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About a Study for the Body of Mandibler Bone by Transfer-Matrix Method

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***Abstract*—This work present an original approach for calculus of body mandible, through a model of a semicircular spring, embedded at the two ends, charged perpendicular of its plane, in two cases: first, charged with concentrated vertical loads and in second case, charged with an uniformly distributed load along the entire length of the semicircle. This is an original approach and very easy to program.**

Keywords—mandible bone; body mandible model; semicircular spring; density of charge; state vector; Transfer-Matrix.

I. Introduction

Mandible is an unpaired mobile bone and very important for 3 functions in which it takes part: mastication, phonation and aesthetics, which results from numerous researches and publications.

The mandible bone consists of a body and two branches, at branches ends is the temporo-mandibular joint.

In this article, we present a study of mandible body by Transfer-Matrix Method (TMM).

Due to the important role of the mandible bone, there are many studies and research on it.

[9] presents methods for a bone mandible supply in case of severe mandible atrophy and in [6] we have some methods and techniques for reconstruction in orthodontics.

[10] gives some bone substitutes and validation.

In [13] it is presented a study of etiopathogenesis and effects of the occlusal trauma.

[7] studies evolution of bone resorption in restorative prosthetic therapy.

[16] gives a study on the position of the mandible channel for total edentation.

In [11] it is presented some methods for bone engineering.

[14] presents a very interesting of clinical use of the fibular flap.

[12] gives some grafting systems in implanto-prosthodontics rehabilitation.

In [15] it is presented some uses of modern methods of imaging exploration in orthodontics and [8] gives studies with Finite Elements Method (FEM) for transmission of masticator forces to the bone substrate via titanium and zirconium implants.

[1] presents some photoelasticimetry applications in biomechanics and other applications in orthopedics field is gives in [2] and [3].

Classical analytical spring calculus is presented in [5].

Basics of TMM calculus are given in [4] and in [17] we have general formulas and applications of TMM.

II. Premises of Mandible Body Calculus as a Semicircular Spring Stressed Perpendicular to Its Plane

We consider a mandible bone (Fig 1).

Fig. 1. Mandible bone

The model for study was built by isolating the body of the mandible (Fig. 2).

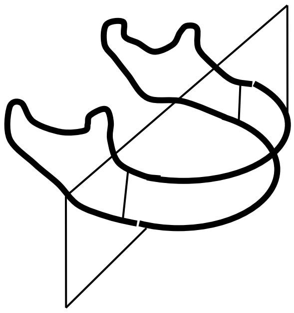

Fig. 2. Separating the body of the mandible from the two branches

The mandible body is assimilated with a semicircular spring, embedded at both ends (Fig. 3).

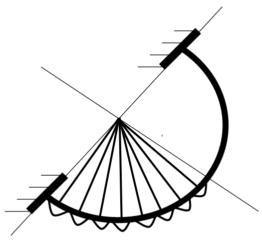

Fig. 3. Model of mandible body as a semicircular spring

Transfer-Matrix Method (TMM) allows the application of calculus for a semicircular spring, used an algorithm based on Dirac's and Heaviside's functions and operators, [4].

It is considered that the spring is stressed perpendicular to its plane, in two cases:

- loaded with concentrated vertical loads corresponding to each tooth and, for simplification, we consider all forces equal to F (Fig. 4), the body of the mandible is loaded with teeth, numbering 16, eight on a quarter of circle and another 8 on the other quarter of circle.

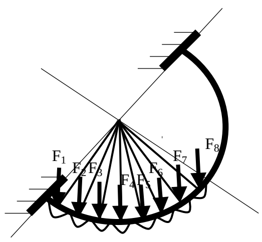

Fig. 4. Semicircular spring with concentrated loads

- loaded with a vertical distributed force, which, for simplification, will be considered uniformly distributed (Fig. 5).

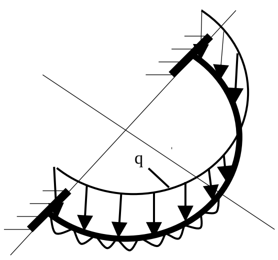

Fig. 5. Semicircular spring with uniformly distributed load

TMM allows a much easier approach to the calculus of circular springs, which means that our application for mandible body model will be relatively easy and original.

