Multicriteria Decision Making Under Conditions of Uncertainty for Selection and Allocating Resources among R&D Projects

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***Abstract*—In this study, a general multicriteria decision-making (MCDM) methodology was used as a tool for selecting and allocating resources among research and development (R&D) projects. The input to the MCDM tool is the quantitative and qualitative information regarding the relevant criteria for evaluating the alternatives. Additionally, the presence of uncertainties associated with parts of the decision-making process is also considered. As a result, the solutions found can be classified as robust. The article describes both the previous decision-making method and also its use with the MCDM tool. Finally, after modeling the initial data, a robust solution was found for the problem of allocating resources among R&D projects.**

Keywords— Multicriteria decision making (MCDM), Portfolio management, Research and Development (R&D).

I. Introduction

Research and development (R&D) and innovation activities are considered important factors to guarantee competitiveness and sustainable development in an organization [1]. According to Kashyap and Garg [2], they are referred to as a process of searching for new knowledge that can be used for innovating or creating products, technologies, or systems, either for own use or for sale. With the dynamism and rapid evolution of the market, organizations increasingly rely on projects and innovations so that their competitiveness can be enhanced. According to Ribeiro and Alves [3], this growth is not limited only to the number of projects, but also to their complexity. To deal with this growth, a tool called project portfolio management was created to ensure that a collection of projects can be evaluated with the aim of prioritizing the allocation of resources, in line with organizational strategies [4].

Project portfolio management is an integral part of a company's overall strategic plan. While project and program management focuses on "getting the job done right", the goal of portfolio management is "getting the right job done" [4]. Companies that seek profitable opportunities to allocate resources to projects must consider many factors, such as demand being higher than the resources available and differences in assessment, considering qualitative and quantitative criteria in the decision-making process. Such criteria can be conflicting and make the choice difficult, especially when this is being done in a group.

According to Sitorus *et al.* [5] in the mining sector, changes in production processes must consider criteria such as economic matters, the environmental impact, policy and regulation. Thus, choosing between projects that fit within a company's norms and regulations becomes a multidisciplinary task. For this task, decision-makers (DMs) need a tool that incorporates quantitative and qualitative analyses in a scientific way, instead of relying only on intuition and experience [5].

According to Bistline [6], the success of an R&D project can reduce the number of years required to achieve the specified technical or cost goals, thus justifying more elaborate methods for obtaining the best possible allocation of resources. Considering that unsatisfactory results in the decision-making process are undesirable, the entire decision-making process must be rethought, starting from the search for a better definition of the problem, choosing and applying a methodology that allows DMs to consider the alternatives in a coherent way, to favor the process when choosing the most suitable alternative.

Therefore, the main objective of this study is to allocate resources among R&D projects by modeling the decision-making problem, taking into account the uncertainties of the initial data, and transforming them into uncertainty intervals. Having obtained these intervals, balanced scenarios can be created, by mixing pessimistic and optimistic situations between the variables, in order to select the representative combinations of the initial data. By doing so, the alternative with the best level of satisfaction among the criteria can be chosen.

This article is organized as follows. Section II describes the decision-making methodology used to date. Then, in Section III, decision-making questions are presented that assist in understanding the discussions presented in the subsequent sections. In Section IV, a brief review of the papers related to this study is presented. Sections V and VI present the general decision-making methodology and its application, respectively. Then, in Section VII, a case study is described using the general decision-making methodology to find the robust solution to the problem. Finally, some conclusions are drawn and the contribution of this study to the literature is summarized in Section VII.

II. Description of the Problem

A Brazilian mining company annually invests part of its net operating revenue in financing Research and Development (R&D) projects. The Innovation and Technology sector of this company, which is in charge of managing R&D projects, collects demands for innovation from the other sectors after defining the key performance indicators (KPI) for evaluating proposals. Individual proposals have a limited budget. After the proposals are collected, they are subjected to the following questions that may result in disqualification:

- Is the project R&D?
- Is the proposal duplicated?
- Does the proposal fit into any company's strategic roadmap?
- Does the project represent a risk to the company's image?

In the next stage, the eligible proposals are ranked considering the score obtained by the following criteria:

- scalability (Is it possible to replicate it in some other similar installation in the company?);
- degree of maturity (Is the project incipient? Is it applied research to solve an existing problem?);
- secondary earnings (Does the project have secondary KPIs with relevant earnings? What are they and how many are there?);
- impact on implantation (What is the negative interference of implementing the project in the production?);
- financial return on the project;
- there is a multi-annual budget earmarked for R&D;
- alignment of KPIs with the company's strategy;
- alignment of KPIs with the Risk Management Table.

