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Performance Analysis Of M-Ary Amplitude Shift Keying Digital Modulation Scheme

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***Abstract*— Performance analysis of M-Ary Amplitude Shift Keying (M-ary ASK) digital modulation scheme is presented. The focus is on the determination of the Bit Error Probability (BER) of M-ary ASK modulation schemes using the complementary error function (erfc). A generalized analytical expression for the determination of the BER for the M-ary ASK modulation is presented. The generalized analytical expression has two requisite coefficients, Aerfc and Berfc which are presented in this paper. The corresponding bit error probability or bit error rate (BER) for the M-ary ASK modulations with M in the range 2 to 256 are computed. At (the modulation index, M = 2 , Aerfc = ½ and Berfc = 3/3 . Also, at Eb/No value of 2 dB, the BER for the M-ary ASK modulation scheme with M =2 is 3.7506 x 10^{-2}. In general, the results show that for any given BER, the required Eb/No increases as M (the modulation index) increases. Also, the results for the required signal power, S versus BER for the M-Ary ASK modulation schemes for No = 2 x10^{-10}-10 W Hz and Rb = 1000000 bps and Eb/No(dB) are show that for any given BER, the required Eb/No increases as M increases. For instance, for M =16, at S = 0.0524 mW, a bit error rate of 1x10^{-1} can be achieved whereas at S = 7.9621 mW, a bit error rate of 1x10^{-2} can be achieved. In all, the analysis presented in this paper will enable users to appropriately apply the various M-ary ASK (M-ASK) modulation scheme in their design and in the selection of the appropriate M-ASK modulation that best fits the purpose.**

Keywords*— *Digital Modulation Scheme, Bit Error Probability, Modulation, Complementary Error Function, Modulation Index, Amplitude Shift Keying, Multi-Level Modulation

I. INTRODUCTION

Amplitude Shift Keying (ASK) is a type of digital modulation scheme in which digital data is represented as shifts in the amplitude of a carrier wave while the frequency and phase of the carrier are not altered [1,2,3,4,5,6]. The M-ary Amplitude Shift Keying, M-ary ASK (also known as M-ary Pulse Amplitude Modulation, M-ary PAM) is a multi-level modulation version of the ASK where the amplitude of the carrier wave is modulated with M different amplitudes [7,8,9,10,11,12,13,14,15]. The modulation and demodulation processes for M-ary ASK modulation scheme is relatively simple and inexpensive when compared to the other modulation schemes [16,17,18]. As such, the lower order M-ary ASK is employed in transmission of data over optical fiber. They are also applied in low frequency RF devices, home automation devices, remote keyless entry and in wireless base stations.

Generally, the performance of the modulation scheme is expressed in terms of probability of bit error and bandwidth efficiency [19,20,21,22,23,24]. In this paper, the performance analysis of M-ary ASK modulation scheme is presented. The focus is on the determination of the Bit Error Probability (BER) of the M-ary ASK modulation schemes using the complementary error function (erfc). A generalized analytical expression for the determination of the BER for the M-ary ASK modulation is also presented. Also, the study on the required signal power and BER versus Eb/No which is the Energy per Bit (Eb) to the Spectral Noise Density (No) ratio is also presented. Sample numerical computations are also presented.

II. ANALYTICAL EXPRESSIONS FOR PROBABILITY OF BIT ERROR AND PROBABILITY OF SYMBOL ERROR FOR M-ARY ASK MODULATION

A. The Probability of Bit Error for M-Ary ASK Modulation

The probability of bit error for M-ary ASK modulation based on Q function is denoted as $P_{bM-ASK}(Qfn)$ where;

$$P_{bM-ASK}(Qfn) = \left(\frac{2(M-1)}{M(\mathrm{Log}_2(M))}\right) Q\left(\sqrt{\left(\frac{6(\mathrm{Log}_2(M))}{M^2-1}\right)\left(\frac{\varepsilon_b}{N_0}\right)}\right) \quad (1)$$

Where modulation order is denoted as M, the energy per bit is denoted as ε_b and noise power density is denoted as N_0. The Q function and the complementary error function ($erfc$)are related as follows;

$$Q(x) = \left(\frac{1}{2}\right) erfc\left(\sqrt{\frac{x}{2}}\right) \quad (2)$$

