State observer design for a family of nonlinear systems with or without exponential nonlinearity

Yeong-Jeu Sun
Department of Electrical Engineering,
I-Shou University, Kaohsiung, Taiwan 840, R.O.C.
Email: yjsun@isu.edu.tw; Fax: 886-7-6577205

***Abstract*—In this paper, a family of nonlinear systems is proposed and the state observation problem of such systems is explored. Based on the time-domain approach with differential inequalities, a suitable state observer for a family of nonlinear systems is proposed to ensure the global exponential stability of the resulting error system. At the same time, the guaranteed exponential decay rate can be accurately calculated. Finally, several numerical simulations are offered to verify the correctness and effectiveness of the obtained results.**

Keywords—observer design; nonlinear systems, Li's chaotic system; exponential decay rate

I. INTRODUCTION

Due to the defects of the measuring instrument or the uncertainties of the system, not all state variables are just measurable for a linear or nonlinear system. At this time, the design of the state observer is very important and needs to be seriously faced. The state observer has come to take its pride of place in system identification and filter theory. Meanwhile, the state observer design for the state reconstruction of dynamic systems with chaos is in general not as easy as that without chaos. Based on the above-mentioned reasons, the state observer design of chaotic systems is quite meaningful and crucial.

In recent years, the design and methodology of state observers for many systems have been widely explored and discussed; see, for example, [1]-[10] and the references therein. Undoubtedly, once the state estimator of the system is designed, the controller design of the system will be more diverse and easier. The above reasons have prompted more researchers to devote themselves to the related research of state observer.

In this paper, the observability problem for a family of nonlinear systems is explored. By using the time-domain approach with differential inequalities, a suitable state observer for such nonlinear systems is offered to ensure the global exponential stability of the resulting error system. Furthermore, the guaranteed exponential decay rate can be precisely calculated. Finally, some numerical simulations are given to demonstrate the effectiveness of the obtained result.

II. PROBLEM FORMULATION AND MAIN RESULTS

Nomenclature

$\Re^n$	the n-dimensional real space
A^T	the transport of the matrix A
$\|x\|$	the Euclidean norm of the vector $x \in \Re^n$

In this paper, we consider a family of nonlinear systems:

$$\dot{x}_1(t) = -a_1 x_1(t) + f_1(x_2(t)), \tag{1a}$$
$$\dot{x}_2(t) = f_2(x_1(t), x_2(t), x_3(t)), \tag{1b}$$
$$\dot{x}_3(t) = -a_2 x_3(t) + f_3(x_2(t)), \tag{1c}$$
$$y(t) = \alpha x_2(t), \quad \forall\, t \geq 0, \tag{1d}$$
$$[x_1(0) \quad x_2(0) \quad x_3(0)]^T = [x_{10} \quad x_{20} \quad x_{30}]^T, \tag{1e}$$

where $x(t) := [x_1(t) \quad x_2(t) \quad x_3(t)]^T \in \Re^3$ is the state vector, $y(t) \in \Re$ is the system output, $[x_{10} \quad x_{20} \quad x_{30}]^T$ is the initial value, f_1, f_2, and f_3 are smooth functions, and $a_1, a_2, \alpha \in \Re$ represent the parameters of the system with $a_1 > 0, a_2 > 0$, and $\alpha \neq 0$. It should be noted that the Li's chaotic system [11] is a special case of system (1) with $a_1 = 10, a_2 = 1, f_1 = x_2$, $f_2 = 8x_2 + x_1 x_3$, and $f_3 = -e^{x_2^2}$. It is a well-known fact that since states are not always available for direct measurement, particularly in the event of sensor failures, states must be estimated. The objective of this paper is to search a suitable state observer for the system (1) such that the global exponential stability of the resulting error systems can be ensured.

Before presenting the main result, the state reconstructibility is offered as follows.

