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Fuzzy δ-metacompactness Spaces in Fuzzy Topological Space

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Abstract : In this paper, we study and introduce some types of fuzzy open cover space, then we turn to be study fuzzy compact space on fuzzy δ-open set in fuzzy topological space on fuzzy sets, we contain some theorems and propositions of fuzzy δ-compact space, finally we study some propositions and theorems of fuzzy metacompact space , fuzzy δ-metacompact space , fuzzy P-metacompact space , fuzzy S-metacompact space, fuzzy δP-metacompact space , fuzzy δS-metacompact space .

Ee study some types of fuzzy open cover in fuzzy topological space and study the relationships between some types of fuzzy open cover .

Also, we shall recall some basic definitions, propositions and theorems about fuzzy δ-compact space

And, we introduce and study some propositions and theorems of fuzzy δ-metacompact space in fuzzy topological space. And study the relationships between fuzzy δ-compact and fuzzy δ-metacompact space.

Introduction:

In 1969 Fletcher gave the definition P-open cover and S-open cover , in 1983 Fora and Hdieb introduced the definition of P-Lindelöf , S-Lindelöf spaces in analogue manner .

In this chapter, we study and introduce some types of fuzzy open cover space, then we turn to be study fuzzy compact space on fuzzy δ-open set in fuzzy topological space on fuzzy sets, we contain some theorems and propositions of fuzzy δ-compact space, finally we study some propositions and theorems of fuzzy metacompact space , fuzzy δ-metacompact space , fuzzy P-metacompact space , fuzzy S-metacompact space, fuzzy δP-metacompact space , fuzzy δS-metacompact space .

on some types of fuzzy δ-covering (1.0):

In this section ,we study some definitions, remarks and propositions about

fuzzy δ- compact space in fuzzy topological spaces.

Definition (1.1):

A fuzzy topology space $(\widetilde{A},\widetilde{T})$ is fuzzy δ- compact space iff every fuzz δ -open cover of $\widetilde{A}$ has a finite sub cover.

Example (1.2):

1) Any finite fuzzy topology space is fuzzy δ-compact.

2) The indiscrete fuzzy space is fuzzy δ-compact space.

3) The discrete fuzzy space$(\widetilde{A},\widetilde{D})$ is not fuzzy δ-compact space if $\tilde{A}$ is any infinite fuzzy set.

4) The cofinite fuzzy space $(\widetilde{A},\widetilde{T}_{co})$ is fuzzy δ -compact space such that $\tilde{A}$ is any finite fuzzy set.

Proposition (1.3):

Every fuzzy δ -compact space is fuzzy compact space.

Proof:

Let $(\tilde{A},\tilde{T})$ be a fuzzy compact space

Let $\{ \tilde{V}_\alpha : \alpha \in \Lambda \}$ be a fuzzy open cover to $\tilde{A}$

$\{ \tilde{V}_\alpha : \alpha \in \Lambda \}$ is a fuzzy δ -open cover to $\tilde{A}$

But $\tilde{A}$ is fuzzy δ -compact space .

So that $\mu_{\tilde{A}}$ (x) = max { $\mu_{\tilde{V}_{\alpha i}(x)}$: i=1,2,.n}.

$\tilde{A}$ is fuzzy compact space.

Corollary (1.4):

Every fuzzy δ -closed subset of a fuzzy δ -compact space is fuzzy

δ - compact.

Proof:

Let $(\tilde{A},\tilde{T})$ be a fuzzy δ - compact space.

And let $\widetilde{B}$ be a fuzzy δ -closed subset of $\tilde{A}$.

We have to show that $\tilde{B}$ is fuzzy δ - compact ,

Let $\{\tilde{V}_\alpha\}_{\alpha\in\Lambda}$ be a fuzzy δ -open cover for $\tilde{B}$.

