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Axial strength estimation of reinforced concrete columns confined through fiber reinforced polymer (FRP) jackets

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***Abstract*—Basic assumptions made in available design models aimed at estimation of the axial strength of axially loaded reinforced concrete columns wrapped with fiber reinforced polymer jackets are discussed. The performance of representative available design models is assessed based on their ability to accurately predict the axial strength of 101 specimens from the literature. A simple design model is proposed and described in detail, which has resulted from a combination of Eurocode provisions in such a way that the predictions are good and also safe.**

Keywords—axial strength; reinforced concrete columns; FRP jacket; transvesre reinforcement; design model; confinement

I. Introduction

There is general consensus that fibre-reinforced polymer (FRP) jackets provide significant confinement in columns subjected to axial compressive load, are easy to apply and do not alter the cross section, and hence the stiffness, of the structural elements. The enhancement of the axial strength of plain concrete columns wrapped with FRP fabrics has been extensively investigated, both analytically and experimentally. Respective research on reinforced concrete (RC) columns strengthened with FRP is significantly more limited. Given that FRP jackets are a good solution for upgrading existing substandard RC columns in older structures in seismic prone areas, this paper deals with axially loaded RC columns fully wrapped with carbon and glass FRP (CFRP and GFRP, respectively), with circular, square, and rectangular cross-section. The basic assumptions made in the design models for the estimation of the axial strength of FRP-jacketed columns are discussed. A simple design model is proposed and described in detail, which has resulted from a combination of Eurocode provisions in such a way that the predictions are good and also safe. Furthermore, three models from the literature, selected among others because of their good agreement with the experimental data, are also presented and some of their aspects are discussed. The models are applied to 101 specimens tested in the literature. The criteria for selecting the models presented here were their simplicity in application, combined with their accuracy in predicting the strength of the tests in the database.

II. Confining Action of Stirrups and Frp Jacket

Columns subjected to axial compression tend to expand laterally. Restriction of expansion by the presence of a confining means in the perimeter of the cross section results in the exertion of lateral pressure in the plane of the cross section, i.e. perpendicular to the axis of the column (Fig. 1). As a result, the element is subjected to a tri-axial state of compression, which, in turn, results in enhancement of the ultimate characteristics at failure (Fig. 2). The peak axial strength and the corresponding strain appear considerably increased as compared to the respective values of the unconfined element, are designated as “confined characteristics”, and usually are indicated by a subscript “c” that follows the symbols used for the unconfined characteristic; f_{cc}, ε_{cc}.

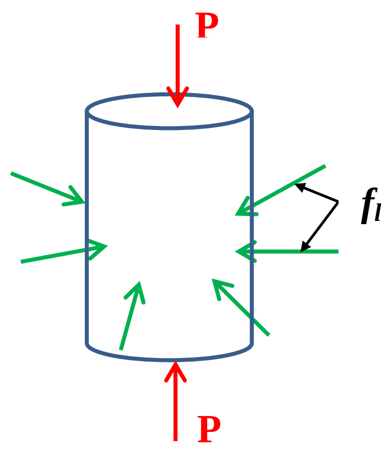

Fig. 1. *Lateral stresses, f_l, generated along an axially loaded element when the element is restricted (confined) to expand in the plane perpendicular to the axial load.*

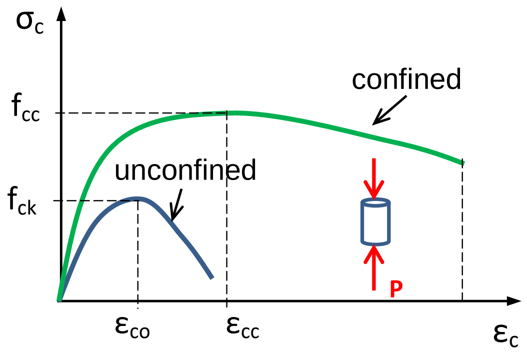

Fig. 2. *Unconfined and confined stress-strain relation of a concrete element subjected to axial compression.*

The majority of the models that estimate the increase in peak axial compressive strength because of the presence of lateral confinement are based on the Richart, Brandtzaeg and Brown model (1) which was derived by tests on plain concrete specimens subjected to hydrostatic pressure [1], [2].

$$f_{cc} = f_c \left(1 + k_1 \frac{f_l}{f_c}\right) \quad (1)$$

where f_{cc} and f_c are the compressive strengths of the confined and unconfined concrete; f_l is the lateral hydrostatic pressure, and $k_1 = 4.1$ is a coefficient depicting the efficiency of confinement.

Numerous models have been proposed for the prediction of the stress-strain characteristics of FRP-confined concrete columns, since the early 1980's, including [3] to [10]. The comparative performance assessment of models has also been studied, e.g. [11] to [13].

Lateral confinement in RC columns wrapped by FRP jackets is the combined result of both the effect of steel stirrups plus the FRP jacket.

Basic differences between steel- and FRP-confinement of concrete columns are: (a) confinement by steel stirrups increases linearly up to yielding of steel and remains constant after yielding, while the lateral pressure induced by the FRP jackets increases linearly up to rupture of the jacket; (b) stirrups induce lateral stresses f_l through the bent parts of the transverse reinforcement, both in the plane of the section and along the axis of the column, while FRP jackets induce lateral stresses (b1) in-plane of the section through the chamfered corners of the section and (b2) in-height along the total height of the FRP jacket, in case of continuous wrapping; (c) confinement through FRP jackets is influenced by the application method and quality of work and strain localizations [14] to [17], and the overlap length [18] to [20], contrary to steel stirrups that are not sensitive to similar parameters; (d) FRP jackets may rupture prematurely in the event of buckling of the longitudinal reinforcement of the column (e.g. [21], [22], [23]).

Effectiveness or efficiency of a confining scheme is usually defined as the percentage of the confined volume in respect to the total volume of the element.

A number of assumptions have to be decided regarding different issues concerning the calculation of the compressive axial strength of a concrete column confined by steel stirrups and FRP jackets. Those assumptions include the estimation of the following:

- Estimation of the effectively confined concrete because of steel stirrups in relation to the geometry of the cross section and the layout of stirrups, in the horizontal plane of the cross-section and in height (Figs. 3, 4). In Figs. 3 and 4 the shaded area corresponds to the effectively confined area of the cross section. Different expressions are used to calculate the effectiveness of steel stirrups in confinement. Most often the approach proposed by [24] and later by [25] is adopted.

