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Analytical Computation Of The Error Probability Of Coherent M-Ary Frequency Shift Key Modulation Scheme

Isaac A. Ezenugu
Department of Electrical/Electronic Engineering,, Imo State University (IMSU), Owerri, Nigeria
isaac.ezenugu@yahoo.com

Abstract— **Analytical computation of the error probability of coherent M-ary frequency shift key (M-co-FSK) modulation scheme in additive white Gaussian noise channel is studied. The analytical expression for the symbol error probability and bit error probability are presented in terms of Q function, erf function and erfc function, as well as a generalized formula that clearly identifies the coefficient values for computing the bit error rate (BER) for different modulation index in the M-co-FSK. Specifically, the erfc was used in this study to evaluate the bit error probability of various coherent M-Ary FSK , from M=2 to M = 256. The results show that the values of the efrc-based BER coefficients, A and B increase with increase in modulation index, M, from A = 0.5 and B =0.5 at M=2 to A = 64 and B = 4 at M=256. The results of the BER versus Eb/No (dB) results show that the higher the order of the M-co-FSK the better the BER performance for a given Eb/No. In particular, at Eb/No = 6 dB , the BER of M-co-FSK for M = 256 is 1.0667E-06 whereas that of M-co-FSK for M = 2 is 2.3007E-02. The ideas presented in this paper will assist communication system designers in the selection of the coherent M-ary frequency shift key modulation index that suits their design.**

Keywords— Frequency Shift Key, Bit Error Rate , Modulation, Symbol Error Rate, Modulation Index, Coherent FSK

1. Introduction

Frequency shift keying (FSK) is one of the digital modulation schemes that are widely used in various communication systems [1,2,3,4,5]. In frequency shift keying (FSK), the amplitude of the signal is held constant but the carrier frequency is changed in discrete levels in keeping with the digital signal input. In the basic binary FSK (known as BFSK), the frequency is changed in two (2) levels but in higher order FSK referred to as M-ary FSK or MFSK, the carrier frequency is changed in M levels, where M is the modulation factor [5,6,7,8,9,10,11]. In the BFSK , one of the two frequencies (usually the higher frequency) is nominated as the “mark” frequency (binary 1) while the second frequency (usually the higher frequency) is the “space” frequency (binary 0). The difference between the mark frequency and the space frequency if term the “shift” and it is about 50 Hz to 1000 Hz [5,12,13].

In practice, the M-ary quadrature amplitude modulation (MQAM) and the M-ary phase shift keying (MPSK) are used for bandwidth limited applications, where there is sufficient power but limited bandwidth [14,15,16,17]. However, in the case of power-limited applications such as wireless sensor networks, deep space application, underwater networks and ultra-wide band applications, MFSK modulation scheme is preferred [14,18,19,20]. Other applications of the MFSK include portable satellite terminals and battery- powered cellular terminals which are power limited due to retractions on the transmit power capabilities of such devices for health reasons **[14].**

There are two forms of MFSK, namely coherent MFSK and non-coherent MFSK. In coherent MFSK , each mark or space frequency maintains a fixed phase relationship with respect to a reference [5,21,22]. On the other hand, non-coherent MFSK does not maintain any phase relationship with respect to a reference. In all, coherent MFSK do perform better than the non-coherent MFSK but it is more difficult to generate and use. This has limited number of published works on the coherent MFSK [5,21,22]. Consequently, in this paper, analytical computation of the error probability of coherent M-ary frequency shift key (M-co-FSK) modulation scheme in additive white Gaussian noise channel is studied.

2. Calculation of the Bit Error Probability and Symbol Error Probability for Coherent M-Ary FSK (M-co-FSK) Modulation

2.1 The Symbol Error Probability of Coherent M-Ary FSK (M-co-FSK) Modulation

Let the modulation order be denoted as M , energy per symbol be denoted as ε_s , the energy per bit be denoted as ε_b and

If the symbol error probability for coherent M-Ary FSK (M-co-FSK) modulation is denoted as $\mathrm{P}_{\mathbf{sM-co-FSK}}(Qfn)$ then with respect to Q function , $\mathrm{P}_{\mathbf{sM-co-FSK}}(Qfn)$ is defined as;

$$\mathrm{P}_{\mathbf{sM-co-FSK}}(Qfn) = \quad (M-1)Q\left(\sqrt{(\mathrm{Log}_2(M))\left(\frac{\varepsilon_b}{N_0}\right)}\right) \qquad (1)$$

