

# Development Of Seeded Bisection Iteration Method Using Perturbation-Based Mechanism

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**Abstract**— In this paper, development of seeded bisection iteration method using perturbation-based mechanism is presented. The seeded bisection method uses one initial guess root value provided by the user to generate the two initial values for the classical Bisection iteration method. The flowchart for the perturbation-based seeded bisection iteration method (PSBIM) was presented and two functions were chosen and used for sample numerical examples to show the applicability of the new concepts presented in this paper. The simulation was conducted in Matlab software. The results showed that the initial seed value for the first function was 0.6253353000 but the generated two initial values for the PSBIM were iterations are 0.6253353006 and 0.6961310521. Also, the convergence cycle of the PSBIM is 11 for the first function. In the same way, the results showed that the initial seed value for the first function is 1.0764738270 but the generated two initial values for PSBIM iterations are 1.0764738281 and 1.2084326042 while the convergence cycle of the PSBIM was 15 for the second function.

**Keywords**— *Seeded Bisection, Perturbation-Based Mechanism, Convergence Cycle, Initial Root, Seed Value, Tolerance Error*

## 1. INTRODUCTION

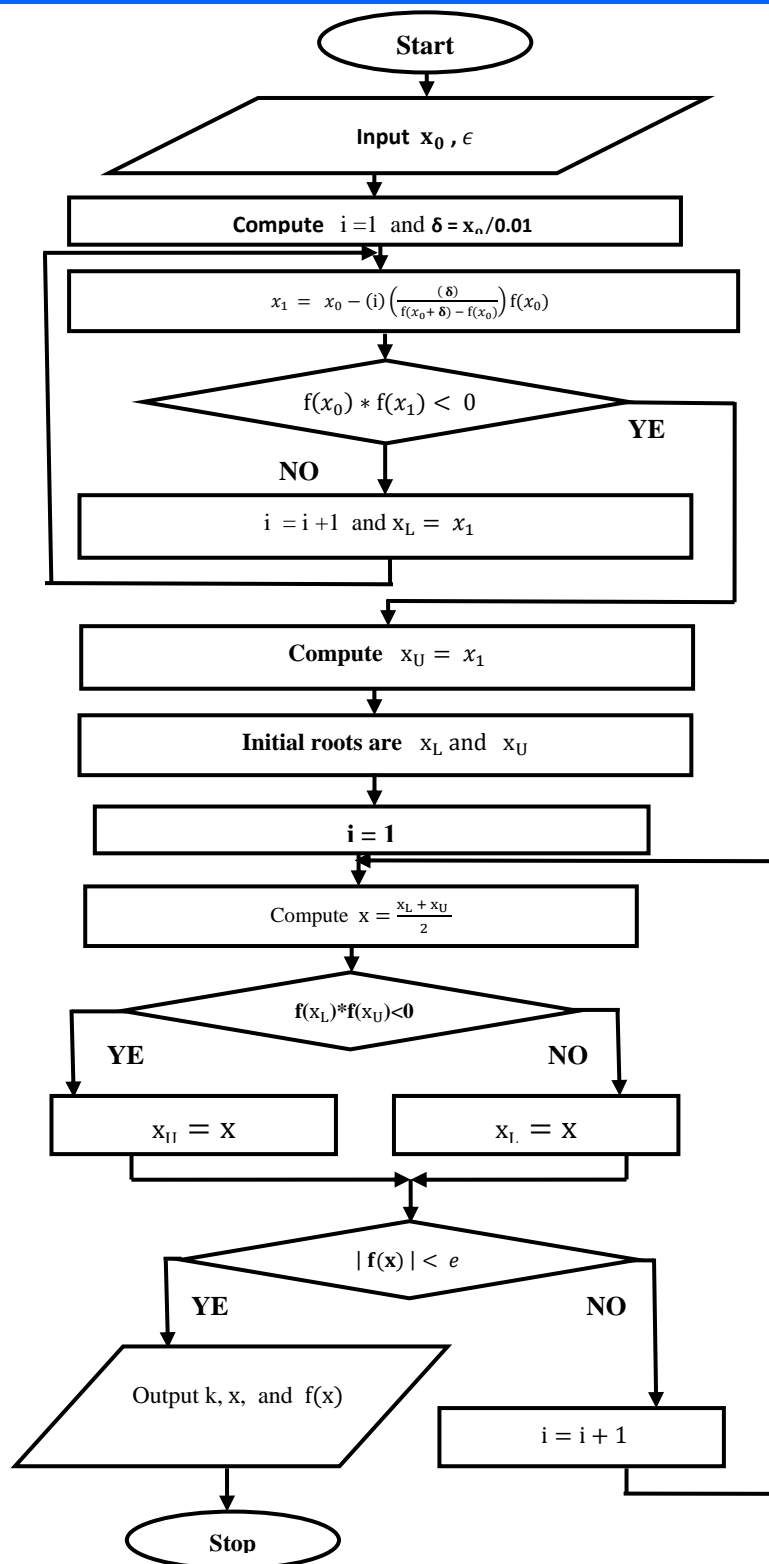
Classical bisection iteration method is one of the most popular iteration method because of its simplicity [1,2,3,4,5,6,7]. However, the drawback of the Bisection method is the slow rate which gives rise to high