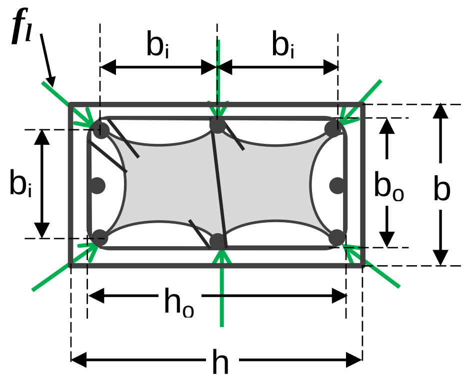

Fig. 3. *Effectively confined area in the plane of a rectangular section confined by steel stirrups and ties.*

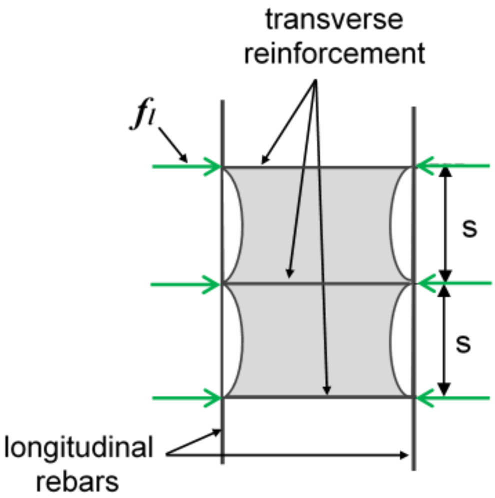

Fig. 4. *Effectively confined area along the height of a column confined by steel stirrups spaced at center-to-center distances equal to s.*

- The effectively confined concrete because of the FRP jacket in relation to the geometry of the cross section and the layout of FRP, in the plane of the cross section and in height when the wrapping in not continuous. Expressions similar to the ones for transverse steel are used. For rectangular sections, the radius of the chamfered corner of the section is also taken into account. Fig. 5 illustrates as shaded area the confined concrete of a rectangular section with external dimensions b×h and sectional corners chamfered at a radius R_c.

- The confinement efficiency of FRP jackets strongly depends on the assumed activation of the FRP at failure. The maximum confinement provided is expressed by the value of the strain of the FRP jacket assumed at rupture of the FRP which is used to calculate the lateral pressure exerted by the FRP. Two issues regarding the actual behavior have to be considered:

(a) Strains are not uniform along the perimeter of a section [26] to [29]. Generally, an average lateral pressure, corresponding to the maximum strained section at mid-height, is assumed. Lateral strains vary particularly in rectangular sections in which maximum hoop strains are typically measured at the axes of the flat sides or at the transition points between the end of the chamfered corner and the beginning of the flat side of the section [30].

(b) Maximum strain at failure of the FRP jacket is less than the one provided by the manufacturer, which is derived from flat coupon tests. This is shown by measurements of FRP strains along the circumference of the section on the FRP at the point of FRP rupture. Most researchers assume an “effective strain at rupture” ε_{ju}, of the FRP which is calculated from (2), by multiplying the ultimate strain of the fibers provided by the manufacturer, ε_{fu}, by a reduction factor k_ε.

$$\varepsilon_{ju} = k_\varepsilon \cdot \varepsilon_{fu} \tag{2}$$

It has been observed that k_ε is material dependent. Furthermore, k_ε depends also on the overall characteristics of the specimens and, therefore, the values reported by different researchers vary considerably, usually within a range of $k_e = 0.50 - 0.90$ (e.g. [19], [23], [31]). For hybrid FRP jackets when the GFRP jacket is outside the CFRP jacket the hoop rupture strain measured on the GFRP jacket may exceed ε_{fu} of the inner CFRP jacket [18].

Different values of FRP rupture strain are adopted in different design equations. Indicative values proposed for CFRP jackets are: $k_\varepsilon = 0.586$ [9], $k_\varepsilon = 0.85$ [32], $\varepsilon_{ju} = 0.003$ [33]. Several models include an equation for k_ε, as in [34] that relates k_ε to the amount of longitudinal reinforcement. Other researchers use the actual effective strain at rupture ε_{ju}, from the measured lateral FRP strains in the tests in order to verify the validity of a proposed model, e.g. [35].

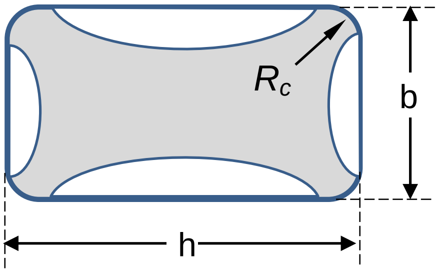

Fig. 5. *Effectively confined area in a FRP-confined rectangular section with sectional (chamfered) corner radius R_c.*

It is worth noting that the maximum hoop strains measured on the FRP jacket do not seem to be always related to the confinement efficiency of the jacket [18], [28], [30].

- Models adopt different analytical procedures for combining both the effects of steel and FRP to confinement when calculating the final compressive concrete strength. For example (a) the confined compressive strengths f_{cc} because of stirrups and because of FRP are calculated separately using different formulas in each case and the results are added [32], or (b) the lateral stresses f_l because of stirrups and FRP are calculated separately and added to obtain the total lateral pressure for the two confining means, and this value is introduced in a single equation similar to (1), e.g. [34], [36].

III. Applied Design Equations for the Estimation of Confined Concrete Strength Models

A. *Proposed model*

Eurocodes do not explicitly describe how to estimate the ultimate axial capacity of FRP-confined RC columns. Eurocode 8, part 3 [37] offers equations that allow separate calculation of the confinement effect of steel and FRP. It is observed that alternative procedures are offered to calculate the lateral confinement pressure due to FRP.

The proposed model is the result of the combination of provisions in [37], Eurocode 8 part 1 [38], and Eurocode 2 part 1-1 [39]. The calculation procedure, which is presented in detail in the following, is easy to apply and leads to good and safe predictions for the specimens in the database.

The stress-strain relation of confined concrete compressive strength, f_{cc} in relation to the effective lateral compressive stress, σ_2, and the characteristic compressive cylinder concrete strength of concrete at

28 days, f_{ck}, is calculated according to (3a) and (3b) from [39]:

For $\sigma_2 \leq 0.05 f_{ck}$

$$f_{cc} = f_{ck}(1.0 + 5.0 \cdot \sigma_2 / f_{ck}) \quad (3a)$$

For $\sigma_2 > 0.05 f_{ck}$

$$f_{cc} = f_{ck}(1.125 + 2.5 \cdot \sigma_2 / f_{ck}) \quad (3b)$$

Lateral stress σ_2 is calculated by adding the respective lateral pressures acting on concrete by the FRP jacket, $f_{l,f}$ and by the steel stirrups, $f_{l,s}$.

The effective lateral stress due to confinement applied by the FRP continuous sheet, $f'_{l,f}$ is calculated from (4) by multiplying the lateral pressure $f_{l,f}$ from the FRP jacket by the effectiveness factor for confinement k_s:

$$f'_{l,f} = k_s \cdot f_{l,f} \quad (4)$$

$$f_{l,f} = 2E_f \varepsilon_{ju} t_f / D \quad (4a)$$

where E_f is the FRP elastic modulus, ε_{ju} is the ultimate strain of FRP at failure of the specimen, t_f, is the total thickness of the FRP jacket, and D is the diameter in case of a circular cross-section, or the larger cross-section width in case of rectangular cross-sections.