Also, with respect to erf function the symbol error probability, $\mathrm{P}_{\mathbf{sM-co-FSK}}(erfc)$ is defined as;

$$\mathrm{P}_{\mathbf{sM-co-FSK}}(erfc) = \left(\frac{M-1}{2}\right) Q\left(\sqrt{\left(\frac{\mathrm{Log}_2(M)}{2}\right)\left(\frac{\varepsilon_b}{N_0}\right)}\right) \quad (2)$$

Also, with respect to erf function the symbol error probability , $\mathrm{P}_{\mathbf{sM-co-FSK}}(erf)$ is defined as;

$$\mathrm{P}_{\mathbf{sM-co-FSK}}(erf) = \left(\frac{M-1}{2}\right)\left(1 - Q\left(\sqrt{\left(\frac{\mathrm{Log}_2(M)}{2}\right)\left(\frac{\varepsilon_b}{N_0}\right)}\right)\right) \quad (3)$$

2.2 The Bit Error Probability of for Coherent M-Ary FSK (M-co-FSK) Modulation

Let $\mathrm{P}_{\mathbf{bM-co-FSK}}(Qfn)$ be the coherent M-Ary FSK (M-co-FSK) modulation bit error probability derived with respect to Q function . Then,

$$P_{bM-co-FSK}(Qfn) = \left(\frac{M}{2}\right) Q\left(\sqrt{(Log_2(M))\left(\frac{\varepsilon_b}{N_0}\right)}\right) \quad (4)$$

Similarly, $\mathrm{P}_{\mathbf{bM-co-FSK}}(erfc)$ the coherent M-Ary FSK (M-co-FSK) modulation bit error probability with respect to $erfc$ is given as;

$$\mathrm{P}_{\mathbf{bM-co-FSK}}(erfc)) = \left(\frac{M}{4}\right) erfc\left(\sqrt{\left(\frac{Log_2(M)}{2}\right)\left(\frac{\varepsilon_b}{N_0}\right)}\right) \quad (5)$$

Also, the bit error probability of coherent M-Ary FSK (M-co-FSK) modulation with respect to erf is;

$$\mathrm{P}_{\mathbf{bM-co-FSK}}(erf)) = \left(\frac{M}{4}\right)\left(1 - erfc\left(\sqrt{\left(\frac{Log_2(M)}{2}\right)\left(\frac{\varepsilon_b}{N_0}\right)}\right)\right) \quad (6)$$

2.3 Generalized Form of Bit Error Probability for Coherent M-Ary FSK (M-co-FSK) Modulation

The coherent M-Ary FSK (M-co-FSK) modulation bit error can generally be expressed with respect to the complementary error function (erfc) as;

$$\mathrm{P}_{\mathbf{bM-co-FSK}}(erfc)) = (A)erfc\left(\sqrt{(B)\left(\frac{\varepsilon_b}{N_0}\right)}\right) \quad (7)$$

Where

$$A = \frac{M}{4} \quad (8)$$

$$B = \frac{Log_2(M)}{2} \quad (9)$$

4.0 Results and Discussion

The erfc was used in this study to evaluate the bit error probability of various coherent M-Ary FSK , from M=2 to M = 256. The values derived for the coefficients in the generalized expressions in Eq 7, Eq 8 and Eq 9 are given in Table 1.The values of the coefficients A and B increase with increase in M, from A = 0.5 and B =0.5 at M=2 to A = 64 and B = 4 at M=256. The results of the BER (or $\mathrm{P}_{\mathrm{bM-co-FSK}}$(erfc))) versus Eb/No (dB) for the coherent M-Ary FSK (M-co-FSK) modulation are given in Table 2 and Figure 1 and Figure 2. The results show that the higher the order of the M-co-FSK the better the BER performance for a given Eb/No. In Figure 2 , Figure 1 and Table 2 , at Eb/No = 6 dB , the BER of M-co-FSK for M = 256 is 1.0667E-06 whereas that of M-co-FSK for M = 2 is 2.3007E-02.

