefore $|Fix_{\bar{C}}\ (f_6)|=6$

Therefore Number N of all distinct fuzzy subgroups of S_3

$=\frac{1}{6}(10+2\times4+3\times6)=6$

The number of distinct fuzzy subgroups of D_8

$D_8=<a,b|a^4=b^2=1, ab=ba^{-1}>$

Clearly $D_8=\{1,a,a^2,a^3,b,ab,a^2b,a^3b\}$

We have ten subgroups of D_8,those are

$\{1\}$,$\{1,b\}$,$\{1,a^2\}$,$\{1,a^2b\}$,$\{1,ab\}$,$\{1,a^3b\}$,$\{1,a^2,b,a^2b\}$,$\{1,a,a^2,a^3\}$, $\{1,a^2,ab,a^3b\}$ and D_8 itself.

$|Aut\ D_8|=n\varphi(n)=4\varphi(4)=4\times2=8$

Automorphism group of D_8 is well known, namely

$Aut\ D_8=\{f_{\alpha,\beta}|0\le\alpha\le4-1\ with\ (\alpha,4)=1, 0\le\beta\le4-1\}$

$=\{f_{1,0}, f_{1,1}, f_{1,2}, f_{1,3}, f_{3,0}, f_{3,1}, f_{3,2}, f_{3,3}\}$

Where $f_{\alpha,\beta}(a^i)=a^{\alpha i}$

$f_{\alpha,\beta}(a^ib)=a^{\alpha i+\beta}b$ for all $0\le i\le n-1=4-1$

1) $f_{1,0}(a^i)=a^i$
$f_{1,0}(a^ib)=a^ib$
Take $f_{1,0}=f_1$

$f_1: D_8$ —— D_8

1 —— 1

a —— a

a^2 —— a^2

a^3 —— a^3

b —— b

ab —— ab

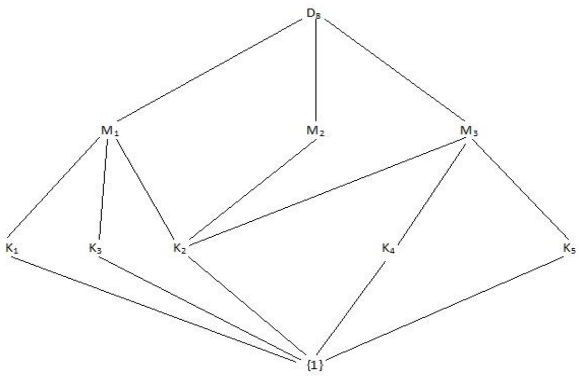
a^2b —— a^2b

a^3b —— a^3b

Fix $f_1=\{H\le D_8|f_1(H)=H\}$

$=\{\{1\},\{1,b\}=K_1,\{1,a^2\}=K_2,\{1,a^2b\}=K_3,\{1,ab\}=K_4,\{1,a^3b\}=K_5,\{1,a^2,b,a^2b\}=M_1,\{1,a,a^2,a^3\}=M_2,\{1,a^2,ab,a^3b\},D_8\}$

Lattice of these subgroups is as follows



$Fix_{\bar{C}}\ (f_1)=\{C\epsilon\bar{C}|f_1(C)=C\}$
Clearly from above lattice diagram

$|Fix_{\bar{C}}\ (f_1)|=32$
2) $f_{1,1}(a^i)=a^i$
$f_{1,1}(a^ib)=a^{i+1}b$
Take $f_{1,1}=f_2$