The methodology previously used in the mining company was to evaluate alternatives with weights of 1, 3 or 5 for each criterion, 1 being used for “bad” or “absent”, 3 for “good” and 5 for “very good”. Weights 1, 3 and 5 are assigned by consensus, considering the opinions of 5 DMs (leaders, governance, directors). Consensual scores are entered on a spreadsheet to compute the weighted sum. If two or more alternatives tie, then the following tiebreaker criteria are applied:

- strategic action (Which strategic area does the project have the most correlation with?);
- *status quo* (What is the state of the project's development?);
- multi-annual budget (Can the project have, or does it have a multi-annual budget? What is the sector of origin?).

At the end of the process of decision-making between the proposals, the DMs publish a list with the results containing the new projects to be incorporated into the company's R&D portfolio. Projects that have lesser priority or that do not have a multi-annual budget are archived that year.

On analyzing the decision-making process described, issues arise that may improve the result. If it is a problem of resource allocation based on criteria that involve multidisciplinary knowledge, is it possible that the scores awarded by the specialists are weighted in relation to their hierarchical level or to their knowledge and experience? When there is no consensus, is it possible to measure the DMs’ uncertainty by using an interval between the scores awarded? If there are different formats for scoring between the DMs, how can they be aggregated into a single assessment? Therefore, it is understood that a methodology that addresses these issues can be applied in the case study of this article.

III. Theoretical Foundation

The systematization of the decision-making process in an organization with effective methodologies, certainly, results in benefits. However, to make proper use of methodologies, DMs must be aware of the technical terms used in decision-making processes. In brief, the following issues need to be reviewed:

- What do you want to do, find, define?
- What alternatives are available?
- What criteria are relevant for evaluating alternatives?
- What are the constraints?
- How are the alternatives classified?
- Do uncertainties affect the outcome of the decision-making process?

According to Hammond *et al.* [14], the way a problem is stated guides the decision-making process, as this influences how alternatives are defined and how they are evaluated. Considering that satisfaction with the results is the goal, defining the problem can start from the desired results. Alternatively, this can start from the unwanted results if they are formalized in terms of constraints. The desired results can be used to draw up the scope of the problem, for which different techniques can be used, e.g., technical brainstorming. For Hammond *et al*. [14], setting out constraints can be a way to describe the problem, as it allows focus and avoids wasting the time that would be spent evaluating alternatives that meet feasible or satisfactory results.

After defining the decision-making problem, the next step is to set relevant criteria or objectives that support the choice process. For Hammond *et al.* [14], objectives help the DM to determine the information he/she seeks. Additionally, the objectives can serve as arguments that justify the choice, the prioritization or, moreover, in order to show the importance, the consequences, the attention and the effort of a given alternative.

The alternatives are the responses available to the decision-making problem. Identifying alternatives can occur naturally when, for example, the problem is disclosed with an invitation to interested parties to present alternatives. For Welch [15], alternatives can also emerge from whenever a previous decision has been made. There are decision problems for which the availability of alternatives is directly or indirectly time dependent. Consider, for example, the decision to regard an investment as an asset; asset pricing may depend on supply and demand. Therefore, understanding the availability of the alternative or the variation of its characteristics with time, space, use, etc., is fundamental to the decision-making process.

For Silva *et al*. [16] what must be raised in the description of the problem are the technical limitations of the system and the relationships of this system with a view to making a critique of the validity of possible solutions. Technical limitations can be interpreted as constraints of the problem. In [17], Drucker says that a DM should start with discerning the preference for what is right by what is acceptable considering that there will always be a commitment to fulfill it in the end. However, if all the alternatives satisfy the boundary conditions, then it is difficult to distinguish what is right from what is acceptable.

Usually, the alternatives are evaluated, in some way, according to the weighting of the criteria. The way weighting is applied and what mechanisms are used to reduce redundancy errors differentiate decision-making methods. Regardless of the method, the best-ranked alternatives are prioritized.

While exploring what alternatives there may be in the decision-making process or even when measuring the values of attributes and criteria of each alternative, DMs may encounter numerous sources of uncertainty that can influence the outcome of the decision. Although it is possible that most of the uncertainties in a problem do not influence the result, neglecting these could be an imprudent and inconsequential action because, depending on the case, this can cause losses of various kinds to the organization. Hammond *et al.* [14] "uncertainties add a new layer of complexity to decision-making". The first step is to become aware of the existence of uncertainties and then to try to understand the various results that this can cause, by evaluating the probabilities and the possible impacts.

