Three-dimensional flow in a square-section conduit filled with a porous metal foam

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***Abstract*— This study is a numerical simulation of flow in a three-dimensional conduit of a square-section filled with porous metallic foam , the objective from this study is to demonstrate that the flow velocities between 0.05m /s and 0.1 m/s is quasi uniform with a pressure drop that varies linearly with the velocity.**
The problem is modelled by the continuity and momentum equations with their appropriate initial and boundary conditions. When the conduit is filled with a porous metal foam, a Darcy-Forchheimer-Brinkman flow model is used to describe the flow field. The model equations are numerically solved by a second order accurate finite volume method.
The results of the numerical simulation allow for a mathematical modelisation of the flow that has an exact analytical solution. The results of the exact solution are similar to those of the numerical simulation.

Keywords— Conduit; Porous medium; Metallic foam; finite volumes

I. Introduction

The flow of the fluids in the porous metallic foam is a very important domain that has drawn lots of research focus. It has a variety of different fields of applications. They are know to be widely used in the problems of water purification, gas and oil extraction. Some examples of the flow studies in the metallic foam filled- are presented below.

Simone Mancin, Claudio Zilio, Andrea Diani and Luisa Rossetto [1] have experimentally studied the pressure drop of the air flow. They used five samples of copper foam with different pore diameters. They found that for a constant porosity, the pressure drop increases with the diameter of the pores. And for a constant pore diameter, the pressure drop decreases as the porosity decreases.

Simone Mancin, Claudio Zilio, Luisa Rossetto and Alberto Cavallini [2] have experimentally studied the pressure drop of an air flow. They used two aluminum foams of equal porosity and different heights. The results show that the pressure drop is very high for both foams.

Bhattacharya, V.V. Calmidi and R.L. Mahajan [3] experimentally determined the permeability they used samples of the high-porosity metal foams. They find the permeability increases with the diameter of the pores.

P. Khayargoli, V. Loya, L. P. Lefebvre and M. Medraj [4] experimentally determined the permeability and the coefficient of drag of NCX (Nickel-Chromium extra strong) samples and measure the pressure drop for speeds between 1m/s and 15m/s, the authors found that the pressure drop is well modelised in the quadratic model of Hazen-Dupuit-Darcy.
They also found that the pressure drop increases with the square of the speed and decreases with increasing pore size, for speeds below 1m/s, the pressure drop increases linearly with the velocity and is independent of pore size.

II. Mathematical Model

The physical properties of the fluid are assumed constant. The fluid is Newtonian and the flow is assumed laminar.
Therefore the flow field is modelled by a Darcy-Forchheimer-Brinkman model. Notice that if the conduit is not filled with a metallic foam the Navier-Stokes equation are obtained by eliminating the terms containing the Darcy number (Da *)* and setting the porosity equal to 1 . The coefficient C_f in the Forchheimer terms is set equal to 0.0598. The number C_f is calculated with the data from the reference [4] for a sample with a pores diameter equals 1.4 mm.

$C_f = C\,\sqrt{K}$.

Coefficient of drag

$C = 360\ \mathrm{m}^{-1}$

Permeability

$K = 27.6\ \ 10^{-9}\ \mathrm{m}^2$

Extractig the dynamic and geometric symmetry, the study is done on the quarter of the cross section of the conduit

The flow is modelled as follows

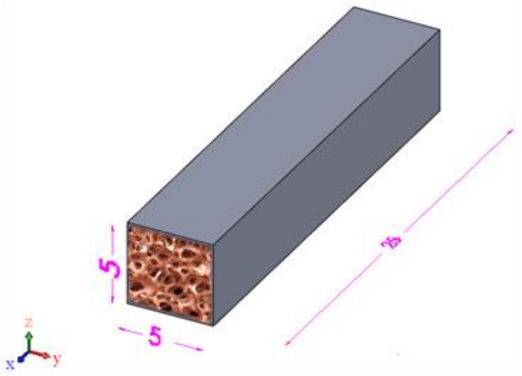

Fig 1.schematic view of the geometry

A. Equations

The flow is modelled as follows

The initial conditions

At $t = 0,\ U = 1,\ V = 0,\ W = 0$

The continuity equation

$$\frac{\partial U}{\partial x}+\frac{\partial V}{\partial y}+\frac{\partial W}{\partial z}=0 \quad (1)$$