III. Spring Loaded Perpendicular to Its Plane (Fig. 4 and Fig. 5)

We consider a state vector with six components for a section θ:

$$\{SV\}_\theta = \{T(\theta), M_r(\theta), M_t(\theta), v(\theta), \omega(\theta), \varphi(\theta)\}^{-1} \quad (1)$$

where:

- θ is the current angle at the center of semicircular spring
- $\{SV\}_\theta$ is the state vector corresponding at the section θ, with 6 components
- T(θ) force resultant perpendicular to the spring plane at the section θ
- $M_r(\theta)$ is the radial moment at the section θ
- $M_t(\theta)$ is the tangential moment at the section θ
- v(θ) is the arrow at the section θ
- ω(θ), φ(θ) are angular deformations at the section θ.

Also, we consider a state vector with six components for origin section 0:

$$\{SV\}_0 = \{T_0, M_{r0}, M_{t0}, v_0, \omega_0, \varphi_0\}^{-1} \quad (2)$$

with the same meaning of the components as in (1), but referring to origin face 0.

Generally, the passage from face 0 to face 1, for the first element, is made by a growth vector $\{\ddot{A}V\}_1$, whose expression is:

$$\{\ddot{A}V\}_1 = [TM]_1\cdot\{SV\}_0 + [TM_{ext}]_1\cdot\{V_{ext}\}_1 \quad (3)$$

and, for the face 1, we have the state vector:

$$\{SV\}_1 = \{SV\}_0 + \{\Delta V\}_1 \quad (4)$$

where:

- $[TM]_1$ is the Transfer-Matrix, 6x6, between the origin section 0 and the section 1
- $\{SV\}_1$ is the state vector corresponding at the face 1
- $[TM_{ext}]_1$ is a matrix of external forces coefficients corresponding at the element 1
- $\{V_{ext}\}_1$ is the vector of external forces.

(4) can still write:

$$\{SV\}_1 = \{SV\}_0 + [TM]_1\cdot\{SV\}_0 + [TM_{ext}]_1\cdot\{V_{ext}\}_1 \quad (5)$$

because:

$$[T]_1 = [TM]_1 + [1] \quad (6)$$

With (6), (5) becomes:

$$\{SV\}_1 = [T]_1\cdot\{SV\}_0 + [TM_{ext}]_1\cdot\{V_{ext}\}_1 \quad (5)$$

The growth vector $\{\ddot{A}V\}_2$ for the second element is:

$$\{\ddot{A}V\}_2 = [TM]_2\cdot\{SV\}_1 + [TM_{ext}]_2\cdot\{V_{ext}\}_2 \quad (7)$$

and the state vector for la face 2 (the right section of the second element) is:

$$\{SV\}_2 = \{SV\}_1 + \{\Delta V\}_2 \quad (8)$$

Replacing (7) in (8), we obtain:

$$\{SV\}_2 = [T]_2\cdot[T]_1\cdot\{SV\}_0 + [T]_2\cdot[TM_{ext}]_1\{V_{ext}\}_1 + [TM_{ext}]_2\cdot\{V_{ext}\}_2 \quad (9)$$

and so on, obtaining the following general relations for the state vector of the face è, [17]:

$$\{SV\}_\theta = \left(\prod_{j=1}^{j=\theta}[T]_j\right)\cdot\{SV\}_0 + \\ + \sum_{m=1}^{m=\theta-1}\left[\left(\prod_{j=m+1}^{j=\theta}[T]_j\right)\cdot[TM_{ext}]_m\{V_{ext}\}_m\right] + \\ + [TM_{ext}]_\theta\cdot\{V_{ext}\}_\theta \quad (10)$$

and with notations:

$[T]_{\theta+1}$= [1] (11)

$\{V_{ext}\}_0$= $\{SV\}_0$ (12)

$[TM_{ext}]_0$=[1] (13)

(12) becomes much simpler:

$$\{SV\}_\theta = \sum_{m=1}^{m=\theta+1}\left[\left(\prod_{j=m}^{j=\theta+1}[T]_\theta\right)\cdot[TM_{ext}]_{m-1}\{V_{ext}\}_{m-1}\right] \quad (14)$$

The general Transfer-Matrix $[TM]_\theta$ for a spring loaded in its plane, [4], has expression (15):

$$[T]_\theta = \begin{bmatrix} t_{11} & t_{12} & t_{13} & t_{14} & t_{15} & t_{16} \\ t_{21} & t_{22} & t_{23} & t_{24} & t_{25} & t_{26} \\ t_{31} & t_{32} & t_{33} & t_{34} & t_{35} & t_{36} \\ t_{41} & t_{42} & t_{43} & t_{44} & t_{45} & t_{46} \\ t_{51} & t_{52} & t_{53} & t_{54} & t_{55} & t_{56} \\ t_{61} & t_{62} & t_{63} & t_{64} & t_{65} & t_{66} \end{bmatrix} \quad (15)$$