IV. RELATED STUDIES

In this Section, studies related to this research are presented. In Subsection A, these concern MCDM and in Subsection B, they address using decision-making for selecting and classifying R&D projects.

A. MCDM

Abdel-Basset *et al* [7] presented the Analytical Hierarchy Process (AHP) method using the theory of neutrosophic sets for group decision-making. The proposed model aimed to solve the difficulty of assigning deterministic assessment values to the judgments of comparisons due to a DM's limited knowledge or differences in DMs' individual judgments. To demonstrate the effectiveness of the method, the results obtained were compared to the traditional AHP method.

Marttunen, Lienert and Belton [8] present a bibliographic review that covered eight Problem-Structuring Methods (PSM) and seven methods of multicriteria decision analysis (MCDA). They found that combining PSMs with MCDA produces a richer view of the decision situation and allows more effective support for different phases of the decision-making process. It was observed that the most used decision-making method is Analytic Hierarchy Process (AHP).

Guo and Zhao [9] extended the Best-Worst Method (BWM) of decision-making to the fuzzy environment. The reference comparisons for the best and worst criteria were described by the DMs' linguistic terms, which can be expressed in triangular fuzzy numbers. The results showed that the proposed fuzzy BWM, in addition to obtaining a reasonable preference rating for alternatives, also has a higher consistency of comparison than the BWM.

Parreiras *et al* [10] present consensus' methodology based on fuzzy models to deal with the contribution of several experts in multicriteria decision making. In this study, a methodology was introduced that consists of combining three evaluation of DMs, each one applying different aggregation approaches in order to construct collective results. As a result, each specialist can cooperate with their respective capabilities.

Pereira *et al* [11] presented multicriteria decision-making under conditions of uncertainty. The general methodology focuses on using quantitative information to reduce regions of decision uncertainty, and, if this does not allow unique solutions to be obtained, the general schema presupposes the use of qualitative information. This modification avoids contradictory solutions being obtained that do not belong to the Pareto set.

Ramalho *et al* [12] addressed multicriteria decision-making under conditions of uncertainty applied to resource allocation. In short, this study combines several techniques applied to decision-making problems in conditions of uncertainty, namely: it uses the Bellman-Zadeh approach, the AHP method, the attribution of values for preferences and their different presentation formats, and conversions between relations and preference formats. By doing so, it was possible to formulate and solve resource allocation problems using quantitative and qualitative information at the same level of analysis.

B. *Decision making in the selection of R&D projects*

Lizarralde and Ganzarain [1] used a methodology based on choosing the MIVES method (Integrated Value Model for Sustainability Assessments) for decision-making with regard to selecting and evaluating technology in an R&D center. Critical factors were identified, by conducting a bibliographic review, so as to construct the decision tree or hierarchy, which was later refined by experts so that the decision-making method selected could be applied.

Kashyap and Garg [2] described a Multi-Criteria Decision-Making (MCDM) technique with a Euclidean distance-based approach (EDBA) so as to evaluate and select various R&D projects. The EDBA methodology is applied to determine the composite distance of alternative R&D projects from the optimal option, which is an option created with the ideal values for all selection criteria. The method was applied to solve an MCDM problem, but it did not deal with the uncertainty in the weighting of the criteria.

Ribeiro and Alves [3] used the AHP method to support decision-making for selecting scientific research projects in an educational institution. As a research procedure, a case study was used, based on exploratory research, followed by a quantitative modeling approach. Additionally, documentary research and unstructured interviews were used to improve a better understand the research context and institutional objectives.

Cheng, Liou and Chiu [13] used the analytical network process based on fuzzy preference relationships (CFPR-ANP) to select R&D projects. The CFPR-ANP was developed to calculate the preference weights of the criteria based on the derived structure of the network. The COmplex PRoportional ASsessment of assessments with Grey relations (COPRAS-G) method was applied with fuzzy relationships to resolve conflicts arising from differences in information and opinions provided by different stakeholders on selecting the most appropriate R&D projects.

As a solution to the problem of this paper, the approach described in [12] was chosen. By using it, we can deal with the uncertainty of the problem and analyze quantitative and qualitative criteria at the same level of analysis.

V. MCDM CLASSIC TOOL

In decision-making problems, it is common to find difficulties in obtaining sufficient volumes of initial information with the reliability needed to construct models. To address this type of uncertainty, Ekel *et al.* [18] developed an approach that combines two branches of mathematics that deal with uncertainties: game theory and fuzzy set theory. This approach is a generalization of the classic approach described in [11] and [19].