Table 1 Modulation order versus values of A and B parameter of the generalized bit error probability for Coherent M-Ary FSK (M-co-FSK) modulation based on error function (erfc)

$$\mathrm{P}_{\mathbf{bM-co-FSK}}(erfc)) = (A)erfc\left(\sqrt{(B)\left(\frac{\varepsilon_b}{N_0}\right)}\right) = \left(\frac{M}{4}\right) erfc\left(\sqrt{\left(\frac{Log_2(M)}{2}\right)\left(\frac{\varepsilon_b}{N_0}\right)}\right)$$

Modulation Order, M	Bits/Symbol , K	A	B
2	1	2/4	1/2
4	2	4/4	2/2
8	3	8/4	3/2
16	4	16/4	4/2
32	5	32/4	5/2
64	6	64/4	6/2
128	7	128/4	7/2
256	8	256/4	8/2

Table 2 The results of the BER (or $P_{bM-co-FSK}(erfc)$) versus Eb/No (dB) for the Coherent M-Ary FSK (M-co-FSK) modulation

Signal Levels or Modulation Order, M	2	4	8	16	32	64	128	256
K bits/symbol	1	2	3	4	5	6	7	8
Eb/No (dB)	**M-co-FSK BER For M=2**	**M-co-FSK BER For M=4**	**M-co-FSK BER For M=8**	**M-co-FSK BER For M=16**	**M-co-FSK BER For M=32**	**M-co-FSK BER For M=64**	**M-co-FSK BER For M=128**	**M-co-FSK BER For M=256**
0	1.587E-01	1.573E-01	1.665E-01	1.820E-01	2.028E-01	2.289E-01	2.608E-01	2.994E-01
2	1.040E-01	7.501E-02	5.844E-02	4.723E-02	3.902E-02	3.271E-02	2.771E-02	2.366E-02
4	5.650E-02	2.500E-02	1.210E-02	6.102E-03	3.154E-03	1.656E-03	8.800E-04	4.716E-04
6	2.301E-02	4.777E-03	1.097E-03	2.637E-04	6.510E-05	1.635E-05	4.157E-06	1.067E-06
8	6.004E-03	3.818E-04	2.714E-05	2.027E-06	1.557E-07	1.218E-08	9.649E-10	7.718E-11
10	7.827E-04	7.744E-06	8.641E-08	1.016E-09	1.230E-11	1.510E-13	3.553E-15	0.000E+00
12	3.430E-05	1.801E-08	1.074E-11	6.661E-15	0.000E+00	0.000E+00	0.000E+00	0.000E+00
14	2.695E-07	1.362E-12	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
16	1.399E-10	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
18	9.992E-16	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
20	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
22	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
24	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00

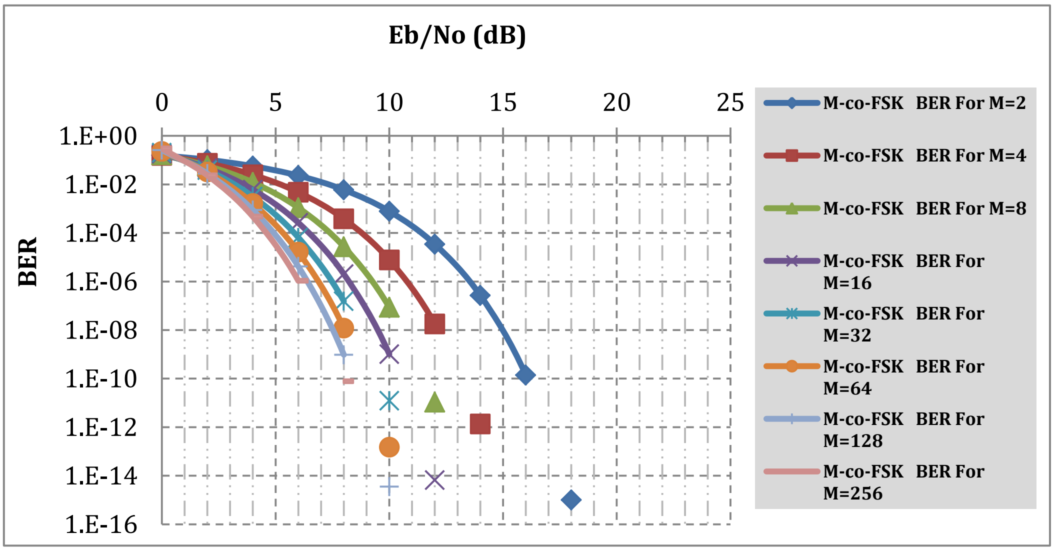

Figure 1 The results of the BER (or $P_{bM-co-FSK}(erfc)$) versus Eb/No (dB) for the Coherent M-Ary FSK (M-co-FSK) modulation