Then $\mu_{\tilde{B}}$ (x) $\leq$ $\cup_{\alpha\in\wedge} \mu_{\tilde{V}_\alpha}(x)$

$\widetilde{V}_\alpha$ is a fuzzy δ -open set in $\tilde{A}$, $\forall\alpha$.

since $\mu_{\tilde{A}}$(x) = max{ $\mu_{\tilde{B}}$(x) , $\mu_{\tilde{B}^c}$(X) }= max {{ $\cup_{\alpha\in\wedge} \mu_{\tilde{H}_\alpha}(x)$}, {$\mu_{\tilde{B}^c}$(X)}}.

so max {{ $\mu_{\tilde{V}_\alpha}(X)\}_{\alpha\in\Lambda}$ }, {$\mu_{\tilde{B}^c}$(X) }} is a fuzzy δ -open cover of $\tilde{A}$ which is fuzzy δ -compact space.

Then there exists $\alpha_1, \alpha_2, ., \alpha_n$ such that

$\mu_{\tilde{A}}$(x) = max { {$\cup_{i=1}^{n} \mu_{\tilde{V}_{\alpha i}}(X)$} ,{$\mu_{\tilde{B}^c}$(X)}}.

Hence , $\mu_{\tilde{B}}$ (x) $\leq \cup_{i=1}^{n} \mu_{\tilde{V}_{\alpha i}}(X)$.

so every fuzzy δ -open cover of B̃ has a finite sub cover

Which means that $\tilde{B}$ is fuzzy δ -compact set .

Proposition (1.5):

Every fuzzy δ -compact subset of fuzzy Hausdorff space is fuzzy closed set.

Proof:

Let B̃ be a fuzzy δ -compact subset of fuzzy Hausdorff space ($\tilde{A}$,$\tilde{T}$) .

To prove $\tilde{B}$ is fuzzy closed set .

Let x_r be fuzzy point and μ_{x_r}(x) ≤ $\mu_{\tilde{B}}$ (x) .

Since ($\tilde{A}$,$\tilde{T}$) is a fuzzy Hausdorff space then

for each μ_{y_t}(x) ≤ $\mu_{\tilde{B}^c}$ (x)

Which is different from x_r , there exists disjoint fuzzy open sets Ũ and Ṽ of x_r and y_t respectively

such that: μ_{x_r}(x) ≤ $\mu_{\tilde{U}}$ (x) and μ_{y_t} (x) ≤ $\mu_{\tilde{V}}$ (x) .

Since, min { $\mu_{\tilde{U}}$ (x) , $\mu_{\tilde{V}}$ (x) } =0

by proposition (4.1.2)

The collection{$\tilde{F}_\alpha$: $\alpha \in \Lambda$ }is fuzzy open cover of B̃

Then the collection{$\tilde{F}_\alpha$: $\alpha \in \Lambda$} is fuzzy δ-open cover of fuzzy δ-compact

Hence ,there exists a finite sub cover which covering B̃ which belong to{ $\tilde{F}_\alpha$: $\alpha \in \Lambda$ }, such that $\mu_{\tilde{B}}$ (x) ≤ max { $\mu_{\tilde{F}_{\alpha i}}$ (x)}.

Let $\mu_{\tilde{E}}$ (x) = min { $\mu_{\tilde{U}_{\alpha i}}$ (x)} , and $\mu_{\tilde{G}}$ (x)= max { $\mu_{\tilde{F}_{\alpha i}}$ (x)}.

Then Ẽ is a fuzzy open set containing x_r ,

thus min { $\mu_{\tilde{E}}$ (x), $\mu_{\tilde{G}}$ (x) } = 0

since $\mu_{\tilde{B}}$ (x) ≤ $\mu_{\tilde{G}}$ (x) thus, we have min{$\mu_{\tilde{E}}$ (x), $\mu_{\tilde{B}}$ (x)}= 0

then $\tilde{E} \cap \tilde{B} = \tilde{\emptyset}$

which implies that $\mu_{\tilde{E}}$ (x) ≤ $\mu_{\tilde{B}^c}$ (x) .

Therefore , $\tilde{B}^c$ is fuzzy open set .