The horizontal momentum equation

$$\frac{1}{\emptyset}\frac{\partial U}{\partial t}+\frac{1}{\emptyset^2}\frac{\partial (UU)}{\partial x}+\frac{1}{\emptyset^2}\frac{\partial (VU)}{\partial y}+\frac{1}{\emptyset^2}\frac{\partial (WU)}{\partial z} = -\frac{\partial P}{\partial x}-Cf\frac{U}{Re\,Da}-\frac{\sqrt{U^2+V^2+W^2}\,U}{\sqrt{Da}} + \frac{1}{Re\,\emptyset}\left[\frac{\partial}{\partial x}\left(\frac{\partial U}{\partial x}\right)+\frac{\partial}{\partial y}\left(\frac{\partial U}{\partial y}\right)+\frac{\partial}{\partial z}\left(\frac{\partial U}{\partial z}\right)\right] \quad (2)$$

The vertical momentum equation

$$\frac{1}{\emptyset}\frac{\partial V}{\partial t}+\frac{1}{\emptyset^2}\frac{\partial (UV)}{\partial x}+\frac{1}{\emptyset^2}\frac{\partial (VV)}{\partial y}+\frac{1}{\emptyset^2}\frac{\partial (WV)}{\partial z} = -\frac{\partial P}{\partial y}-\frac{V}{Re\,Da}-Cf\frac{\sqrt{U^2+V^2+W^2}\,V}{\sqrt{Da}} + \frac{1}{Re\,\emptyset}\left[\frac{\partial}{\partial x}\left(\frac{\partial V}{\partial x}\right)+\frac{\partial}{\partial y}\left(\frac{\partial V}{\partial y}\right)+\frac{\partial}{\partial z}\left(\frac{\partial V}{\partial z}\right)\right] \quad (3)$$

The vertical momentum equation

$$\frac{1}{\emptyset}\frac{\partial W}{\partial t}+\frac{1}{\emptyset^2}\frac{\partial (UW)}{\partial x}+\frac{1}{\emptyset^2}\frac{\partial (VW)}{\partial y}+\frac{1}{\emptyset^2}\frac{\partial (WW)}{\partial z} = -\frac{\partial P}{\partial z}-\frac{W}{Re\,Da}-Cf\frac{\sqrt{U^2+V^2+W^2}W}{\sqrt{Da}} + \frac{1}{Re\,\emptyset}\left[\frac{\partial}{\partial x}\left(\frac{\partial W}{\partial x}\right)+\frac{\partial}{\partial y}\left(\frac{\partial W}{\partial y}\right)+\frac{\partial}{\partial z}\left(\frac{\partial W}{\partial z}\right)\right] \quad (4)$$

The boundary conditions

At $x = 5,\quad \frac{\partial U}{\partial x}=\frac{\partial V}{\partial x}=\frac{\partial W}{\partial x}= 0$

At $y = 0,\ U = 0,\ V = 0,\ W = 0$

At $y = 0.5,\ \frac{\partial U}{\partial y} = 0,\ V = 0, \frac{\partial W}{\partial y} = 0$

At $z = 0,\quad U = 0,\ V = 0,\ W = 0$

At $z = 0.5\,, \frac{\partial U}{\partial z} = 0,\ \frac{\partial V}{\partial z} = 0, W = 0$

The aspect ratio

$\frac{L}{H} = 5$

The Reynolds number

$Re = \frac{U_e^* H}{\nu}$

If the d is filled with a metallic foam we add to the previous parameters

The Darcy number

$$Da = \frac{K}{H^2} = 1.104 \times 10^{-5}$$

The porosity

$\phi = 0.9$

the number C_f is calculated with the data from the reference [4] for a sample with a pores diameter equals 1.4 mm

III. Numerical Method

The model equations are solved by a finite volume method. The used discretization method is second order accurate in space and time. The time derivatives are discretized with the second order Euler backward approximation. The temporal discretization of all advective and non-linear terms is approached by the Adam-Bashforth second-order scheme. However, the time discretization of the Darcy terms, the pressure terms and all the diffusive terms is totally implicit. For the spatial discretization, we used the second order accurate central difference scheme. The used mesh is uniform in both spatial directions with 82 points in the x direction , 43 points in the y direction and 43 points in the direction z. The sequential solution of the systems of discretization equation of the computed variables follows the Simpler Algorithm discussed by S. Patankar [5]. The systems of discretization equations are solved by the sweeping method [5]. Starting from the initial conditions, the time integration (time marching) with a time step equal to10^{-4} is continued until the steady state solution is obtained.