The effectiveness factor for FRP-confinement, k_s used in (4) is calculated as follows:

For circular cross-section:

$$k_s = 1 \quad (5a)$$

For rectangular cross-section:

$$k_s = 2R_c / D \quad (5b)$$

where R_c is the radius of the rounded corner section (Fig. 5), and D is the larger section width.

It is noted that in [37] ε_{ju} is "the adopted FRP jacket ultimate strain, which is lower than the ultimate strain ε_{fu} of FRP" provided by the manufacturer. Further on it is specified elsewhere that the limit strain ε_{fu} is equal to 0.0015 for CFRP (carbon fiber reinforced polymer) or AFRP (aramid fiber reinforced polymer), and equal to 0.02 for GFRP (glass fiber reinforced polymer).

However, since no effectiveness factor is explicitly mentioned in the code, the model has been applied in this work assuming the strain of the FRP jacket at failure:

$$\varepsilon_{ju} = \varepsilon_{fu} \quad (6)$$

A basic reason why this assumption is made, is that lateral strains measured on the FRP jacket in column tests show considerable variation along the same cross-section, both in RC elements ([28], [30], [34]), as well as in plain concrete specimens ([18], [19], [20], [26], [31]), with the divergence of strains depending on the different section characteristics. Therefore, it is difficult to decide which value of the FRP rupture strain is more representative to adopt for the calculation of the confining lateral pressure at the ultimate load, i.e. the maximum or the average. Furthermore, the value of ε_{ju} at rupture of the FRP jacket is very uncertain.

It is worth noting, however, that despite the different assumptions made by researchers, e.g. [9] adopt $\varepsilon_{ju} = 0.586\varepsilon_{fu}$ for plain concrete CFRP confined columns, while [32] assume for RC CFRP-confined columns $\varepsilon_{ju} = 0.85\varepsilon_{fu}$, the resulting performance of the respective models proposed is good.

To account for the increased axial strength the assumption (6) entails, it was decided not to include the contribution of longitudinal reinforcement in calculating the ultimate axial strength. In other models, for the calculation of the contribution of the longitudinal bars usually the yield stress of longitudinal reinforcement is assumed [40], [41], while in [30] the measured strains of the steel bars were introduced in the design model.

The lateral confining pressure due to steel stirrups $f_{l,st}$ is calculated in the proposed model by (7a), (7b) (among different alternatives supplied in [38]):

$$f_{l,st} = \alpha \cdot \rho_{sx} \cdot f_{yw} \quad (7a)$$

$$f_{l,st} = \alpha \cdot \rho_{sy} \cdot f_{yw} \quad (7a)$$

where f_{yw} = stirrup yield strength (in MPa), $\rho_{sx} = A_{sx} / b_x s$ = ratio of transverse steel parallel to the direction x of loading, b_x = the respective width of the cross-section along x direction, $\rho_{sy} = A_{sy} / b_y s$ = ratio of transverse steel parallel to the direction y of loading, b_y = the respective width of the cross-section along y direction, s = stirrup spacing, and α = the confinement effectiveness factor, calculated from (8).

The confinement effectiveness factor, α, is calculated as follows [38]:

$$\alpha = \alpha_n \cdot \alpha_s \quad (8)$$

where α_n is the confinement effectiveness factor in the plane of the section (Fig. 3), calculated from (9a) and (9b), and α_s is the confinement effectiveness factor along the height of the column (Fig. 4), calculated from (10a), (10b) and (10c):

For circular cross-section:

$$\alpha_n = 1 \tag{9a}$$

For rectangular cross-section:

$$\alpha_n = (1 - \sum_n b_i^2 / 6b_o h_o) \tag{9b}$$

For circular cross-section with spiral hoops:

$$\alpha_s = (1 - s / 2D_o) \tag{10a}$$

For circular cross-section with circular hoops:

$$\alpha_s = (1 - s / 2D_o)^2 \tag{10b}$$

For rectangular cross-section:

$$\alpha_s = (1 - s / 2b_o)(1 - s / 2h_o) \tag{10c}$$

where D_o = diameter of the confined circular core (to the centreline of hoops), h_o and b_o = the depth and width, respectively, of the confined rectangular core (to the centreline of hoops), s = spacing of hoops, and b_i = distance between consecutive longitudinal bars, engaged by a tie or a stirrup (Fig. 3).

The total lateral confinement pressure σ_2 is calculated by (11), as sum of the lateral pressure due to FRP and stirrups, from (4) and (7), respectively:

$$\sigma_2 = f'_{l,f} + f_{l,st} \tag{11}$$

The confined concrete strength, f_{cc}, is calculated using (3a) or (3b), with the total lateral confinement pressure σ_2 being calculated from (11).

B. *Pellegrino and Modena model [34]*

The model is applicable to columns with and without steel, for all types of cross-sections. It has been calibrated against a vast number of specimens with and without steel reinforcement and takes explicitly into account a number of parameters that other models do not consider. Details about the derivation of the parameters in the model may be found in [34].

The authors propose an empirical formula (12a), (12b) to calculate the coefficient of efficiency k_ε of the FRP, in order to determine the effective FRP strain at rupture ε_{ju} from the ultimate strain of FRP ε_{fu} : $\varepsilon_{ju} = k_\varepsilon \cdot \varepsilon_{fu}$ (2).

For FRP-confined columns without steel reinforcement:

$$k_\varepsilon = 0.25 + 0.25(2R_c / b) \tag{12a}$$

where R_c is the corner radius of rectangular sections (Fig. 5), and b is the minimum dimension of the cross section. For circular sections $2R_c / b = 1$ and $k_\varepsilon = 0.50$.

For RC columns, [34] concluded that k_ε is not significantly affected by $2R_c / b$ because the presence of stirrups at the corners of the section reduces the stress concentration in the FRP jacket. On the other hand, the advent of buckling of the longitudinal reinforcement should be considered in k_ε, especially for sparsely spaced stirrups. For FRP-confined columns with steel reinforcement the authors propose (12b) for the calculation of k_ε where the effect of FRP stiffness on restraining steel buckling is considered by parameter C that expresses the ratio of mechanical steel percentage $(E_{y,long} \cdot \rho_{y,long})$ to the mechanical FRP percentage $(E_f \cdot \rho_f)$:

$$k_\varepsilon = \gamma \cdot C^{-0.7} = \gamma \frac{E_{y,long} \cdot \rho_{y,long}}{E_f \cdot \rho_f} \leq 0.8 \tag{12b}$$

where $\gamma = 0.7$ for CFRP jackets and $\gamma = 1.5$ for GFRP jackets. Furthermore, $E_{y,long}$ and $\rho_{y,long}$ are the elastic modulus of the longitudinal steel reinforcement and the longitudinal steel ratio, respectively, and E_f is the elastic modulus of FRP. The FRP ratio ρ_f is calculated from (13a) and (13b).