A. *Modeling Uncertainties and defining a robust solution*

When using the Bellman-Zadeh approach [20], the entire objective function Fp(X) is replaced by a fuzzy objective function or a fuzzy set

$$A_p = \{X, \mu_{Ap}(X)\}, X \in L,\ p = 1,2, \dots, q \qquad (1)$$

$\mu_{Ap}(X)$ being a membership function of Ap, *p* the number of the objective function and *q* the number of objectives of the decision-making problem [21], [22].

To obtain (1), it is necessary to build membership functions $\mu_{Ap}(X)$, p = 1,...,q that reflect the degree of reach of the proper end by the corresponding $F_p(X)$, $X \in L,\ p = 1,2, \dots, q$. This condition is satisfied by using membership functions

$$\mu_{Ap} = \left[\frac{F_p(X) - min_{X \in L} F_p(X)}{max_{X \in L} F_p(X) - min_{X \in L} F_p(X)}\right]^{\lambda_p} \qquad (2)$$

for maximized objective functions, or by using membership functions

$$\mu_{Ap} = \left[\frac{max_{X \in L} F_p(X) - F_p(X)}{max_{X \in L} F_p(X) - min_{X \in L} F_p(X)}\right]^{\lambda_p} \qquad (3)$$

for minimized objective functions. In (2) and (3), λ_{p}, p = 1,...,q, are importance factors of the corresponding objective functions.

The fuzzy solution *D*, based on (1), is given by the membership function

$$\mu_D(X) = min_{p=1,\dots,q}\ \mu_{Ap}(X),\ X \in L \qquad (4)$$

Its use enables a solution to be obtained that proves the maximum degree of pertinence to the fuzzy *D* solution. Thus, from a formal point of view, the multicriteria problem is replaced by the max min problem.

$$\max \mu_D(X) = \max\ min_{\ p=1,\dots,q}\ \mu_{Ap}(X),\ X \in L \qquad (5)$$

Therefore, the problem is reduced in looking for a robust solution defined by:

$$X^0 = \arg\max \mu_D(X) \qquad (6)$$

B. Classical Approach

In [11], 6 steps necessary for applying the classical approach are listed:

- formulate the problem mathematically;
- select representative combinations of the initial data (also called states of nature or scenarios);
- propose alternative solutions;
- construct payoff matrices;
- analyze the payoff matrices and choose the rational alternative for the solution;
- select the best scored solution.

Starting with the mathematical formulation of the problem, uncertainty in the initial data requires the corresponding description of the coefficients. Taking this into account, the objective functions can be written as follows:

$$F_p(X) = \sum_{i=1}^{n}[C'_{pi}, C''_{pi}]x_i, p = 1, 2, \dots, q \quad (7)$$

C'_{pi} and C''_{pi} being the minimum and maximum values of the uncertainty interval for , $, p = 1, 2, \dots, q$ (number of objectives) and $i = 1, 2, \dots, n$ (number of projects), respectively. To select the representative combinations of the initial data, in step 2, in [10] the use of the sequence LP_τ is suggested in order to create balanced scenarios that mix pessimistic and optimistic situations between the variables.

The sequence LP_τ é is a method for generating almost random numbers and its use allows a more uniform sampling of the search space. This method allows uniformly distributed points $Q_s, s = 1, 2, \dots, S$ with coordinates $Q_{st}, t = 1, 2, \dots, T$ to be determined in the unitary hypercube Q^T. The C_{st} coefficients for each scenario can be obtained by using (8).

$$C_{st} = C_t^{min} + (C_t^{max} - C_t^{min})q_{st} \quad (8)$$

As an example, Table 1 shows the coordinates of the points Q_s for *S* = 7 e *T* = 6.

As seen in [12], after the mathematical formulation of the problem, the next stage consists of solving S multi-criteria problems and obtaining alternative solutions. To propose the solution alternatives, a vector of objective functions $F(X) = \{F_1(X), \dots, F_q(X)\}$ is considered, and the problem consists of the simultaneous optimization of all objective functions:

$$F_p(X) \rightarrow extr_{X \epsilon L}, \; p = 1, \dots, q \quad (9)$$

Considering a given number S of the representative combination of scenarios, the coordinates of the points calculated in Table I serve to construct S multi-objective optimization problems with deterministic coefficients. Considering this, based on (7), for example, the objective functions for each scenario $Y_s, s = 1, 2, \dots, S$ can be written as follows:

$$F_p(X, Y_s) = \sum_{i=1}^{n}[C'_{pis}]x_i, p = 1, 2, \dots, q, s = 1, 2, \dots, S \quad (10)$$