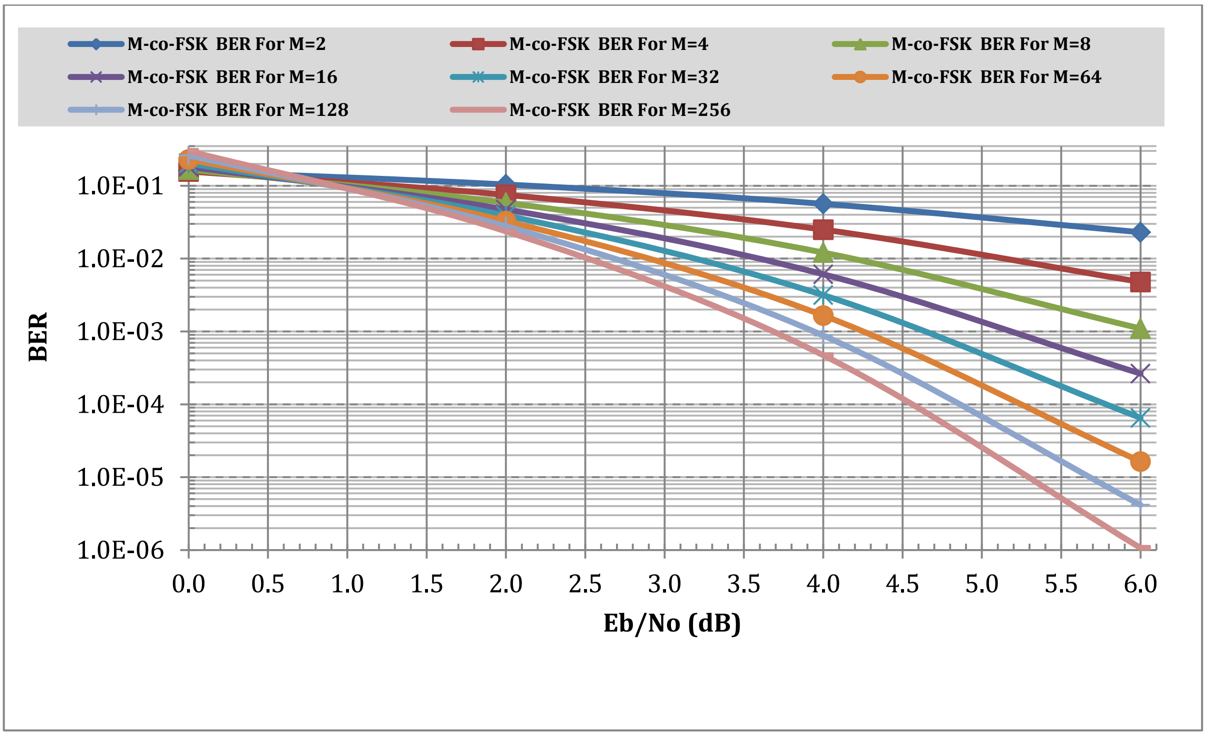

Figure 2 The results of the BER (or $P_{bM-co-FSK}(erfc)$) versus Eb/No (dB) for the Coherent M-Ary FSK (M-co-FSK) modulation with maximum Eb/No = 6 dB

5.0 Conclusion

The bit error rate of the coherent M-Ary FSK (M-co-FSK) modulation scheme is studied. The analytical expressions for the determination of the symbol error probability and bit error probability are presented. A generalized mathematical expression for the bit error probability (BER) is also presented. The BER for M-co-FSK for M = 2 to M =256 was computed. The results show that for a given Eb/No, the BER decreases (improves) as M increases.