Hence B̃ is fuzzy closed set .

corollary (1.6):

Every fuzzy δ -compact subset of fuzzy Hausdorff space is fuzzy δ - closed set.

Proof: Obvious**.**

Theorem (1.7):

Let ($\tilde{A}$,$\tilde{T}$) be a fuzzy topology space if B̃ and C̃ are two fuzzy δ-compact subsets of $\tilde{A}$, then $\tilde{B} \cup \tilde{C}$ is also fuzzy δ -compact.

Proof:

Let $\{\tilde{H}_\alpha:\ \alpha \in \Lambda\}$be a fuzzy open cover of max $\{\mu_{\tilde{B}}(x)\ ,\ \mu_{\tilde{C}}(x)\}$.

Then, max $\{\mu_{\tilde{B}}(x)\ ,\ \mu_{\tilde{C}}(x)\}$ ≤ sup { $\mu_{\tilde{H}_\alpha}(x)$ }

Since, $\mu_{\tilde{B}}(x) \leq \mu_{\tilde{B} \cup \tilde{C}}(x)$

Also, $\mu_{\tilde{C}}(x) \leq \mu_{\tilde{B} \cup \tilde{C}}(x)$

It is follows that $\{\tilde{H}_\alpha: \alpha \in \Lambda\}$ is fuzzy δ- open cover of B̃ and fuzzy

δ- open cover of $\tilde{C}$

Since, B̃ and C̃ are two fuzzy δ-compact sets ,

Then, there exists a finite sub cover{$\tilde{H}_{\alpha 1}$, $\tilde{H}_{\alpha 2}$,. $\tilde{H}_{\alpha n}$} which covering B̃ ∈{$\tilde{H}_\alpha$: $\alpha \in \Lambda$}

Then, $\mu_{\tilde{B}}(x)$ ≤ max { $\mu_{\tilde{H}_{\alpha i}}(x)$ }

Hence $\tilde{B} \leq \cup_{i=1}^{n} \tilde{H}_{\alpha i}$

and there exists a finite sub cover {$\tilde{H}_{\alpha 1}$, $\tilde{H}_{\alpha 2}$,. $\tilde{H}_{\alpha m}$}which is covering $\tilde{C}$ ∈{$\tilde{H}_\alpha$: $\alpha \in \Lambda$}.

Then, $\mu_{\tilde{C}}(x)$ ≤ max { $\mu_{\tilde{H}_{\alpha i}}(x)$ }.

Hence, $\tilde{C} \leq \cup_{i=1}^{m} \tilde{H}_{\alpha i}$

Then, $\tilde{B} \cup \tilde{C} \leq \cup_{i=1}^{n+m} \tilde{H}_{\alpha i}$

Thus, $\tilde{B} \cup \tilde{C}$ is fuzzy δ -compact.

proposition (1.8):

A fuzzy δ-compact space of fuzzy Hausdorff space is fuzzy δ-regular space.

Proof:

Let (Ã,T̃) be a fuzzy δ-compact and fuzzy Hausdorff space

To prove (Ã,T̃) is fuzzy δ-regular space.

Let, $\mu_{x_r}(X) \leq \mu_{\tilde{A}}(X)$ and B̃ be a fuzzy δ-closed set in Ã

Such that, $\mu_{x_r}(X) \geq \mu_{\tilde{B}}(X)$

then $\mu_{x_r}(X) \neq \mu_{y_t}(X)$,∀ $\mu_{y_t}(X) \leq \mu_{\tilde{B}}$ (x).