Table I - Points on the Hypercube Q^T

S \ T	1	2	3	4	5	6
1	0.5	0.5	0.5	0.5	0.5	0.5
2	0.25	0.75	0.25	0.75	0.25	0.75
3	0.75	0.25	0.75	0.25	0.75	0.25
4	0.125	0.625	0.875	0.875	0.625	0.125
5	0.625	0.125	0.375	0.375	0.125	0.625
6	0.375	0.375	0.625	0.125	0.875	0.875
7	0.875	0.875	0.125	0.625	0.375	0.375

To find alternative solutions, the Bellman-Zadeh approach is used. The next step is to build and analyze the payoff matrices, represented by Table II. For the construction it is necessary to solve S multi-criteria optimization problems found by applying (10). Of these solutions, a subset of K different solutions ($X_K, k = 1, 2, \dots, K$) is selected to construct the matrices. The so-called payoff matrices reflect the effects (or consequences) of a given action $x_K, k = 1, 2, \dots, K$, in a scenario $Y_s, s = 1, 2, \dots, S$.

Table II –Payoff Matrix

	Y_1	...	Y_s	...	Y_S
X_1	$F(X_1, Y_1)$	...	$F(X_1, Y_s)$	...	$F(X_1, Y_S)$
...	...	...	...	...	...
X_k	$F(X_k, Y_1)$	...	$F(X_k, Y_s)$	...	$F(X_k, Y_S)$
...	...	...	...	...	...
X_K	$F(X_K, Y_1)$	...	$F(X_K, Y_s)$	...	$F(X_K, Y_S)$

For the analysis of the payoff matrices and the choice of alternatives for the solution of the decision-making problem, the selection criteria are used according to [11]. The selection criteria used: a) Wald; b) Laplace; c) Savage; and d) Hurwicz. These criteria are based on applying, in each line of the payoff matrix, the estimates of maximum values, minimum values, average values and the maximum level of risk.

The level of risk $R(X_k, Y_s)$is the overspending that occurs under the combination of the Y_s scenario and the choice of the X_k solution alternative over the alternative solution that is locally optimal for the Y_s data. To determine the $R(X_k, Y_s)$ risks, what must be defined are the minimum value of the objective function (if minimized), or the maximum value (if maximized) for each combination of the state of nature Y_s (for each column of the payoff matrix). The risk for any X_k alternative solution and any state of nature Y_s can be assessed as

$$R(X_k, Y_s) = F(X_k, Y_s) - F^{min}(Y_s) \quad (11)$$

and

$$R(X_k, Y_s) = F^{max}(Y_s) - F(X_k, Y_s)\ , \quad (12)$$

by considering the minimization and maximization of the objective functions respectively.

According to [19], the equations for the selection criteria are:

- Wald

$$\min_{1 \le k \le K} F_p^{max}(X_k) = \min_{1 \le k \le K} \max_{1 \le s \le S} F_p(X_k, Y_s) \quad (13)$$

- Laplace

$$\min_{1 \le x \le k} \bar{F}(X_k) = \min_{1 \le k \le K} \frac{1}{S} \sum_{s=1}^{s} F(X_k, Y_s) \quad (14)$$

- Savage

$$\min_{1 \le k \le K} R_p^{max}(X_k) = \min_{1 \le k \le K} \max_{1 \le s \le S} R_p(X_k, Y_s) \quad (15)$$

- Hurwicz

$$\min_{1 \le k \le K} \left[\alpha F_p^{\max}(X_k) + (1-\alpha) F_p^{\min}(X_k) \right]$$
$$= \min_{1 \le k \le K} \left[\alpha \max_{1 \le s \le S} F_p(X_k, Y_s) + (1-\alpha) \min_{1 \le s \le S} F_p(X_k, Y_s) \right] \quad (16)$$

$\alpha \in [0,1]$ being the pessimism-optimism index defined by the DM.

Thus, in [11] it is proposed to construct the membership functions for (9), using (2) or (3), and the solution of the problem by using (5Therefore, considering the characteristic estimates of the choice criteria, the corresponding modified matrices of these characteristic estimates can be constructed. These modified matrices can be aggregated, and the obtained aggregated matrix is processed on the basis of (6) to generate the problem solution. The multi-objective analysis, carried out in this way, is effective in dealing with uncertainty and guarantees the choice of rational solution alternatives according to the Pareto boundary principle.