For circular columns:

$$\rho_f = 4t_f / D \tag{13a}$$

For columns with rectangular cross-sections:

$$\rho_f = 2t_f (b + h) / bh \tag{13b}$$

where D, b, h are the geometric dimensions of the cross-section, t_f is the total width of the FRP jacket.

Peak strength of an axially loaded confined column wrapped by a continuous FRP jacket is given by (14)

$$\frac{f_{cc}}{f_{co}} = 1 + k_1 \cdot \frac{\sigma_2}{f_{co}} \tag{14}$$

$$k_1 = k_A \cdot k_R \tag{14a}$$

with:

$$k_R = 1 - 2.5(0.3 - 2R_c / b) \quad \text{for} \quad 2R_c / b < 0.3 \tag{14b}$$

$$k_R = 1 \quad \text{for} \quad 2R_c / b \geq 0.3 \tag{14c}$$

$$k_A = A \cdot \left(\frac{\sigma_2}{f_{co}} \right)^{-\alpha} \tag{14d}$$

where parameters A and α were determined through a regression analysis on a test database, and differ according to the type of section:

For circular sections:

- without steel: $A = 3.55$ and $\alpha = 0.15$
- with steel: $A = 2.95$ and $\alpha = 0.40$

For rectangular sections:

- without steel: $A = 2.25$ and $\alpha = 0.25$
- with steel: $A = 1.35$ and $\alpha = 0.50$

The lateral pressure σ_2 is attributed to both the FRP – and steel-confinement and is derived from (15):

$$\sigma_2 = f_{lf}^{'} + f_{l,st} \cdot A_{cc} / A_g \tag{15}$$

The effective lateral pressure induced by the FRP jacket is calculted according to (16).

$$f_{lf}^{'} = \frac{1}{2} k_f \cdot \rho_f \cdot E_f \cdot \varepsilon_{ju} \tag{16}$$

For circular sections:

$$k_f = 1 \tag{16a}$$

For rectangular sections:

$$k_f = 1 - \frac{(b - 2R_c)^2 + (h - 2R_c)^2}{3bh} \tag{16b}$$

The lateral pressure induced by the steel stirrups is calculted from (17), as proposed by [25]. The coefficent of efficinecy for the transverse steel k_s is calualted from (18). The symbols used in [34] are kept in the following:

$$f_{ls} = \frac{1}{2} k_s \cdot \rho_{st} \cdot f_{y,st} \tag{17}$$

$$k_s = k_v \cdot k_{es} \tag{18}$$

where $k_v =$ coefficient of vertical efficiency for transverse confining steel, calculated from (19a) to (19c), $k_{es} =$ coefficient of horizontal efficiency for transverse confining steel, calculated from (20a) and (20b), $\rho_{st} =$ ratio of transverse confining steel, and $f_{y,st} =$ yield stress of the transverse steel reinforcement.

For circular section confined with circular spirals:

$$k_v = \frac{(1 - s^{'} / 2D_o)}{1 - \rho_{cc}} \tag{19a}$$

For circular cross-section with circular stirrups:

$$k_v = \frac{(1 - s^{'} / 2D_o)^2}{1 - \rho_{cc}} \tag{19b}$$

For rectangular cross sections:

$$k_v = \frac{(1 - s^{'} / 2b_o)(1 - s^{'} / 2h_o)}{1 - \rho_{cc}} \tag{19c}$$

For rectangular sections:

$$k_{es} = \frac{1 - \sum_n b_i^2 / 6b_o h_o}{1 - \rho_{cc}} \tag{20a}$$

For circular sections:

$$k_{es} = 1 \tag{20b}$$

where $\rho_{cc} =$ ratio of area of longitudinal reinforcement to area of core of section, the area of core section being defined by the center lines of the perimeter spiral or hoops, b_o and h_o = the depth and width, respectively, of the confined rectangular core (to the centreline of hoops), $s^{'} =$ net distance between two spirals or hoops, D_o = diameter of the confined circular core (to the centreline of hoops).

C. Rousakis and Karabinis model [42]

The model is applicable to circular and square cross-sections.

To overcome the uncertainty of determining the value of lateral FRP strain at failure of the specimen, ε_{ju}, the authors provide an expression, derived from regression analysis, that relates ε_{ju} to the normalized axial rigidity of the FRP jacket, which is substituted into a Richart, Brandtzaeg and Brown type model (1). Hence, the FRP-rupture strain is not required for the calculation of the confined concrete strength.

The equations for the calculation of the confined concrete strength of a plain concrete element confined by an FRP jacket, $f_{cc,FRP}$ are: (21) for circular columns, and (22) for square columns.

$$\frac{f_{cc,FRP}}{f_{ck}} = \frac{\rho_f E_f}{f_{ck}} \left(-\frac{0.4142 E_f \cdot 10^{-6}}{E_{f\mu}} + 0.0248 \right) + 1 \tag{21}$$

$$\frac{f_{cc,FRP}}{f_{ck}} = \frac{2\rho_f E_f}{f_{ck}} \left(-\frac{0.4142 E_f \cdot 10^{-6}}{E_{f\mu}} + 0.0248 \right) \left(\frac{2R_c}{b} \right) + 1 \tag{22}$$

where f_{ck} is the compressive concrete strength of standard cylinders, E_f is the FRP jacket modulus of elasticity, $E_{f\mu} = 10MPa$, $\rho_f = 4t_f / D$ and D, b, and t_f are the diameter of round section, external dimension of square section, and the total width of the jacket, respectively, while R_c is the corner radius of the chamfered corners in square cross sections (Fig. 5).

The total confined strength of the element is calculated as sum of the independent contribution of FRP, of stirrups and of longitudinal reinforcement, calculated from (23):

$$f_{cc} = f_{cc,FRP} + \Delta f_{cc,stirrup} + f_{cc,bars} \qquad (23)$$

where $f_{cc,FRP}$ is calculated from (21) and (22) and $f_{cc,bars}$ is the respective load carrying capacity of the longitudinal reinforcement bars.

For the calculation of the enhancement of the concrete strength because of the steel stirrups, $\Delta f_{cc,stirrup}$ the authors propose to use the equations (24a) and (24b) of Model Code 1990 [43].

For $\sigma_2 \le 0.05 f_{ck}$

$$\Delta f_{cc,stirrup} = f_{ck}(1.0 + 2.5 \cdot \alpha \cdot \omega_w) - f_{ck} \qquad (24a)$$

For $\sigma_2 > 0.05 f_{ck}$

$$\Delta f_{cc,stirrup} = f_{ck}(1 + 1.25 \cdot \alpha \cdot \omega_w) - f_{ck} \qquad (24b)$$

where α is the confinement effectiveness factor, which can be calculated from (8), and ω_w is the volumetric percentage of confining steel.