with elements:

t_{11}=1
t_{12}=0
t_{13}=0
t_{14}=0
t_{15}=0
t_{16}=0
t_{21}=R·sin θ
t_{22}=cos θ
t_{23}=sin θ
t_{24}=0
t_{25}=0
t_{26}=0
t_{31}= - R·(1-cosθ)
t_{32}= - sin θ
t_{33}=cos θ
t_{34}=0
t_{35}=0
t_{36}=0
t_{41}= - a·R^2/2[(1-k)(sinθ - θ·cosθ)+2k(1 - cosθ)]
t_{42}= - aR/2[(1-k)(1 - cosθ) – (1–k)(θ sinθ - cosθ – 1)]
t_{43}= - aR/2(1-k)(sinθ - θcosθ)
t_{44}=1
t_{45}= - Rsinθ
t_{46}= R(1-cos θ)
t_{51}=aR/2[(1-k) θsinθ-2k(1-cos θ)]
t_{52}= a/2[(1-k)sin θ-(1-k) θcos θ]
t_{53}= a/2(1-k) θsin θ
t_{54}=0
t_{55}= cos θ
t_{56}= - sin θ
t_{61}= aR/2(1-k)(sin θ - θcos θ)
t_{62}= a/2(1-k) θsin θ
t_{63}= a/2[(1-k)sin θ-(1-k) θcos θ]
t_{64}=0
t_{65}= sin θ
t_{66}= cos θ

with, [4]:

$$k = \frac{E\cdot I}{G\cdot J} \quad (16)$$

where:

- E is Young modulus
- I is the moment of inertia of the section in rapport to the horizontal axis
- G is transversal modulus
- J is moment of inertia for torsion

and:

$$a = \frac{R}{E\cdot I} \quad (17)$$

where:

- R is the rayon of the circle.

The general expression for the state vector for exterior load for element θ is:

$$\{V_{ext}\}_\theta = \begin{Bmatrix} -R\cdot q_1(\theta) \\ -R^3\cdot r(\theta) \\ R^2\cdot t(\theta) \\ -aR^3\int_0^\theta l(\theta)d\theta \\ aR^2\cdot l(\theta) \\ -aR^2(1-k)\cdot h(\theta) \end{Bmatrix} \quad (18)$$

where elements are function of mode and type loading.

IV. Model for Mandible Body as a Semicircular Spring, Loaded Perpendicular to Its Plane with Concentrated Forces

The first model for mandible body, studied by TMM, is a mandible body as a semicircular spring, loaded with vertical concentrated forces, perpendicular to its plane.

To simplify the study, we make some work hypotheses:

- the semicircular spring is considered to be embedded at both ends, those that have been sectioned and isolated the body of the mandible from its two branches;
- we consider the vertical concentrated forces in number of 16, corresponding to the 16 teeth on the body of the mandible;
- the vertical concentrated forces are equal to each other and equal to F;
- the angular distances between the forces are equal to each other and equal to a center angle of 11.25°;
- the angular distances between the two forces near the two embedded ends is 5,625° (Fig. 4).

As in relation to the load, the semicircular spring is symmetrical, it can be considered an element of the spring, starting from the left support, which represents the origin.

For applying TMM, we must divide the semicircular spring into 16 identical elements. Each element is loaded with concentrated vertical load F.

Each element is bordered by two faces, a left face and a right face, so that the right face of one element becomes the left face of the next element. For the first element, the left face is the origin face, the face 0 at the angle θ=0°, or the section 0. For the last element, element, number 16, the right face is the last face, or the last section, for angle θ=180°, or θ = π.

The right section of the element, at an angle of 11.25° rapport to section 0 (the origin), is considered to be the final section for this element.

We can study this element and, after, the right face of this first element (the right section) is the origin section (the left section) for the second element, identical to the first element.

Calculus can continue up to the 16th element, thus traversing the entire semicircular model of mandible body, applying (13).

Matrix relation (13) must be used successively for each element of spring, starting from element 1 (bordered by faces 0 and 1), then to element 2 and so on until the last element, element 16.

For the face which is at a certain angle θ, we can write matrix relationship (13), with the Transfer-Matrix $[TM]_\theta$ (15) and its components given below.

The vector corresponding at the exterior load at the angle θ - $\{V_{ext}\}_\theta$ is given by (18).

For concentrated vertical force, the expression of the charge density, for an element, is, [4]:

$$q(\theta) = - F\delta(\theta - \theta_0) \qquad (19)$$

and, [4]:

$$T(\theta) = T_0 + F\, Y(\theta - \theta_0) \qquad (20)$$

where $Y(\theta - \theta_0)$ is the Heaviside function, [4].