convergence cycle number [8,9,10,11,12,13,14]. In addition, the Bisection method requires two initial guess root values [15,16,17] for the function and the guessed root values must be such that one of the values must be above the actual root while the other guess root must be below the actual root. This requirement makes it difficult to apply seeded iteration concept to the Bisection method. In seeded iteration methods, only one guess root is required to conduct the iteration.

Consequently, in this paper, a seeded Bisection method with perturbation-based mechanism is presented. The perturbation-based seeded bisection iteration method (PSBIM) requires users to supply only one seed (guess root value) and the PSBIM procedure will generate the required two initial roots and hence complete the Bisection iteration for finding the root of the function. The relevant flowchart for the PSBIM is presented along with numerical examples.

## II. THE PERTURBATION-BASED SEEDED BISECTION METHOD

In the perturbation-based seeded bisection iteration method (PSBIM), the assumption is that the function  $f(x)$  has at least a root  $x$  and a seed point,  $x_0$  can be selected by the user such that when the PSBIM is implemented for the function based on  $x_0$ , two other initial root values,  $x_L$  and  $x_U$  will be generated such that  $x_L$  and  $x_U$  will be generated such that  $f(x_L) * f(x_U) < 0$ . The flowchart for the perturbation-based seeded Bisection method is given in Figure 1.



**Figure 1 : The flowchart for the Perturbation-Based Seeded Bisection Method**

### III. RESULTS AND DISCUSSIONS

The following two functions were chosen for the numerical example on the application of the perturbation-based seeded bisection method;

- 1)  $f(x) = x^{-0.5} - \ln(1 + x^{0.5}) + 0.5\ln(1 - x) = 0$
- 2)  $f(x) = x^2 - \sin(x) - 0.5 = 0$

Mathlab software was used to determine the roots of the two chosen functions using the perturbation-based seeded bisection iteration method (PSBIM) with error tolerance of  $f(x_U) < 10^{-5}$ . The initial seed (guess root value) was selected for each function and the iteration was conducted until the error tolerance threshold was satisfied. The r two initial roots generated from the single guess root for the first function is shown in Table 1 while Table 2 shows the results of the PSBIM iterations and the convergence cycle. The results in Table 1 shows that the initial seed value for

the first function is 0.6253353000 but the generated two initial values for PSBIM iterations are  $x_L = 0.6253353006$  and  $x_U = 0.6961310521$ . Also, Table

2 results show that the convergence cycle of the PSBIM is 11 for the first function.