Hence, the BER for M-ary ASK modulation based on $erfc$ is denoted as $P_{bM-ASK}(erfc)$ where;

$$P_{bM-ASK}(erfc) = \left(\frac{(M-1)}{M(\mathrm{Log}_2(M))}\right) erfc\left(\sqrt{\left(\frac{3(\mathrm{Log}_2(M))}{M^2-1}\right)\left(\frac{\varepsilon_b}{N_0}\right)}\right) \quad (3)$$

The $erfc$ is related to the error function (erf) as follows;

$$erfc(x) = 1 - erf(x) \quad (4)$$

Hence, the BER for M-ary ASK modulation based on erf is denoted as $P_{bM-ASK}(erf)$ where;

$$P_{bM-ASK}(erf) = \left(\frac{(M-1)}{M(\mathrm{Log}_2(M))}\right)\left(1 - erf\left(\sqrt{\left(\frac{3(\mathrm{Log}_2(M))}{M^2-1}\right)\left(\frac{\varepsilon_b}{N_0}\right)}\right)\right) \quad (5)$$

B. The Probability of Symbol Error for M-Ary ASK Modulation

The energy per symbol (ε_s) and energy per bit (ε_b) are related as follows;

$$\varepsilon_s = \varepsilon_b(\mathrm{Log}_2(M)) \quad (6)$$

The probability of symbol error or symbol error probability (SEP) for M-ary ASK modulation based on Q function is denoted as $\mathrm{P_{sM-ASK}}(Qfn)$ where;

$$\mathrm{P_{sM-ASK}}(Qfn) = (\mathrm{P_{bM-ASK}}(Qfn))(\mathrm{Log}_2(M)) = \left(\frac{2(M-1)}{M}\right) Q\left(\sqrt{\left(\frac{6}{M^2-1}\right)\left(\frac{\varepsilon_s}{N_0}\right)}\right) \quad (7)$$

The SEP for M-ary ASK modulation based on $erfc$ is denoted as $\mathrm{P_{sM-ASK}}(erfc)$ where;

$$\mathrm{P_{sM-ASK}}(erfc) = \mathrm{P_{bM-ASK}}(erfc)(\mathrm{Log}_2(M)) = \left(\frac{(M-1)}{M}\right) erfc\left(\sqrt{\left(\frac{3}{M^2-1}\right)\left(\frac{\varepsilon_s}{N_0}\right)}\right) \quad (8)$$

The SEP for M-ary ASK modulation based on erf is denoted as $\mathrm{P_{sM-ASK}}(erf)$ where;

$$\mathrm{P_{sM-ASK}}(erf) = \mathrm{P_{bM-ASK}}(erf)(\mathrm{Log}_2(M)) = \left(\frac{(M-1)}{M}\right)\left(1 - erf\left(\sqrt{\left(\frac{3}{M^2-1}\right)\left(\frac{\varepsilon_s}{N_0}\right)}\right)\right) \quad (9)$$

C. The Generalized Form of Probability of Bit Error for M-Ary ASK Modulation

In this paper, the analysis of probability of bit error for M-ary ASK modulation is based on $erfc$ and it is expressed in this form;

$$\mathrm{P_{bM-ASK}}(erfc) = (A_{erfc})erfc\left(\sqrt{(B_{erfc})\left(\frac{\varepsilon_b}{N_0}\right)}\right) \quad (10)$$

Where

$$A_{erfc} = \frac{(M-1)}{M(\mathrm{Log}_2(M))} \quad (11)$$

$$B_{erfc} = \frac{3(\mathrm{Log}_2(M))}{M^2-1} \quad (12)$$

The M-ary ASK modulation probability of bit error is captured as Bit Error Rate (BER) and the analysis is conducted for selected values of M =2,4,8,16,32,64,128 and 256. The values of A_{erfc} and B_{erfc} are provided for the various values of M and along with the BER for each M. Also, the BER is computed for different signal power for each of the listed modulation order, M.

III. RESULTS AND DISCUSSION

In order to determine the BER of M-ary ASK modulations using the erfc , the coefficients need to be determined based on the analytical expression in Eq 8 and generalized in Eq 10. The coefficients, Aerfc and Berfc in Eq 11 and Eq12 are computed for the various M-ary ASK modulations with M in the range 2 to 256 and the results are shown in Table 1. The corresponding BER for the M-ary ASK modulations with M in the range 2 to 256 are shown in Table 2 and Figure 1. The results in Table 2 and Figure 1 show that for any given BER, the required Eb/No increases as M increases.