Components of vector (18), for a concentrated vertical load are:

$$\begin{cases} V_{ext_1} = -Rq_1(\theta) = FY(\theta-\theta_0) \\ V_{ext_2} = -Rr(\theta) = F\sin(\theta-\theta_0)Y(\theta-\theta_0) \\ V_{ext_3} = R^2 t(\theta) = -FRY(\theta-\theta_0)[1-\cos(\theta-\theta_0)] \\ V_{ext_4} = aR^3\int_0^\theta l(\theta)d\theta = -aFR^2Y(\theta-\theta_0)\{k[\theta-\theta_0-\sin(\theta-\theta_0)]+ \\ +\frac{1-k}{2}[-(\theta-\theta_0)\cos(\theta-\theta_0)+\sin(\theta-\theta_0)]\ \} \\ V_{ext_5} = aR^2 l(\theta) = -aFR\left(k\theta+\frac{1-3k}{2}\sin\theta-\frac{1-k}{2}\theta\cos\theta\right) \\ V_{ext_6} = -aR^2(1-k)h(\theta) = aR(1-k)\left(1-\cos\theta-\frac{\theta}{2}\sin\theta\right) \end{cases} \qquad (21)$$

If in last expression (13), applied successively for the 16 elements of the semicircular spring, we replace θ = π in the state vector corresponding to the section 16, we obtain:

$$\{SV(\pi)\}_\pi = [T(\pi)]_\pi \cdot \{SV\}_0 + \{V_{ext}(\pi)\}_\pi \qquad (22)$$

That is a relation between the state vector of the origin section 0 and the state vector of the final section for θ = π.

In (22) we can put the conditions for the two embedded ends. We obtain a linear system of equations whose solutions give the other unknowns from the two extremities of the semicircular spring, meaning the model of mandible body studied.

Replacing the solutions in the matrix relation (13), for the last state vector, at the right end, we can now calculate all components for all state vectors for all spring sections.

V. Model for Mandible Body as a Semicircular Spring, Loaded Perpendicular to Its Plane with Uniformly Distributed Load (Fig. 5)

The second model for mandible body, studied by TMM, is a mandible body as a semicircular spring, loaded with vertical uniformly distributed load along the entire length of the semicircle and perpendicular to its plane (Fig. 5).

We consider the semicircular spring to be embedded at both ends, those that have been sectioned and isolated the body of the mandible from its two branches.

The uniformly distributed force materializes the action of continuous pressing on the teeth along the entire length of the semicircular spring.

In this case, the density charge is:

$$q(\theta) = q\delta(\theta - \theta_0) \qquad (23)$$

and:

$$q_1(\theta) = q\,\theta \qquad (24)$$

Also, (1) to (13) remain valid for uniformly distributed load.

In (18), elements corresponding to the exterior loads become (25):

$$\begin{cases} V_{ext_1} = -Rq_1(\theta) = Rq\theta \\ V_{ext_2} = -Rr(\theta) = Rq(1-\cos\theta) \\ V_{ext_3} = R^2 t(\theta) = -R^2 q(\theta-\sin\theta) \\ V_{ext_4} = aR^3\int_0^\theta l(\theta)d\theta = -aR^3 q\left[\frac{k\theta^2}{2}+(1-2k)(1-\cos\theta)-\frac{1-k}{2}\theta\sin\theta\right] \\ V_{ext_5} = aR^2 l(\theta) = -aR^2 q\left(k\theta+\frac{1-3k}{2}\sin\theta-\frac{1-k}{2}\theta\cos\theta\right) \\ V_{ext_6} = -aR^2(1-k)h(\theta) = aR^2 q(1-k)\left(1-\cos\theta-\frac{\theta}{2}\sin\theta\right) \end{cases} \qquad (25)$$

Now, we can write (22) in which are placed the conditions on the two supports and after, we obtain a linear equation system, which the solutions for the state vector in the origin and the state vector in the final section (with all components known).

Replacing in (13) the solutions for state vectors of the right end known, we can calculate in all sections, the state vector with all its elements.

VI. Conclusions

In this work we present an original approach for calculus of body mandible, through a model of a semicircular spring embedded at the two ends, charged perpendicular of its plane, in two cases: first, charged with concentrated vertical loads and in second case, charged with an uniformly distributed load along the entire length of the semicircle.

This approach by TMM is very easy to program and to use for practical cases of quick calculations for shape optimization for the body of the mandible.

In future, we hope to present an experimental validation and a numerical validation by Finite Elements Method (FEM).

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