Definition 1: The system (1) is exponentially state reconstructible if there exist a state observer $f(z, \dot{z}, y) = 0$ and positive numbers κ and α such that

$$\|e(t)\| := \|x(t) - z(t)\| \leq \kappa \exp(-\alpha t), \quad \forall\, t \geq 0,$$

where $z(t)$ expresses the reconstructed state of the system (1). In this case, the positive number α is called the exponential decay rate.

Now, we are in a position to present the main results for the state observer of system (1).

Theorem 1: The system (1) is exponentially state reconstructible. Besides, a suitable state observer is given by

$$\dot{z}_1(t) = -a_1 z_1(t) + f_1(z_2(t)), \tag{2a}$$

$$z_2(t) = \frac{1}{\alpha} y(t), \tag{2b}$$

$$\dot{z}_3(t) = -a_2 z_3(t) + f_3(z_2(t)), \quad \forall t \geq 0. \tag{2c}$$

In this case, the guaranteed exponentia decay rate is given by $\alpha := \min\{a_1, a_2\}$.

Proof. For brevity, let us define the observer error

$$e_i(t) := x_i(t) - z_i(t), \forall i \in \{1,2,3\} \text{ and } t \geq 0. \tag{3}$$

From (1), (2), and (4), one has

$$\begin{aligned}\dot{e}_1(t) &= \dot{x}_1(t) - \dot{z}_1(t)\\ &= -a_1 x_1(t) + f_1(x_2(t)) + a_1 z_1(t) - f_1(z_2(t))\\ &= -a_1[x_1(t) - z_1(t)] + f_1(x_2(t)) - f_1(z_2(t))\\ &= -a_1 e_1(t) + f_1\left(\frac{y(t)}{\alpha}\right) - f_1\left(\frac{y(t)}{\alpha}\right)\\ &= -a_1 e_1(t), \quad \forall t \geq 0.\end{aligned}$$

This implies that

$$e_1(t) = e_1(0) \cdot e^{-a_1 t}, \quad \forall t \geq 0. \tag{4}$$

$$e_2(t) = x_2(t) - z_2(t) = \frac{y(t)}{\alpha} - \frac{y(t)}{\alpha} = 0, \quad \forall t \geq 0. \tag{5}$$

$$\begin{aligned}\dot{e}_3(t) &= \dot{x}_3(t) - \dot{z}_3(t)\\ &= -a_2 x_3(t) + f_3(x_2(t)) + a_2 z_3(t) - f_3(z_2(t))\\ &= [x_3(t) - z_3(t)] + f_3\left(\frac{y(t)}{\alpha}\right) - f_3\left(\frac{y(t)}{\alpha}\right)\\ &= -a_2 e_3(t), \quad \forall t \geq 0.\end{aligned}$$

It results that

$$e_3(t) = e_3(0) \cdot e^{-a_2 t}, \quad \forall t \geq 0. \tag{6}$$

From (4)-(6), we have

$$\begin{aligned}\|e(t)\| &:= \sqrt{e_1^2(t) + e_2^2(t) + e_3^2(t)}\\ &\leq \sqrt{e_1^2(0) + e_3^2(0)} \cdot e^{-\alpha t}, \quad \forall t \geq 0.\end{aligned}$$

Consquently, we conclude that the system (2) is a suitable state observer with the guaranted exponential decay rate $\alpha := \min\{a_1, a_2\}$. This completes the proof.

III. Numerical Simulations

Consider the following Li's chaotic system with exponential nonlinearity:

$$\dot{x}_1(t) = -10 x_1(t) + x_2(t), \tag{7a}$$

$$\dot{x}_2(t) = 8 x_2(t) + x_1(t) x_3(t), \tag{7b}$$

$$\dot{x}_3(t) = -x_3(t) - e^{x_2^2(t)}, \tag{7c}$$

$$y(t) = -1.73 x_2(t), \tag{7d}$$

Comparison of (7) with (1), one has

$$a_1 = 10, a_2 = 1, \alpha = -1.73, \quad f_1 = x_2,$$

$$f_2 = 8x_2 + x_1 x_3, \quad f_3 = -e^{x_2^2},$$

By Theorem 1, we conclude that the system (7) is exponentially state reconstructible by the state estimator