$f_2: D_8$ —— D_8

1 —— 1

a —— a

a^2 —— a^2

a^3 —— a^3

b —— ab

ab —— a^2b

a^2b —— a^3b

a^3b —— b

Fix $f_2=\{H\le D_8|f_2(H)=H\}$

$=\{\{1\},\{1,a^2\},\{1,a,a^2,a^3\},D_8\}$

Lattice of these subgroups is as follows

D_8
|
$\{1,a,a^2,a^3\}$
|
$\{1,a^2\}$
|
$\{1\}$

$Fix_{\bar{C}}\ (f_2)=\{C\epsilon\bar{C}|f_2(C)=C\}$
Members of $Fix_{\bar{C}}\ (f_2)$ are
$\rightarrow D_8$
$\rightarrow \{1,a,a^2,a^3\}\subset D_8$
$\rightarrow \{1,a^2\}\subset D_8$
$\rightarrow \{1,a^2\}\subset_{\{1,a,a^2,a^3\}}\subset D_8$
$\rightarrow \{1\}\subset D_8$
$\rightarrow \{1\}\subset\{1,a^2\}\ D_8$
$\rightarrow \{1,\}\subset_{\{1,a,a^2,a^3\}}\subset D_8$
$\rightarrow \{1\}\subset\{1,a^2\}\subset_{\{1,a,a^2,a^3\}}\subset D_8$
$|Fix_{\bar{C}}\ (f_2)|=8$