Table 1 The Aerfc and Berfc parameter values for the generalized form of probability of bit error for M-Ary ASK modulation based on error function (erfc)

$$\mathrm{P_{bM-ASK}}(erfc) = (\boldsymbol{A_{erfc}})erfc\left(\sqrt{(\boldsymbol{B_{erfc}})\left(\frac{\varepsilon_b}{N_0}\right)}\right)$$

M (Modulation Order)	Aerfc	Berfc
2	1/2	3/3
4	3/8	6/15
8	7/24	9/63
16	15/64	12/255
32	31/160	15/1023
64	63/384	18/4095
128	127/896	21/16383
256	255/2048	24/65535

Table 2 BER versus Eb/No (dB) for the M-Ary ASK modulation

Eb/No(dB)	M-ASK BER For M=2	M-ASK BER For M=4	M-ASK BER For M=8	M-ASK BER For M=16	M-ASK BER For M=32	M-ASK BER For M=64	M-ASK BER For M=128	M-ASK BER For M=256
0	7.8650E-02	1.3916E-01	1.7295E-01	1.7789E-01	1.6741E-01	1.5181E-01	1.3602E-01	1.2182E-01
2	3.7506E-02	9.7559E-02	1.4612E-01	1.6391E-01	1.6068E-01	1.4865E-01	1.3454E-01	1.2113E-01
4	1.2501E-02	5.8618E-02	1.1576E-01	1.4691E-01	1.5230E-01	1.4468E-01	1.3268E-01	1.2025E-01
6	2.3883E-03	2.7871E-02	8.3473E-02	1.2667E-01	1.4194E-01	1.3972E-01	1.3034E-01	1.1915E-01
8	1.9091E-04	9.2472E-03	5.2320E-02	1.0334E-01	1.2925E-01	1.3352E-01	1.2740E-01	1.1776E-01
10	3.8721E-06	1.7542E-03	2.6533E-02	7.7807E-02	1.1395E-01	1.2581E-01	1.2371E-01	1.1602E-01
12	9.0060E-09	1.3866E-04	9.7240E-03	5.2022E-02	9.5984E-02	1.1631E-01	1.1910E-01	1.1383E-01

14	6.8101E-13	2.7632E-06	2.1540E-03	2.9098E-02	7.5707E-02	1.0474E-01	1.1335E-01	1.1108E-01
16	0.0000E+00	6.2502E-09	2.1717E-04	1.2400E-02	5.4235E-02	9.0911E-02	1.0622E-01	1.0763E-01
18	0.0000E+00	4.5222E-13	6.3511E-06	3.4721E-03	3.3663E-02	7.4880E-02	9.7454E-02	1.0332E-01
20	0.0000E+00	0.0000E+00	2.6339E-08	5.0531E-04	1.6819E-02	5.7166E-02	8.6835E-02	9.7950E-02
22	0.0000E+00	0.0000E+00	4.9744E-12	2.6336E-05	6.0244E-03	3.9022E-02	7.4251E-02	9.1307E-02
24	0.0000E+00	0.0000E+00	0.0000E+00	2.7204E-07	1.2877E-03	2.2522E-02	5.9855E-02	8.3171E-02