$$\dot{z}_1(t) = -10 z_1(t) + z_2(t), \tag{8a}$$

$$z_2(t) = \frac{-1}{1.73} y(t), \tag{8b}$$

$$\dot{z}_3(t) = -z_3(t) - e^{z_2^2(t)}, \tag{8c}$$

with the guaranteed exponential convergence rate $\alpha := \min\{10, 1\} = 1$. The typical state trajectories of the systems (7) and (8) are depicted in Figure 1 and Figure 2, respectively. Besides, the time response of error states between the systems (7) and (8) is shown in Figure 3.

Conclusions

In this paper, a family of nonlinear systems has been proposed and the state observation problem of such systems has been investigated. Based on the time-domain approach with differential inequalities, a suitable state observer for a family of nonlinear systems has been established to ensure the global exponential stability of the resulting error system. In addition, the guaranteed exponential decay rate can be accurately calculated. Finally, several numerical simulations have been offered to verify the correctness and effectiveness of the obtained results.

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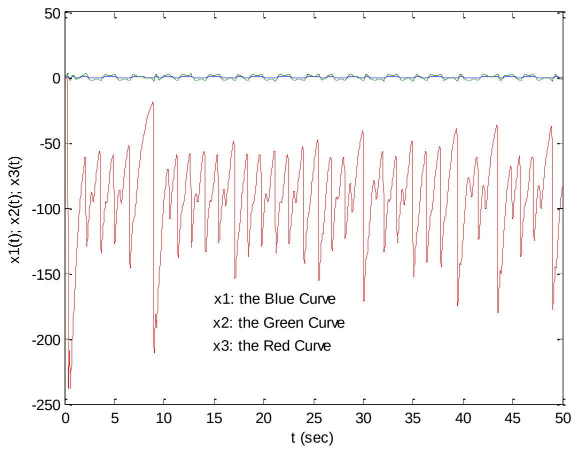

Figure 1: Typical state trajectories of the system (7).

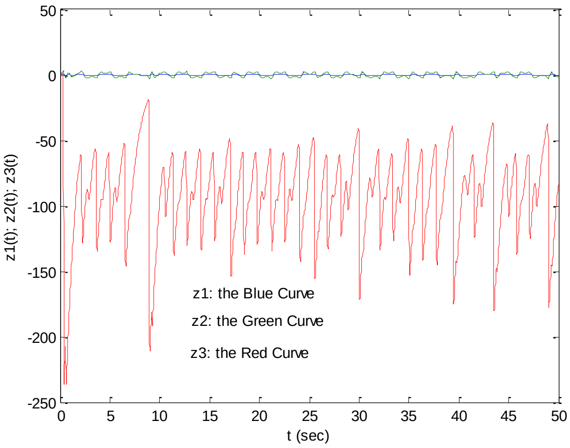

Figure 2: Typical state trajectories of the system (8).

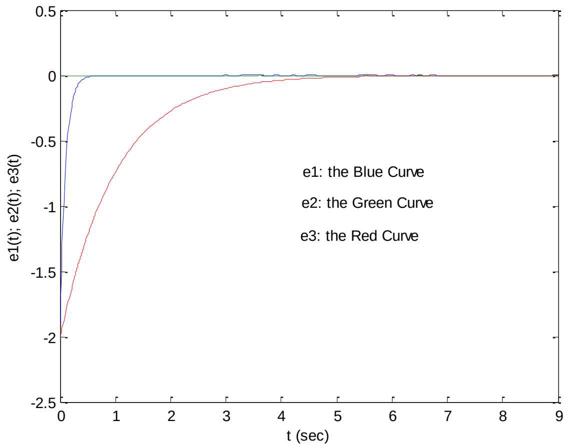

Figure 3: The time response of error states.

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