IV. Results

The results of the numerical simulation show that the metallic foam have a large capacity for the uniformity of the flow, axially the flow develpes rapidely near the inlet, in fact the velocity components along (y, z) Vertical and transversal direction disappear exactly after the outlet. The hydrodynamic length is very low. The flow becomes unidirectional, (with the velocity component (U)) and slightly bidimesional, because the axial velocity varies only with vertical and transversal directions. The Fig. 2 represents the distribution of the axial velocity which is near uniform in the conduit in the case of Reynolds number equates 159.2.

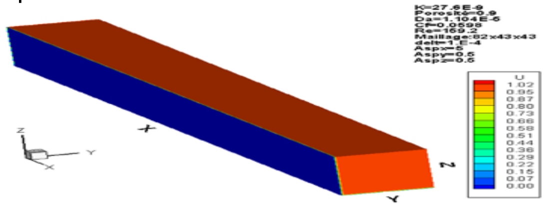

Fig 2 . Horizontal velocity distribution U

At a given cross section, the flow is quasi-uniform on all the aire of the surface ; the variations of the velocity is limited to the very thin wall, (the thickness of the layer is 0.043) close to the walls as illustrated in the figure Fig.3. In these layers, the nul velocity value increases rapidly (at the walls) to its uniform value. These very thin layers of the velocity variations demonstrate that the viscous effect only generates near the walls. The uniformity of the flow is due to the low permeability of metal foams.

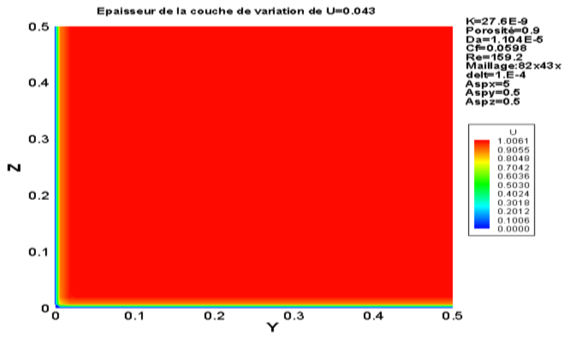

Fig 3 .Distribution of the velocity U

The uniformity of the flow is a great advantage that can be found in flows with heat and/or material transfer. In fact, it has been established that the heat and/or material transfer obtained with a uniform flow is the maximum possible: it is higher than the transfer obtained with non-uniform velocity profiles. This is easily understandable because in the case of uniform flow the velocity is very high near the heat or material exchange walls, and therefore the convective transfer is improved.

However, a disadvantage of flow through metal foams is the relatively high pressure drop. The pressure drop in the direction of flow is considerable when the axial length of the flow is large, which increases the cost of streaming the flow, since the mechanical power of the flow circulation is the product of the volumetric flow rate and the pressure difference between the inlet and outlet of the conduit. The figure Fig. 4 illustrates the pressure drop in the conduit for the Reynolds number case equal to 159.2.

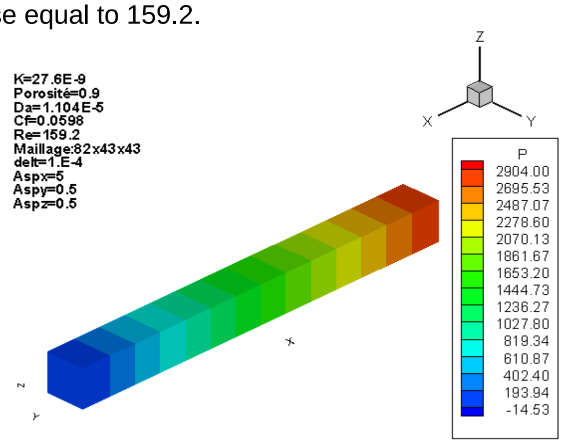

Fig4.non-dimensional pressure drop in the conduit

For the other considered Reynolds numbers, the velocity distributions are qualitatively and quantitatively very close to that of the 159.2 Reynolds number case, but the pressure drop increases linearly with the increasing Reynolds number (proportional to the average flow velocity) as will be demonstrated later.