D. Ilki, Peker, Karamuk, Demir, and Kumbasar model [32]

The model provides equations for the compressive strength and corresponding strain of CFRP wrapped RC columns, and was determined by statistical evaluation of experimental data from tests presented in [32]. The model is adequate for both circular and rectangular columns, for the range of variables considered in the study, i.e. for unconfined concrete standard cylinder strength $f'_{co} = (15-30) MPa$ and for ratios of effective confinement pressure provided by CFRP jacket, $f'_{l,FRP}$ to the unconfined concrete compression strength f'_{co}, i.e. $f'_{l,FRP} / f'_{co}$ (a) for circular columns between 0.20 and 6.25, and (b) for rectangular columns between 0.08 and 1.93.

The model calculates the confined concrete strength as sum of the lateral confinement because of (a) CFRP jacket and (b) internal transverse reinforcement, ITR, according to (25). Equations for the contribution of each confining factor are given in (26) to (28).

$$\left[\frac{f'_{cc} - f'_{co}}{f'_{co}}\right]_{TOTAL} = \left[\frac{f'_{cc}}{f'_{co}} - 1\right]_{CFRP} + \left[\frac{f'_{cc}}{f'_{co}} - 1\right]_{ITR} \qquad (25)$$

The confined concrete strength attributed to the FRP jacket is calculated from (26).

$$f'_{cc,CFRP} = f'_{co} + 2.54 f'_{l,CFRP} \qquad (26)$$

$$f'_{l,CFRP} = \frac{1}{2}\kappa_\alpha \varepsilon_{ju} E_f \qquad (27)$$

where $f'_{l,CFRP}$ = effective lateral confinement stress induced by the FRP jacket, κ_α = efficiency factor, determined as the ratio of effectively confined cross-sectional area of concrete by the FRP jacket to the gross area of the section, and may by calculated according to [44] or [45], E_f = tensile modulus of elasticity of FRP, and ε_{ju} = tensile strain of CFRP at failure of the specimen.

It is noted that tensile rupture CFRP strain is assumed to be 85% of the ultimate strain $\varepsilon_{ju} = 0.85 \cdot \varepsilon_{fu}$, which resulted from the observation that the average lateral strains on the CFRP jacket measured at peak load in the study was 0.0125, which corresponded to 85% of the ultimate CFRP strain provided by the manufacturer.

The confined concrete strength attributed to the effect of transverse stirrups is calculated from (28).

$$f'_{cc,ITR} = f'_{co} + 4.54 f'_{l,ITR} \qquad (28)$$

where $f'_{l,ITR}$ the effective lateral confinement stress provided by the internal reinforcement, calculated according to [46].

Unconfined concrete strength f'_{co} in (25), (26) and (28) is the compressive strength of the same size specimen without confinement at the time of testing. When this strength is not available, [32] assume f'_{co} equal to 85% of the standard cylinder strength the day of testing.

E. Wang and Hsu model [40]

The design model proposed in [40] aims at the calculation of the axial strength of FRP- and steel-confined columns with rectangular and square section. The nominal compressive strength in terms of axial force is determined from (29).

$$P_n = P_{cn} + P_{sn} \qquad (29)$$

where P_{cn} and P_{sn} are the nominal compressive forces carried by the concrete and the longitudinal steel bars, calculated from (30) and (31), respectively.

$$P_{cn} = 0.3 f_{ck} A_{cu} + f_{cc,j} A_{cj} + f_{cc,js} A_{cjs} \qquad (30)$$

$$P_{sn} = f_{sy} A_s \qquad (31)$$

where f_{ck} is the cylinder compressive strength of concrete, $f_{cc,j}$ and $f_{cc,js}$ are the compressive strengths of concrete confined by FRP jacket, and of concrete confined by both FRP jacket and the steel hoops, f_{sy} is the yield strength of the longitudinal reinforcement, A_s is the cross-section area of the longitudinal steel reinforcement, while A_{cu} is the unconfined concrete area of the section, A_{cj} and A_{cjs} are the areas of concrete effectively confined by the FRP jacket, and by both the FRP jacket and the steel hoops, respectively.

The effect of steel confinement to the enhancement of the compressive strength of the column is calculated according to [25].

More details about the model may be found in [40].

IV. Assembled Database From the Literature

A database consisting of 101 reinforced concrete columns wrapped by continuous FRP jackets and subjected to axial compression up to failure has been assembled from the literature. The database is used to assess the predictive capacity of the design equations that are studied.

The range of the characteristics of the specimens in the database is shown in Table I.

The specimens included in the database are assembled from 19 references: [23], [32], [33], [36], [40], [42], and [47] to [59].

V. Predictive Capacity of the Models for the Specimens of the Database

For the specimens in the assembled database, the analytical value of the confined strength because of the combined effect of FRP jacket and transverse steel reinforcement has been calculated using the proposed model and the 4 models discussed, i.e. the models of Pellegrino and Modena [34], Rousakis and Karabinis [42], Ilki, Peker, Karamuk, Demir, and Kumbasar [32], and Wang and Hsu [40].

In the application of Rousakis and Karabinis model the specimens with rectangular cross-section are not included (21 specimens).

The model of Ilki, Peker, Karamuk, Demir, and Kumbasar has not been applied on the GFRP-confined specimens of the database (14 specimens). However, specimens with plain concrete strength higher than the range specified by the authors have been included in the comparison. It should be mentioned that for the majority of specimens the strength of unconfined specimens f'_{co} as defined by [32] was not available. Therefore, f'_{co} was assumed equal to 85% of the standard cylinder strength the day of testing, as alternatively proposed by the authors.

The model of Wang and Hsu has not been applied on specimens with circular section (42 specimens).

Generally, the strengths of materials were entered in the models without reduction factors, since they were obtained experimentally.

TABLE I. Characteristics of specimens in the database

Test parameter	Range
Unconfined concrete cylinder compressive strength (MPa)	15 to 46
Ratio of longitudinal reinforcement, ρ_s	0.0017 to 0.014
Ratio of transverse reinforcement, ρ_w	0.0026 to 0.0164
Ratio of FRP fabric, ρ_f	0.0022 to 0.0847
Number of carbon FRP (CFRP) layers	1 to 6
Number of glass FRP (GFRP) layers	1 to 6
(Stirrup distance, s) / (longitudinal bar diameter, D_s)	3.6 to 47
Yield strength of transverse reinforcement (MPa)	200 to 587
Yield strength of longitudinal reinforcement (MPa)	275 to 620
Diameter of circular cross-section (mm)	150 to 610
Dimensions of rectangular cross-section (mm)	79 to 610
Aspect ratio of sides of rectangular cross section	1 to 2.7

Figs. 6 to 10 depict the comparison between the analytically calculated values of the confined concrete strength ("analytical", V_{anal}) and the measured peak compressive strength of the specimens at testing ("experimental", V_{exp}). Different symbols are used to indicate the different types of cross section, i.e. round, square and rectangular, and also the range of number of FRP plies.