Table 1 The two initial roots generated from the single guess root for the first function,  $f(x) = x^{-0.5} - \ln(1 + x^{0.5}) + 0.5\ln(1 - x) = 0$

i		$x_i$	$f(x_i)$	$f(x_{i-1}) * f(x_i)$	$x_L$	$x_U$
0	$f(x_0)=f(x)$	0.6253353000	0.19105792	1.00000000	0.6253353006	0.6961310521
1	$f(x_1) = f(x+\delta)$	0.6253353006	0.19105792	0.03650313		
2	$f(x_2) = f(1-(\Delta * x))$	0.6961310521	-0.00372126	-0.00071098		
3	$f(x_3) = f(1-(2*\Delta)*x)$	0.7069551029	-0.03458368	0.00012869		

Table 2 The convergence cycle of the perturbation-based seeded bisection method with error tolerance of  $f(x_U) < 10^{-5}$  for the first function;  $f(x) = x^{-0.5} - \ln(1 + x^{0.5}) + 0.5\ln(1 - x) = 0$

n	XL	XU	f(XL)	f(XU)	X	f(X)	f(XL)*f(X)
1	<b>0.6253353</b>	<b>0.696131</b>	1.9105791769E-01	-0.003721259	<b>0.6607332</b>	<b>0.094845</b>	<b>1.8120823801E-02</b>
2	<b>0.6607332</b>	0.696131	9.4844662917E-02	-0.003721259	<b>0.6784321</b>	<b>0.045954</b>	<b>4.3585283625E-03</b>
3	<b>0.6784321</b>	0.696131	4.5954387189E-02	-0.003721259	<b>0.6872816</b>	<b>0.021228</b>	<b>9.7552848926E-04</b>
4	<b>0.6872816</b>	0.696131	2.1228190580E-02	-0.003721259	<b>0.6917063</b>	<b>0.008783</b>	<b>1.8645018494E-04</b>
5	<b>0.6917063</b>	0.696131	8.7831407123E-03	-0.003721259	<b>0.6939187</b>	<b>0.002539</b>	<b>2.2296763527E-05</b>
6	<b>0.6939187</b>	0.696131	2.5385866237E-03	-0.003721259	<b>0.6950249</b>	<b>-0.000589</b>	<b>-1.4962326639E-06</b>
7	<b>0.6939187</b>	0.695025	2.5385866237E-03	-0.000589396	<b>0.6944718</b>	<b>0.000975</b>	<b>2.4753168579E-06</b>
8	<b>0.6944718</b>	0.695025	9.7507677490E-04	-0.000589396	<b>0.6947483</b>	<b>0.000193</b>	<b>1.8815200816E-07</b>
9	<b>0.6947483</b>	0.695025	1.9296122418E-04	-0.000589396	<b>0.6948866</b>	<b>-0.000198</b>	<b>-3.8242425808E-08</b>
10	<b>0.6947483</b>	0.694887	1.9296122418E-04	-0.000198187	<b>0.6948175</b>	<b>-0.000003</b>	<b>-5.0273755205E-10</b>
11	<b>0.6947483</b>	0.694817	1.9296122418E-04	-2.60538E-06	<b>0.6947829</b>	<b>0.000095</b>	<b>1.8366012656E-08</b>
12	<b>0.6947829</b>	0.694817	9.5179810005E-05	-2.60538E-06	<b>0.6948002</b>	<b>0.000046</b>	<b>4.4056532189E-09</b>
13	<b>0.6948002</b>	0.694817	4.6287686629E-05	-2.60538E-06	<b>0.6948088</b>	<b>0.000022</b>	<b>1.0109818971E-09</b>
14	<b>0.6948088</b>	0.694817	2.1841270773E-05	-2.60538E-06	<b>0.6948131</b>	<b>0.000010</b>	<b>2.1006878076E-10</b>
15	<b>0.6948131</b>	0.694817	9.6179742904E-06	-2.60538E-06	<b>0.6948153</b>	<b>0.000004</b>	<b>3.3723540853E-11</b>

In the same way, the results in Table 3 shows that the initial seed value for the first function is 1.0764738270 but the generated two initial values for PSBIM iterations are  $x_L = 1.0764738281$  and  $x_U = 1.2084326042$ . Also, Table 4

results show that the convergence cycle of the PSBIM is 15 for the second function. In all, the perturbation-based approach has been successfully applied in the seeded Bisection iteration method