VI. MCDM APPLIED TOOL

In [12] an approach is proposed for constructing the coefficient estimates for forming Objective Functions based on Experts' preferences and, subsequently, applying them in Resource Allocation Problems.

The approach proposed is divided into four stages:

- attribute preferences;
- standardize preferences in multiplicative relations (RM);
- obtain the weight vectors by using the AHP;
- aggregate preferences and create scenarios.

For the attribution of preferences, experts choose between three formats of preference relationship: a) by ordering; b) by a multiplicative preference relation; and c) by a non-reciprocal fuzzy preference relationship. In the second step, in order to apply the AHP, the preferences must be registered are registered as multiplicative preference relations, it being necessary to convert the preference relation formats for standardization and application of the AHP.

After the preference relation formats have been standardized, it is certified if the matrices have a good level of consistency, that is, they have their maximum eigenvalue λ_{Max} close to the dimension of the respective matrices and thereafter the weight eigenvectors of the weights are found.

Finally, the weighted average operator of the ordered arguments OWA (Ordered Weighted Averaging) is applied to extract the level of pessimism / optimism, finding a vector of coefficients for the objective function. From the minimum values between the weight vectors calculated by the AHP, we find the minimum values of the uncertainty interval of the objective function (7). Likewise, from the maximum values between the weight vectors, the maximum values of the uncertainty interval of the objective function are found.

VII. CASE STUDY

In a mining company, it is desired to allocate 2000 units of value among research and development (R&D) projects for the next quarter. In meetings with the 5 DM managers (h1, h2, ..., h5) and specialists from the R&D project sector, information was collected on the decision-making process used in the company, as well as the criteria for decision-making. During the planning, the following objectives were agreed to

- to prioritize allocating resources to projects that have the greatest scalability;
- to prioritize allocating resources to projects that have the least impact on implantation;
- prioritize allocating resources to projects that generate the greatest financial return;
- prioritize allocating resources to projects that are best aligned with the company's strategy;
- prioritize allocating resources to projects that have a higher degree of maturity;

Among all the projects submitted, 5 were classified that fit within the company's strategic planning. The initial data of the R&D projects are shown in Table III.

Table III - Research and Development projects

Project index	Budget (k U$)	Scalability (%)	Impact on implantation (%)	Financial return
1	400	40	25-30	8.6-9.5
2	550	50	30-40	8.5-10
3	650	30	15-20	10.0-12.0
4	500	35	5-10	8.5-9.8
5	450	30	10-15	9.0-11.0

Among the objectives established in the planning process, two are qualitative, namely: degree of maturity of the project and alignment with the company's strategy. The scale for evaluation between the projects is 1, 3, 5 and 9 when there is no preference, moderate preference, strong and absolute preference, respectively.

Consider that the project's degree of maturity criterion would be assessed by manager h1 who expressed his preference through multiplicative preference relationships as shown in (7) as

$$RM_{h1}(x_k\,,x_l) = \begin{bmatrix} 1 & 1/9 & 1/3 & 1/3 & 1 \\ 9 & 1 & 3 & 3 & 9 \\ 3 & 1/3 & 1 & 1 & 3 \\ 3 & 1/3 & 1 & 1 & 3 \\ 1 & 1/9 & 1/3 & 1/3 & 1 \end{bmatrix}. \qquad (17)$$

The weight normalization step of the AHP method was applied to find the weight vector (18).

$$E_{h1} = [0.059 \;\; 0.529 \;\; 0.176 \;\; 0.176 \;\; 0.059] \qquad (18)$$

As it is the opinion of only one person, there is no need to apply the step of aggregating preferences.

For the criterion of alignment with the Company's strategy, managers h2, h3, h4 and h5 classified the projects by using multiplicative relationships. Their respective maximum eigenvalues λMax are close to the dimension n = 5 of the matrices.