Values below the diagonal indicate unsafe predictions, given that the model estimates higher capacity than the one that was observed in the test. The main objective of the comparison, however, is to determine which model tends to better predict the ultimate axial strength for the range of the test parameters considered.

Table II shows the statistical indices that allow for the evaluation of the competence of the models to predict the confined concrete strength. More particularly, Table II presents: the number of specimens of the database on which each model is applied, the average value of the ratios $V_{anal} / V_{\exp}$, the correlation coefficient (CORREL) and the standard deviation (STDEV) between V_{anal} and $V_{\exp}$, where V_{anal} is the analytically predicted confined compressive strength from a model, and $V_{\exp}$ is the experimental peak compressive strength measured in the test.

The statistical indices are reported both for all specimens to whom the models apply, as well as for the rectangular specimens (square sections being included).

From the statistical indices it is demonstrated that the proposed model for all shapes of cross-section results in the best correlation coefficient and standard deviation values, but in lower average predicted-to-test axial strength ratio, among the models considered. In general, the predictions of this model are on the safe side, hence the lower average predicted-to-test axial strength ratio. For the specimens with rectangular section, Pellegrino and Modena model appears to perform slightly better than the one proposed, while the performance of the model of Wang and Hsu is comparatively inferior to that of the other models.

The predictive capacity of the models for columns confined only with FRP jacket without any steel reinforcement is depicted on Fig. 11. It is noted that those specimens are included in the database as companion specimens to the RC FRP-confined specimens tested in the different experimental studies, for comparison purpose. Fig. 11 demonstrates that the proposed model predicts particularly well the compressive strength of plain concrete FRP-confined specimens

TABLE II. Statistical indices for predicted V_{anal} and experimental V_{exp} axial strengths for the specimens in the database.

Model	**Number of specimens**	**Average** V_{anal} / V_{exp}	**CORREL**	**STDEV**
All sections				
Proposed model	101	0.927	0.945	0.138
Pellegrino and Modena	101	0.940	0.917	0.161
Ilki et al.	87	0.960	0.918	0.151
Rousakis and Karabinis	80	0.972	0.883	0.172
Rectangular sections only				
Proposed model	59	0.885	0.837	0.146
Pellegrino and Modena	59	0.929	0.817	0.166
Wang and Hsu	59	1.046	0.780	0.253
Ilki et al.	46	1.051	0.900	0.147

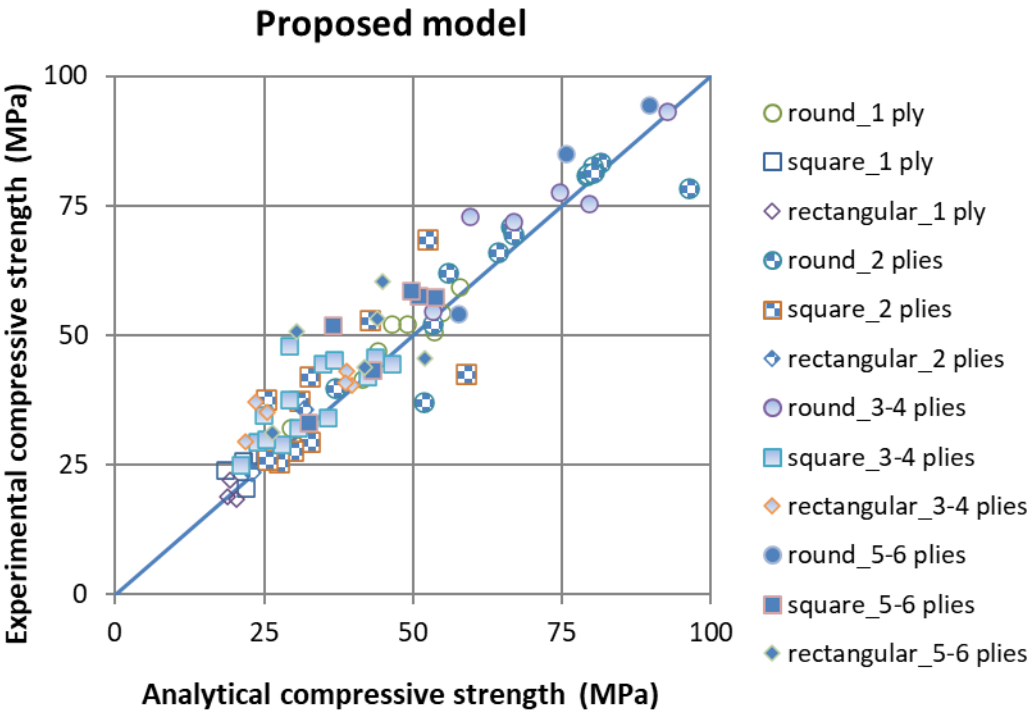

Fig. 6. *Comparison between experimental and analytical compressive strength calculated according to the proposed model for the specimens of the database.*

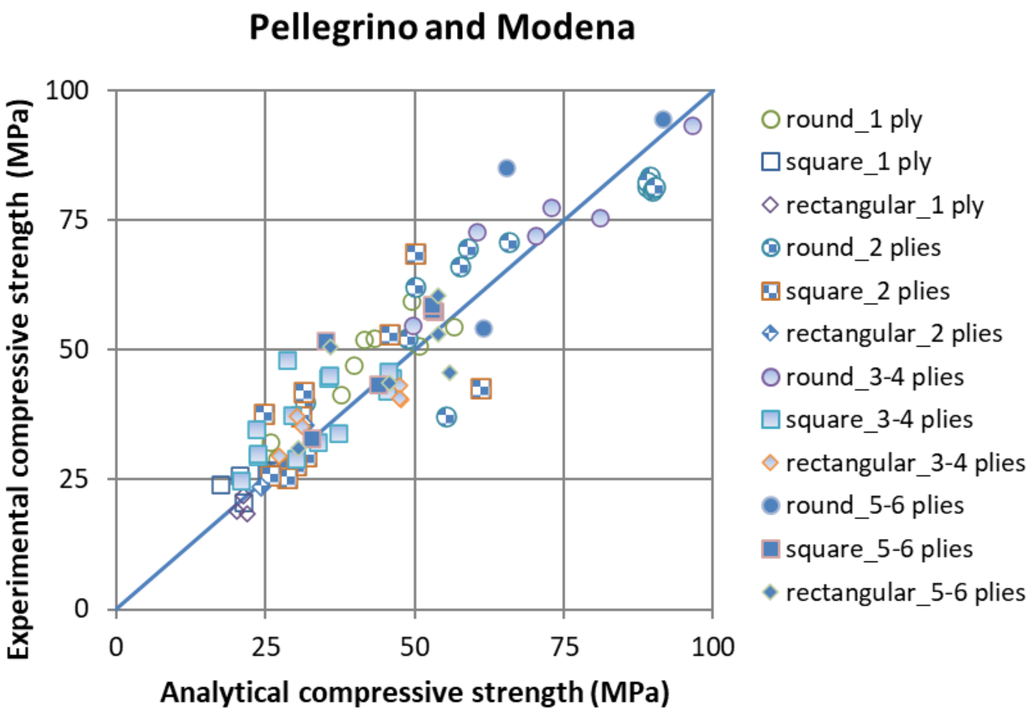

Fig. 7. *Comparison between experimental and analytical compressive strength from the model of Pellegrino and Modena [34] for the specimens of the database.*