In the applications of compact systems where the distance of the flow is short, and therefore the pressure drop is not a big disadvantage. The metallic foams is used to increase the transfer phenomena, because the high thermal conductivity of metallic foams is added to the desirable effect of the uniform flow already mentioned which increases the heat transfers by diffusion.

V Aproximative analytical solution

The results of the numerical three-dimensional solutions, with the considered velocities, have shown that the flow through the sample NCX1116 is essseentially unidirectional (the velocity components V and W are neglected) and bidimesional (the dominant velocity component U is only related to y and z). The pre-mentioned reliance is limited to thin-layered walls.

The Forchheimer has been found to be quite low compared to Darcy term. Theses notes explain the modelization of the flow copared to the differential equation

$$\frac{1}{Re\,\emptyset}\left[\frac{\partial^2 U}{\partial y^2}+\frac{\partial^2 U}{\partial z^2}\right]-\frac{dP}{dx}-\frac{U}{Re\,Da}=0 \qquad (12)$$

$\frac{dP}{dx}$ Is the constant pressure drop

The boundary conditions are

$$y=0\ ,\quad U=0$$
$$y=0.5,\ \frac{\partial U}{\partial y}=0$$
$$z=0\ ,\quad U=0$$
$$z=0.5\ ,\ \frac{\partial U}{\partial z}=0$$

This problem has an exact solution

$$U(y,z)=\frac{\pi^2}{4}\frac{\sum_{m=1,3}^{\infty}\sum_{n=1,3}^{\infty}\left[\frac{\sin(m\pi y)\sin(n\pi z)}{\frac{\emptyset\, mn}{Da}+\pi^2 mn(m^2+n^2)}\right]}{\sum_{m=1,3}^{\infty}\sum_{n=1,3}^{\infty}\left[\frac{1}{\frac{\emptyset\, m^2n^2}{Da}+\pi^2 m^2n^2(m^2+n^2)}\right]} \qquad (13)$$

This exact solution is represented graphically in the figure (Fig.5). the velocity variation is limited to the very thin layers near the walls ; and outide of these layers , the flow is uniform . We notice that the exact solution is similar to the numerical solution at the outlet of the conduit which is represented in the figure Fig.3.

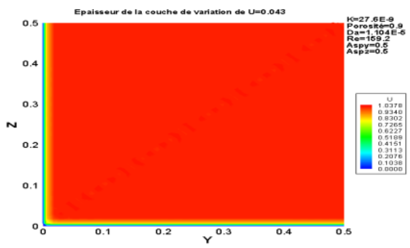

Fig3 .Distribution of the velocity U obtained

With this solution we can calculate the pressure drop through the metallic foam sample and we compare it with pressure drop obtained from the three-dimensional numerical solution.
The dimensional values of the pressure drop, for the considered velocities, are compared to each other in the figure Fig .5. We notice that for the considered velocities, the analytical solution gives results that are pretty much similar to those of our numerical simulation. Also we notice that in the field of considered velocity, the pressure drop increases linearly with the velocity.

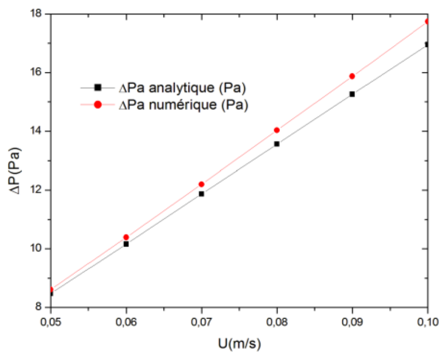

Fig 5 .Analytical and numerical pressure drops

CONCLUSION

For the averarge velocities between 5 cm/s et 10 cm/s , we study the air flow in a three-dimensional conduit, of 25 cm length and a (1 cm x 1 cm) square section filled with the metallic foam (NCX1116). The three-dimensional numerical solution of the Darcy-Forchheimer-Brinkmann model has shown the rapid axial developpent of the flow and the near uniformity of the velocity on the cross section of the conduit.
The numerical simulation has demonstrated the linear increase of the pressure drop.
This linear increase is due to the weakness of the Forchheimer term compared to Darcy term, for the average velocity, the numerical results encouraged us to propose a mathematical model that has an exact solution which gives a velocity distribution and a pressure drop close to those found in the numerical solution.

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