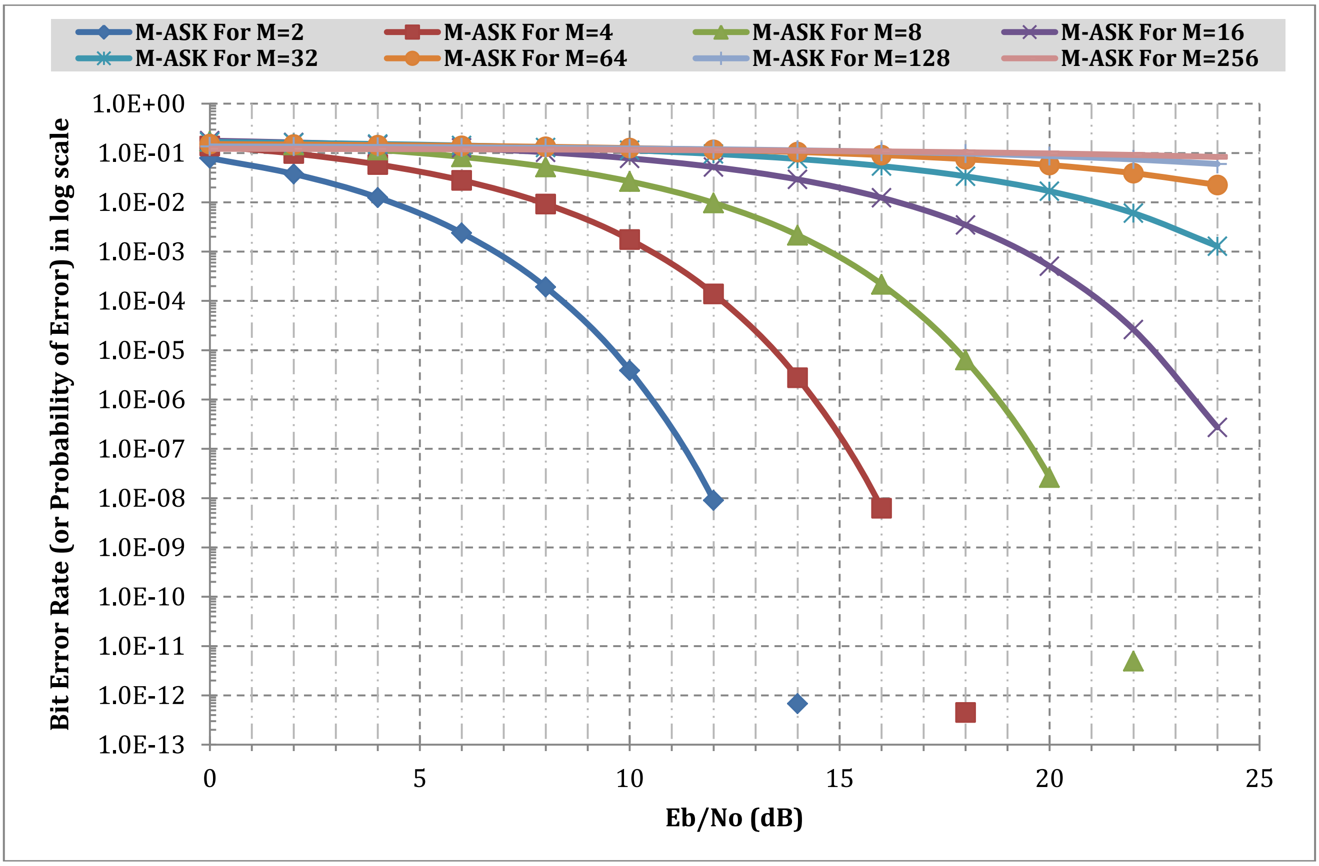

Figure 1 BER versus Eb/No (dB) for the M-Ary ASK modulation

The results for the required signal power, S versus BER for the M-Ary ASK modulation for No = 2×10^{-10}-10 W Hz and Rb = 1000000 bps and Eb/No(dB) are presented in Table 3 and Figure 2, as well as in Table 4 and Figure 3. The results in Table 3, Table 4, Figure 2 and Figure 3 show that for any given BER, the required Eb/No increases as the M increases. For instance, for M =16, at S = 0.0524 mW, a bit error rate of 1×10^{-1} can be achieved whereas at S = 7.9621 mW, a bit error rate of 1×10^{-2} can be achieved.

In all, the analysis presented in this paper will enable users to appropriately apply the various M-ary ASK (M-ASK) modulation scheme in their design and in the selection of the appropriate M-ASK modulation that best fits the purpose.