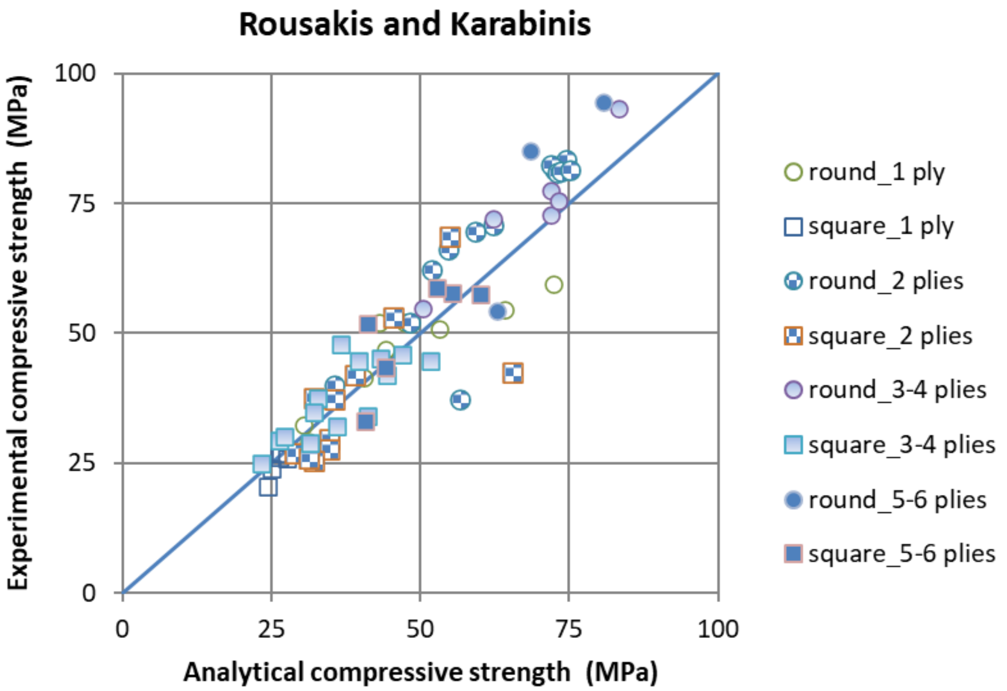

Fig. 8. *Comparison between experimental and analytical compressive strength from the model of Rousakis and Karabinis [42] for the specimens of the database with circular and square section.*

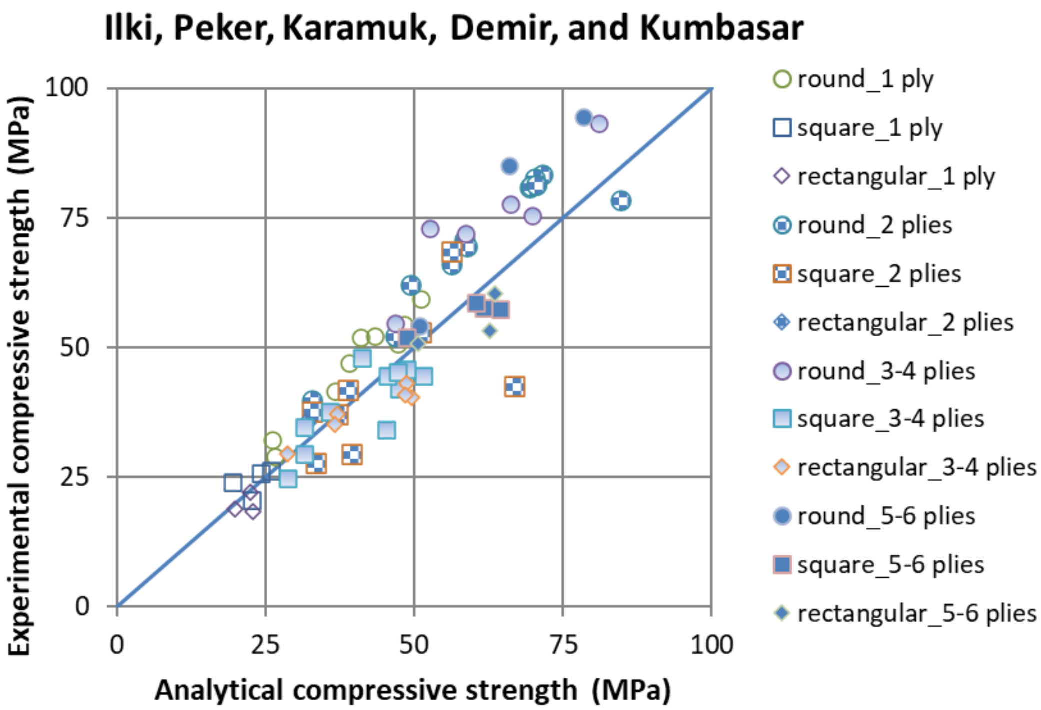

Fig. 9. *Comparison between experimental and analytical compressive strength from the model of Ilki , Peker, Karamuk, Demir, and Kumbasar [32] for the CFRP-confined specimens of the database.*