Table 3 The two initial roots generated from the single guess root for the second function,  $f(x) = x^2 - \sin(x) - 0.5 = 0$

i		$x_i$	$f(x_i)$	$f(x_{i-1}) * f(x_i)$	$x_L$	$x_U$
0	$f(x_0)=f(x)$	1.0764738270	-0.22149444	1.00000000	1.0764738281	1.2084326042
1	$f(x_1) = f(x+\delta)$	1.0764738281	-0.22149444	0.04905979		
2	$f(x_2) = f(1-(\Delta * x))$	1.2084326042	0.02524783	-0.00559225		
3	$f(x_3) = f(1-(2*\Delta)*x)$	-1.6613796806	3.25608258	0.08220901		

Table 4 The convergence cycle of the perturbation-based seeded bisection method with error tolerance of  $f(x_U) < 10^{-6}$  for the second function,  $f(x) = x^2 - \sin(x) - 0.5 = 0$

n	XL	XU	f(XL)	f(XU)	X	f(X)	f(XL)*f(X)
1	<b>1.0764738</b>	<b>1.208433</b>	-2.2149443987E-01	0.025247827	<b>1.1424532</b>	<b>-0.104456</b>	<b>2.3136392076E-02</b>
2	<b>1.1424532</b>	1.208433	-1.0445585943E-01	0.025247827	<b>1.1754429</b>	<b>-0.041194</b>	<b>4.3030041952E-03</b>
3	<b>1.1754429</b>	1.208433	-4.1194474093E-02	0.025247827	<b>1.1919378</b>	<b>-0.008372</b>	<b>3.4487162939E-04</b>
4	<b>1.1919378</b>	1.208433	-8.3717934744E-03	0.025247827	<b>1.2001852</b>	<b>0.008338</b>	<b>-6.9806494091E-05</b>
5	<b>1.1919378</b>	1.200185	-8.3717934744E-03	0.008338296	<b>1.1960615</b>	<b>-0.000042</b>	<b>3.4881971606E-07</b>
6	<b>1.1960615</b>	1.200185	-4.1666067984E-05	0.008338296	<b>1.1981233</b>	<b>0.004142</b>	<b>-1.7258435867E-07</b>
7	<b>1.1960615</b>	1.198123	-4.1666067984E-05	0.004142084	<b>1.1970924</b>	<b>0.002049</b>	<b>-8.5359252084E-08</b>
8	<b>1.1960615</b>	1.197092	-4.1666067984E-05	0.002048651	<b>1.1965769</b>	<b>0.001003</b>	<b>-4.1795372313E-08</b>

9	<b>1.1960615</b>	1.196577	-4.1666067984E-05	0.001003103	<b>1.1963192</b>	<b>0.000481</b>	<b>-2.0025599897E-08</b>
10	<b>1.1960615</b>	1.196319	-4.1666067984E-05	0.000480621	<b>1.1961903</b>	<b>0.000219</b>	<b>-9.1437554418E-09</b>
11	<b>1.1960615</b>	1.196190	-4.1666067984E-05	0.000219453	<b>1.1961259</b>	<b>0.000089</b>	<b>-3.7035936383E-09</b>
12	<b>1.1960615</b>	1.196126	-4.1666067984E-05	8.88875E-05	<b>1.1960937</b>	<b>0.000024</b>	<b>-9.8370284078E-10</b>
13	<b>1.1960615</b>	1.196094	-4.1666067984E-05	2.36092E-05	<b>1.1960776</b>	<b>-0.000009</b>	<b>3.7619503216E-10</b>
14	<b>1.1960776</b>	1.196094	-9.0288105012E-06	2.36092E-05	<b>1.1960856</b>	<b>0.000007</b>	<b>-6.5820962123E-11</b>
15	<b>1.1960776</b>	1.196086	-9.0288105012E-06	7.2901E-06	<b>1.1960816</b>	<b>-0.000001</b>	<b>7.8494430234E-12</b>

#### IV . CONCLUSION

The application of perturbation approach to realize seeded Bisection iteration method is presented. The seeded bisection method uses one initial value to generate the two initial values for the classical Bisection iteration method. Sample numerical examples were used to show the applicability of the new concepts presented in this paper. The simulation was conducted in Matlab software.

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