Table 3 The Required Signal Power, S versus BER for the M-Ary ASK modulation

S (mW)	M-ASK BER For M=2	M-ASK BER For M=4	M-ASK BER For M=8	M-ASK BER For M=16	M-ASK BER For M=32	M-ASK BER For M=64	M-ASK BER For M=128	M-ASK BER For M=256
0.20000	7.8650E-02	1.3916E-01	1.7295E-01	1.7789E-01	1.6741E-01	1.5181E-01	1.3602E-01	1.2182E-01
0.31698	3.7506E-02	9.7559E-02	1.4612E-01	1.6391E-01	1.6068E-01	1.4865E-01	1.3454E-01	1.2113E-01
0.50238	1.2501E-02	5.8618E-02	1.1576E-01	1.4691E-01	1.5230E-01	1.4468E-01	1.3268E-01	1.2025E-01
0.79621	2.3883E-03	2.7871E-02	8.3473E-02	1.2667E-01	1.4194E-01	1.3972E-01	1.3034E-01	1.1915E-01
1.26191	1.9091E-04	9.2472E-03	5.2320E-02	1.0334E-01	1.2925E-01	1.3352E-01	1.2740E-01	1.1776E-01
2.00000	3.8721E-06	1.7542E-03	2.6533E-02	7.7807E-02	1.1395E-01	1.2581E-01	1.2371E-01	1.1602E-01
3.16979	9.0060E-09	1.3866E-04	9.7240E-03	5.2022E-02	9.5984E-02	1.1631E-01	1.1910E-01	1.1383E-01

5.02377	6.8101E-13	2.7632E-06	2.1540E-03	2.9098E-02	7.5707E-02	1.0474E-01	1.1335E-01	1.1108E-01
7.96214	0.0000E+00	6.2502E-09	2.1717E-04	1.2400E-02	5.4235E-02	9.0911E-02	1.0622E-01	1.0763E-01
12.61915	0.0000E+00	4.5222E-13	6.3511E-06	3.4721E-03	3.3663E-02	7.4880E-02	9.7454E-02	1.0332E-01
20.00000	0.0000E+00	0.0000E+00	2.6339E-08	5.0531E-04	1.6819E-02	5.7166E-02	8.6835E-02	9.7950E-02
31.69786	0.0000E+00	0.0000E+00	4.9744E-12	2.6336E-05	6.0244E-03	3.9022E-02	7.4251E-02	9.1307E-02
50.23773	0.0000E+00	0.0000E+00	0.0000E+00	2.7204E-07	1.2877E-03	2.2522E-02	5.9855E-02	8.3171E-02

Note: S (in mW) for No = 2 x10^{-10}-10 W Hz and Rb = 1000000 bps and Eb/No(dB) based on Table 2

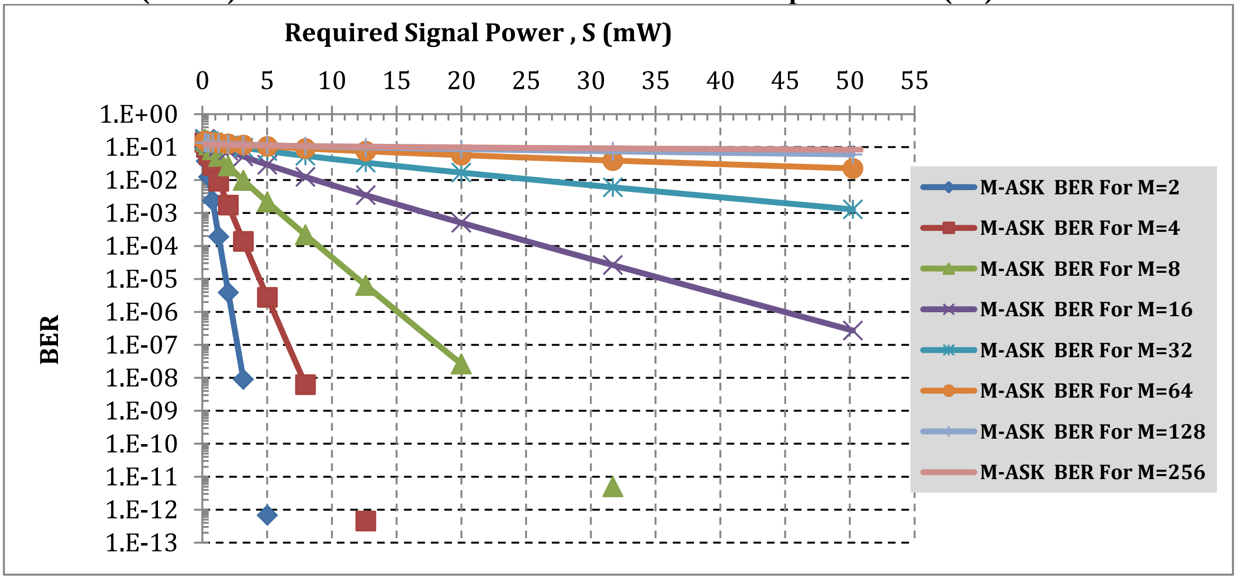

Figure 2 The Required Signal Power, S versus BER for the M-Ary ASK modulation

Table 4 Modulation order , M Versus BER for various Signal Power, S (mW)