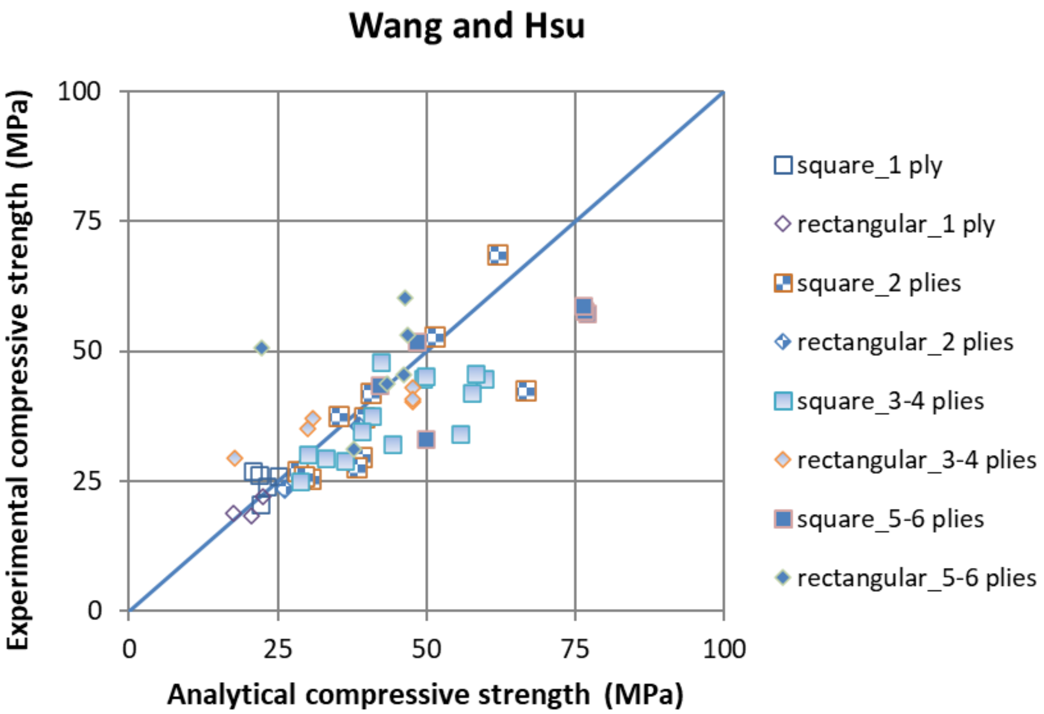

Fig. 10. *Comparison between experimental and analytical compressive strength from the model of Wang and Hsu for the rectangular specimens of the database.*

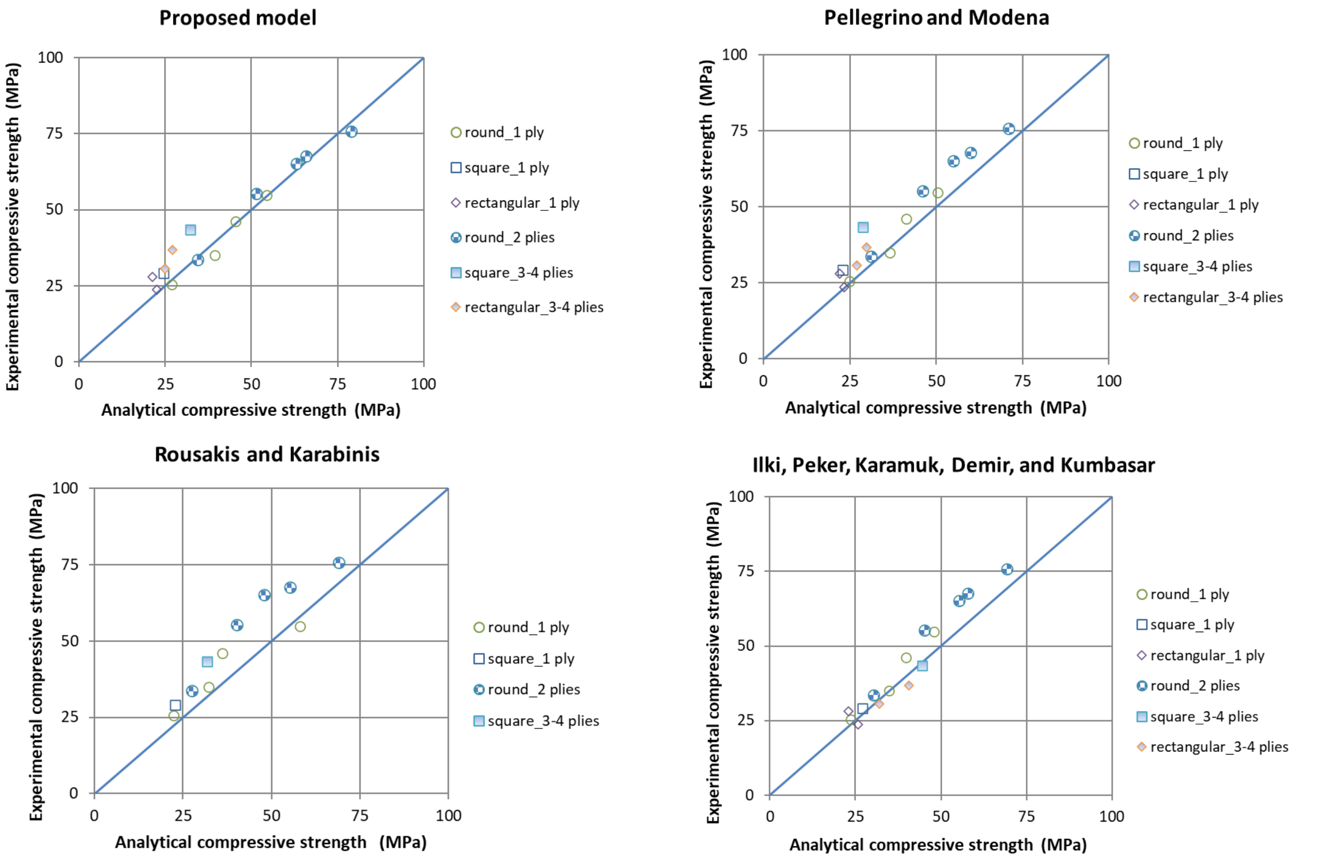

Fig. 11. Comparison between analytical values calculated with the proposed model and 3 other models and the experimental values of the peak compressive concrete stress for FRP-confined specimens without steel reinforcement.

VI. CONCLUSIONS

The paper reviews the basic assumptions that the majority of design models make regarding the confining effect of steel stirrups and FRP jackets on increasing the compressive strength of FRP-jacketed reinforced concrete columns.

Four design models from the literature, selected among others owing to their good predictive performance, are discussed. A new model based on the provisions of the Eurocodes is proposed. All models were assessed on an assembled database consisting of 101 columns from 19 different papers, with a broad range of parameters.

Despite their substantial differences, all models predict rather satisfactorily, though with a varying degree of accuracy, the experimentally measured confined strength of the specimens in the database.

It is alleged, therefore, that it is not so much a matter of which value for a certain parameter is more appropriate to adopt, e.g. regarding the value of the rapture strain ε_{ju} of the FRP jacket for which apparently so many controversial perspectives have been offered by different researchers, but it is rather a matter of competence of the entity of the proposed design procedure. The effectiveness of a design model resides on its ability to accurately predict the confining effect of steel stirrups and FRP in relation to different characteristics of specimens tested.

The proposed model proved to have the best performance among the models considered. It is recommended for use not only because of its slightly superior predictive capacity, but also owing to its simplicity of application and uniform approach to all types of cross-sections and FRP materials. Furthermore, the model tends to lead to safe predictions, and if safety factors for the resistance of materials are applied, the predicted confined strength for practically all specimens in the data base would be on the safe side. Simplicity in application and good and safe predictions are essential prerequisites for a reliable design model.

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