$$RM_{h2}(x_k\,,x_l) = \begin{bmatrix} 1 & 1 & 3 & 9 & 3 \\ 1 & 1 & 3 & 9 & 3 \\ 1/3 & 1/3 & 1 & 3 & 1 \\ 1/9 & 1/9 & 1/3 & 1 & 1/3 \\ 1/3 & 1/3 & 1 & 3 & 1 \end{bmatrix} \qquad (19)$$

$$RM_{h3}(x_k\,,x_l) = \begin{bmatrix} 1 & 3 & 3 & 3 & 9 \\ 1/3 & 1 & 1 & 1 & 3 \\ 1/3 & 1 & 1 & 1 & 3 \\ 1/3 & 1 & 1 & 1 & 3 \\ 1/9 & 1/3 & 1/3 & 1/3 & 1 \end{bmatrix} \qquad (20)$$

$$RM_{h4}(x_k\,,x_l) = \begin{bmatrix} 1 & 3 & 1 & 9 & 9 \\ 1/3 & 1 & 1/3 & 3 & 3 \\ 1 & 3 & 1 & 9 & 9 \\ 1/9 & 1/3 & 1/9 & 1 & 1 \\ 1/9 & 1/3 & 1/9 & 1 & 1 \end{bmatrix} \qquad (21)$$

$$RM_{h5}(x_k\,,x_l) = \begin{bmatrix} 1 & 1 & 1/3 & 3 & 1 \\ 1 & 1 & 1/3 & 3 & 1 \\ 3 & 3 & 1 & 9 & 3 \\ 1/3 & 1/3 & 1/9 & 1 & 1/3 \\ 1 & 1 & 1/3 & 3 & 1 \end{bmatrix} \qquad (22)$$

By using (19), (20), (21) and (22) the corresponding eigenvectors are extracted by means of the AHP, thereby obtaining the weight vectors (23), (24), (25) and (26).

$$E_{h2} = [0.360 \;\; 0.360 \;\; 0.120 \;\; 0.040 \;\; 0.120] \qquad (23)$$

$$E_{h3} = [0.474 \;\; 0.158 \;\; 0.158 \;\; 0.158 \;\; 0.053] \qquad (24)$$

$$E_{h4} = [0.391 \;\; 0.130 \;\; 0.391 \;\; 0.043 \;\; 0.043] \qquad (25)$$

$$E_{h5} = [0.158 \;\; 0.158 \;\; 0.474 \;\; 0.053 \;\; 0.158] \qquad (26)$$

Observing the values obtained, some disagreement between the experts is noticed, and so step 4 must be applied to extract the limits of pessimism-optimism between which scenarios will be created. On extracting the minimum and maximum values between the vectors (23), (24), (25) and (26), we have the minimum and maximum limits, respectively, of the objective function (7).

$$B_{min} = [0.158 \;\; 0.130 \;\; 0.120 \;\; 0.040 \;\; 0.043] \qquad (27)$$

$$B_{max} = [0.474 \;\; 0.360 \;\; 0.474 \;\; 0.158 \;\; 0.158] \qquad (28)$$

Taking into account the uncertainty intervals of the initial data and the vectors (18), (27) and (28) found from the qualitative criteria, the problem was modeled mathematically by objective functions, using (7), for each decision-making criterion as in the equations:

$$F_1(X) = [0.059]x_1 + [0.059]x_2 + [0.176]x_3 + [0.176]x_4 + [0.059]x_5 \rightarrow MAX \qquad (29)$$

$$F_2(X) = [0.16\,,0.48]x_1 + [0.13,0.36]x_2 + [0.12,0.48]x_3 + [0.04,0.16]x_4 + [0.043,0.16]x_5 \rightarrow MAX \qquad (30)$$

$$F_3(X) = [0.4]x_1 + [0.5]x_2 + [0.3]x_3 + [0.35]x_4 + [0.3]x_5 \rightarrow MAX \qquad (31)$$

$$F_4(X) = [0.25,0.3]x_1 + [0.3,0.4]x_2 + [0.15,0.26]x_3 + [0.05,0.1]x_4 + [0.1,0.15]x_5 \rightarrow MAX \qquad (32)$$

$$F_5(X) = [8.6,9.5]x_1 + [8.5,10]x_2 + [10,12]x_3 + [8.5,9.8]x_4 + [9\,,11]x_5 \rightarrow MAX \qquad (33)$$

Subject to the constraints:

$0 \leq x_1 \leq 400$ (34)

$0 \leq x_2 \leq 550$ (35)

$0 \leq x_3 \leq 650$ (36)

$0 \leq x_4 \leq 500$ (37)

$0 \leq x_5 \leq 450$ (38)

$x_1 + x_2 + x_3 + x_4 + x_5 \leq 2000$ (39)

By considering (29) for the degree of maturity of the project, (30) for alignment with the company's strategy, (31) for scalability, (32) for impact on implementation and (33) for the financial return of the project. The constraints from (34) to (38) correspond to the budget demands of the projects and the constraint (39) corresponds to the resources available for allocation. After the mathematical formulation of the problem, the next stage consists of solving *S* multicriteria problems, of obtaining alternative solutions and of building the respective payoff matrices. The scenarios will be constructed using the sequence LP_τ with τ = 25. Making, for this example, S = 5, $f_{s,p}$, S = 1, 2, …, 5; p = 1, 2, …, 5, objective functions are obtained. Treating each of the 5 multicriteria problems, with five functions each, using the Bellman-Zadeh approach, the solution alternatives shown in Table IV are obtained.