Modulation Order, M	BER for S= 0.50238 (mW)	BER for S = 2.0000 (mW)	BER for S = 7.96214 (mW)
2	1.2501E-02	3.8721E-06	0.0000E+00
4	5.8618E-02	1.7542E-03	6.2502E-09
8	1.1576E-01	2.6533E-02	2.1717E-04
16	1.4691E-01	7.7807E-02	1.2400E-02
32	1.5230E-01	1.1395E-01	5.4235E-02
64	1.4468E-01	1.2581E-01	9.0911E-02
128	1.3268E-01	1.2371E-01	1.0622E-01
256	1.2025E-01	1.1602E-01	1.0763E-01

Note: Table 4 is extracted from Table 3

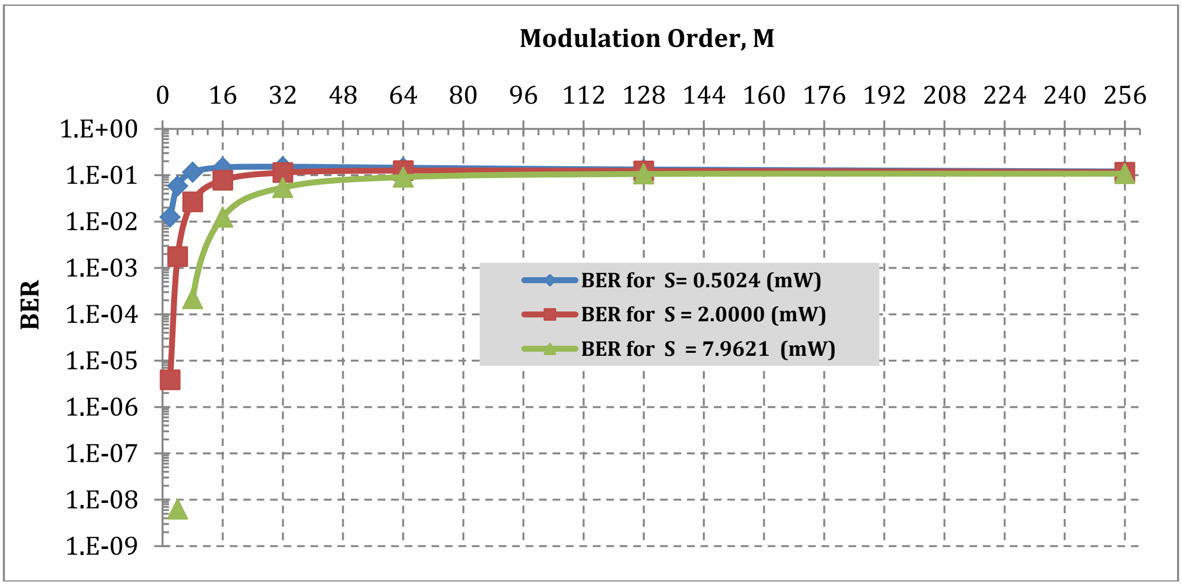

Figure 3 Modulation order , M Versus BER for various Signal Power, S (mW)

IV. CONCLUSION

The M-ary ASK modulation scheme is studied and the analytical expressions for the determination of the bit error probability or bit error rate (BEP or BER) and symbol error probability (SEP or SER) of the various M-ary ASK modulation schemes are presented. Numerical examples of M-ary BER for M-ary ASK schemes for M in the range of 2 to 256 are presented . The results showed that higher order M-ary ASK (with higher value of M) require more power to achieve a given BER when compared to the lower order M-ary ASK modulation schemes. In all, the study of the M-ary ASK scheme is essential to provide communication system designers to know how to select the appropriate M-ary ASK modulation scheme that best fits their system specifications.

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