3) $f_{1,3}(a^i) = a^i$
$f_{1,3}(a^i b) = a^{i+3} b$
Take $f_{1,3}$ = f_3

f_3: D_8 —— D_8

1 —— 1

a —— a

a^2 —— a^2

a^3 —— a^3

b —— $a^3 b$

ab —— b

$a^2 b$ —— ab

$a^3 b$ —— $a^2 b$

Fix f_3={H≤ D_8|f_3(H)=H}

={{1},{1,a^2},{1,a, a^2, a^3},D_8}

Lattice of these subgroups is as follows

D_8
|
{1,a, a^2, a^3}
|
{1,a^2}
|
{1}

Clearly in this case also

|Fix$_{\bar{C}}$ (f_3)|=8

4) $f_{3,1}(a^i) = a^{3i}$
$f_{3,1}(a^i b) = a^{3i+1} b$
Take $f_{3,1}$ = f_4

f_4: D_8 —— D_8

1 —— 1

a —— a

a^2 —— a^2

a^3 —— a^3

b —— ab

ab —— b

$a^2 b$ —— $a^3 b$

$a^3 b$ —— $a^2 b$

Fix f_4={H≤ D_8|f_4(H)=H}

={{1},{1,a^2},{1,a, a^2, a^3},D_8}

Lattice of these subgroups is as follows

D_8
|
{1,a, a^2, a^3}
|
{1,a^2}
|
{1}

Clearly in this case also

|Fix$_{\bar{C}}$ (f_4)|=8

5) $f_{3,3}(a^i) = a^{3i}$
$f_{3,1}(a^i b) = a^{3i+3} b$
Take $f_{3,3}$ = f_5

f_5: D_8 —— D_8

1 —— 1

a —— a^3

a^2 —— a^2

a^3 —— a

b —— $a^3 b$

ab —— $a^2 b$

$a^2 b$ —— ab

$a^3 b$ —— b

Fix f_5={H≤ D_8|f_5(H)=H}

={{1},{1,a^2},{1,a, a^2, a^3},D_8}

Lattice of these subgroups is as follows

D_8
|
{1,a, a^2, a^3}
|
{1,a^2}
|
{1}

Clearly in this case also

|Fix$_{\bar{C}}$ (f_5)|=8

6) $f_{1,2}(a^i) = a^i$
$f_{1,2}(a^i b) = a^{i+2} b$
Take $f_{1,2}$ = f_6

f_6: D_8 —— D_8

1 —— 1

a —— a

a^2 —— a^2

a^3 —— a^3

b —— $a^2 b$

ab —— $a^3 b$

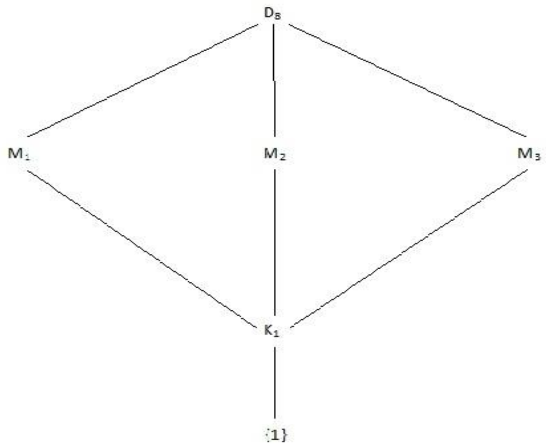
$a^2 b$ —— b

$a^3 b$ —— ab

Fix f_6={H≤ D_8|f_6(H)=H}

={{1},{1,a^2},{1,a^2,b,a^2b},{1,a,a^2,a^3},{1,a^2,ab,a^3b},D_8}

Subgroup lattice diagram of these subgroups is as follows



Clearly in this case

|Fi$x_{\bar{C}}$ (f_6)|=16

7) $f_{3,0}(a^i) = a^{3i}$

$f_{3,0}(a^ib) = a^{3i}b$

Take $f_{3,0}$ =f_7

f_7:D_8——D_8

1 ——1

a—— a^3

a^2——a^2

a^3——a

b——b

ab ——a^3b

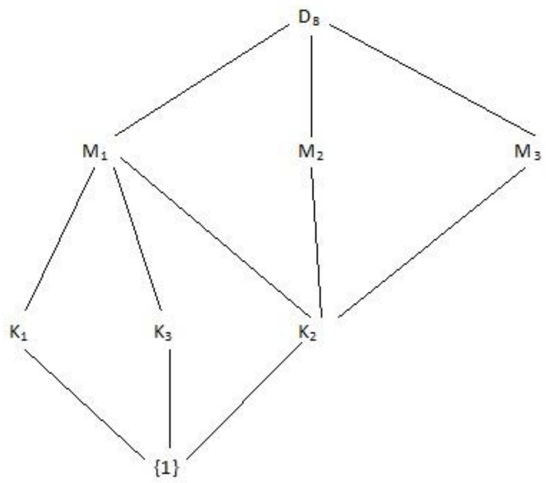
a^2b—— a^2b

a^3b—— ab

Fix f_7={H≤ D_8|f_7(H)=H}

={{1},{1,b},{1,a^2},{1,a^2b},{1,a^2,b,a^2b},{1,a,a^2,a^3},{1, a^2,ab,a^3b},D_8}

Subgroup lattice diagram of these subgroups is as follows



Clearly in this case

|Fi$x_{\bar{C}}$ (f_7)|=24

8) $f_{3,2}(a^i) = a^{3i}$

$f_{3,2}(a^ib) = a^{3i+2}b$

Take $f_{3,2}$ =f_8

f_8:D_8——D_8

1 ——1

a—— a^3

a^2——a^2

a^3——a

b—— a^2b

ab——ab

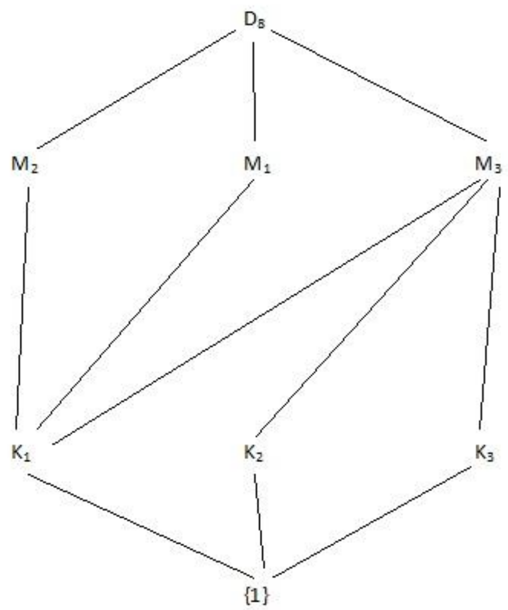
a^2b—— b

a^3b——a^3b

Fix f_8={H≤ D_8|f_8(H)=H}

={{1},{1,a^2},{1,ab},{1,a^3b}, {1,a^2,b,a^2b},{1,a,a^2,a^3},{1,a^2,ab,a^3b},D_8}

Subgroup lattice diagram of these subgroups is as follows



Clearly in this case

$|Fix_{\bar{C}}\ (f_8)|=24$

Therefore N of all distinct fuzzy subgroups of D_8

$=\frac{1}{8}(32+4\times 8+16+2\times 24)=16$

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