Table IV – Alternative Solutions for S=5 scenarios

	x_1^0	x_2^0	x_3^0	x_4^0	x_5^0
X_1	234.50	401.00	650.00	343.50	371.00
X_2	180.50	470.00	650.00	479.00	220.50
X_3	79.00	500.00	650.00	322.00	449.00
X_4	10.00	515.00	650.00	500.00	325.00
X_5	248.00	398.00	650.00	500.00	204.00

Then, these alternative solutions are replaced in the objective functions in order to construct the payoff matrices. One matrix must be constructed for each objective function. For illustrative purposes, only the matrices of the objective function (29), corresponding to the degree of maturity of the project, will be presented. The payoff matrix is represented by Table V.

Table V – Payoff Matrix - objective function F_1

	Y_1	Y_2	Y_3	Y_4	Y_5
X_1	422.71	422.71	422.71	422.71	422.71
X_2	470.99	470.99	470.99	470.99	470.99
X_3	466.72	466.72	466.72	466.72	466.72
X_4	494.60	494.60	494.60	494.60	494.60
X_5	439.61	439.61	439.61	439.61	439.61

Applying the choice criteria of Wald, Laplace, Savage and Hurwicz, represented by equations (13), (14), (15) and (16) respectively, the matrices of the choice criteria estimates are obtained. Table VI shows the example of applying the selection criteria. The α = 0.75 was used in (16).

Table VI - Estimates of the criteria of choice - objective function F_1

	Fw(Xk)	Fl(Xk)	Fs(Xk)	Fh(Xk)
X_1	422.71	422.71	71.89	422.71
X_2	470.99	470.99	23.61	470.99
X_3	466.72	466.72	27.88	466.72
X_4	494.60	494.60	0.00	494.60
X_5	439.61	439.61	54.99	439.61
Fmin	422.71	422.71	0.00	422.71
Fmax	494.60	494.60	71.89	494.60

The next step consists of creating the respective membership functions, obtained by applying (2) and/ or (3) according to the objective to be achieved (minimization or maximization) for the construction of the matrix of modified criteria of choice in Table VII.

Table VII – Modified matrix of the criteria of choice - objective function F1

	μw,A1(Xk)	μl,A1(Xk)	μs,A1(Xk)	μh,A1(Xk)
X_1	0.00	0.00	1.00	0.81
X_2	0.67	0.67	0.33	0.97
X_3	0.61	0.61	0.39	0.30
X_4	1.00	1.00	0.00	0.00
X_5	0.24	0.24	0.76	1.00

Finally, in Table VIII, on applying (6) to the matrix of modified criteria of choice, it is possible to identify as the solution of this step, the alternative that has the maximum relevance value for each of the criteria.

Table VIII – Matrix of aggregated levels of the fuzzy of criteria of choice.

	μD,w(Xk)	μD,l(Xk)	μD,s(Xk)	μD,h(Xk)
X_1	0.000	0.000	0.000	0.000
X_2	0.063	0.000	0.000	0.000
X_3	0.182	0.053	0.000	0.111
X_4	0.000	0.000	0.000	0.000
X_5	0.000	0.000	0.000	0.000

Therefore, using Table IV, the robust solution for allocating resources in each project corresponds to

X_3 ={79.00 500.00 650.00 322.00 499.00}.

VIII. Conclusions

In this study, the MCDM approach by Ekel *et al.* [18] was used as a tool for selecting and allocating resources between R&D projects under conditions of uncertainty in a mining company. The selection of R&D projects was previously carried out by direct application of weights according to the group of managers, with tiebreaker criteria being added. Furthermore, the question of uncertainty was not taken into account. With the application of the MCDM approach, the decision-making process has become more systematic, thereby limiting the interference of decision-makers. As a result, there is a greater reliability of the system and the production of robust solutions.

The case study presented was limited to five projects so that the development of reasoning was objective and clear. The approach is flexible and can be used with numerous projects and with different scenarios. Another important issue is in the nature of the information on the criteria and alternatives that can be both quantitative and qualitative at the same level of the analysis of the problem.

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