References

1. Barnela, M., & Kumar, D. S. (2014). Digital modulation schemes employed in wireless communication: A literature review. *International Journal of Wired and Wireless Communications*, *2*(2), 15-21.
2. Li, C., Hutchins, D. A., & Green, R. J. (2008). Short-range ultrasonic digital communications in air. *IEEE transactions on ultrasonics, ferroelectrics, and frequency control*, *55*(4), 908-918.
3. Anand, P., Gupta, A., & Bhagat, S. (2014). Bit Error Rate Assessment of Digital Modulation Schemes on Additive White Gaussian Noise, Line of Sight and Non Line of Sight Fading Channels. *International Journal of Engineering Science Invention*, *3*(8), 6-10.
4. Bohra, D. D., & Bora, A. (2014). Bit error rate analysis in simulation of digital communication systems with different modulation schemes. *International Journal of Innovative Science*, *1*(3).
5. Watson, B. (1980). FSK: signals and demodulation. *Watkins–Johnson Company Tech–notes*, *7*(5).
6. Olšovský, P., & Podhoranský, P. Design and simulation of frequency hopping technique in MATLAB. *Institute of Electronics and Photonics, Faculty of Electrical Engineering and Information Technology, Slovak University of Technology, Bratislava, available at: http://www. humusoft. cz/ftp/www/papers/tcb12/059_olsovsky. pdf*.
7. Nagarathna, N., Biradar, P. G., & Rao, S. K. (2013). BER Analysis of FSK Transceiver for Cognitive Radio Applications.
8. Rustagi, A., Srow, D., Singh, V. R., & Jain, T. (2014). Implementation of various digital Modulation Techniques Using Low Cost Components.
9. Du, K. L., & Swamy, M. N. (2010). *Wireless communication systems: from RF subsystems to 4G enabling*

technologies. Cambridge University Press.

10. Gaur, P., & Singh, H. G. (2008). *Comparative Performance of Various Hopping Techniques Using Modulation Methods* (Doctoral dissertation).
11. Ahmed, A., Zeeshan, Q., Rehman, A., & Karim, S. (2012). The Performance of Fast Frequency Hopping System in Additive White Gaussian Noise (AWGN). *International Journal of Computer Applications*, *975*, 8887.
12. Ziemer, R. E., & Tranter, W. H. (2014). *Principles of communications*. John Wiley & Sons.
13. Popplewell, P., Karam, V., Shamim, A., Rogers, J., Roy, L., & Plett, C. (2008). A 5.2-GHz BFSK transceiver using injection-locking and an on-chip antenna. *IEEE Journal of Solid-State Circuits*, *43*(4), 981-990.
14. Chandra, A., Poram, S. R., & Bose, C. (2011). SEP calculations for coherent M-ary FSK in different fading channels with MRC diversity. *International Journal of Communication Systems*, *24*(2), 202-224.
15. Khandokar, M. A. H., Rashed, M. G., Yasmin, R., & Hasan, M. A. (2009). Performance Comparison of M-Ary Modulation Schemes for an Efficient Wireless Communication System with Digital Color Image Transmission.
16. Chandra, A., & Bose, C. (2009). Performance of single and multichannel coherent reception under rician fading. *International Journal of Wireless Information Networks*, *16*(1-2), 81-90.
17. Kubankova, A., & Kubanek, D. (2010). Algorithms of digital modulation classification and their verification. *Wseas transactions on communications*, *9*(9), 563-572.
18. Akyildiz, I. F., & Vuran, M. C. (2010). *Wireless sensor networks* (Vol. 4). John Wiley & Sons.
19. Oluge, D. (2014). An Evaluation of Chaos Modulation In Wireless Acoustic Sensor Networks.
20. Guimarães, D. A., & Gomes, G. G. R. (2012). Introduction to Ultra-Wideband Impulse Radio. *Revista Telecomunicacoes*, *14*(1), 49-61.
21. Chandra, A., Bose, C., & Bose, M. K. (2013). Performance of non-coherent MFSK with selection and switched diversity over Hoyt fading channel. *Wireless personal communications*, *68*(2), 379-399.
22. Javed, U., & Hassan, S. A. (2013). SNR estimation for non-coherent M-FSK receivers in Rice fading environment. *IEEE communications letters*, *17*(9), 1786-1789.