But Ã is fuzzy δ-Hausdorff space , so there exist $\tilde{H}_{y_t}$ and $\tilde{G}_{y_t}$

such that, $\mu_{x_r}(X) \leq \mu_{\tilde{H}_{y_t}}$(X) and $\mu_{y_t}(X) \leq \mu_{\tilde{G}_{y_t}}$(X)

with min { $\mu_{\tilde{H}_{x_r}}$(X) , $\mu_{\tilde{G}_{y_t}}$(X)} = $\mu_{\tilde{\emptyset}}$ (x)

And $\mu_{\tilde{B}}$(x) ≤ $\cup_{y_t \in \tilde{B}} \mu_{\tilde{G}_{y_t}}$(x) , then $\{\tilde{G}_{y_t}\}_{y_t \in \wedge}$ is fuzzy δ-open cover of $\tilde{B}$

But $\tilde{B}$ is fuzzy δ-closed set in Ã which is fuzzy δ-compact then $\tilde{B}$ is fuzzy δ-compact by proposition (4.1.6)

So, there exist y_{t_1},., y_{t_n}. such that $\mu_{\tilde{B}}$(x) ≤ $max\{\tilde{G}_{y_t i}\}$

On the other hand , $\mu_{x_r}(X) \leq \{\mu_{\tilde{H}_{x_{ri}}}$(X) } , ∀$i$

Let, $\tilde{G} = \cup \{\tilde{G}_{y_t}\}$ and $\tilde{H} = \cap \{\tilde{H}_{y_t}\}$, $\mu_{\tilde{B}}$ (x) ≤ $\mu_{\tilde{H}}$(x)

and $\mu_{x_r}(X) \leq \mu_{\tilde{H}}(X)$

also, min { $\mu_{\tilde{H}}$(x), $\mu_{\tilde{G}}$(x)}= $\mu_{\tilde{\emptyset}}$(x)

Hence , Ã is fuzzy δ-regular space.

Corollary (1.9)

A fuzzy δ-compact space of fuzzy δ- Hausdorff space is fuzzy δ-$\tilde{T}_3$ space.

Proof:

It proves by proposition (1.8).

Theorem (1.10)

Let B̃, C̃ two fuzzy subset $(\tilde{A},\tilde{T})$, B̃ ⊆ C̃ , $\tilde{C}$ is fuzzy open set of Ã then B̃ is fuzzy δ-compact relative to subspace C̃ iff B̃ is fuzzy δ-compact relative to Ã .

Proof:

(⟹) Suppose that B̃ is fuzzy δ-compact relative to C̃ ,

and $\{\tilde{H}_\alpha : \alpha \in \Lambda\}$ is fuzzy open cover to B̃ , $\tilde{H}_\alpha$ is fuzzy set in $\tilde{A}$

Thus $\tilde{H}_\alpha$ is fuzzy δ-open set in $\tilde{C}$

Since B̃ is fuzzy δ-compact relative to $\tilde{C}$

such that, $\mu_{\tilde{B}}(x) \leq$ max $\{\mu_{\tilde{H}_{\alpha i}}(x)\}$

Hence, B̃ is fuzzy δ-compact relative to $\tilde{A}$.

(⟸) Let B̃ is fuzzy δ-compact relative to Ã

and $\{\tilde{H}_\alpha : \alpha \in \Lambda\}$ is fuzzy open cover to $\tilde{B}$

Such that, $\tilde{H}_\alpha$ is fuzzy δ-open set in $\tilde{C}$

since $\tilde{H}_\alpha$ is fuzzy δ-open set in Ã , then B̃ is fuzzy δ-compact relative to Ã , such that $\mu_{\tilde{B}}(x) \leq$ max $\{\mu_{\tilde{H}_{\alpha i}}(x)\}$.

∴ B̃ is fuzzy δ-compact relative to $\tilde{C}$.

Proposition (1.11)

If $\tilde{B}_1$ and $\tilde{B}_2$ are fuzzy δ-compact sets relative to $(\tilde{A},\tilde{T})$,then

$\tilde{B}_1 \cup \tilde{B}_2$ is fuzzy δ-compact sets relative to $(\tilde{A},\tilde{T})$.

Proof:

W= $\{\tilde{H}_\alpha : \alpha \in \Lambda\}$ is fuzzy open cover of $\tilde{B}_1 \cup \tilde{B}_2$.

Then, max { $\mu_{\tilde{B}_1}$ (x), $\mu_{\tilde{B}_2}$ (x)} ≤ sup $\{\mu_{\tilde{H}_\alpha}(X)\}$

Then, W is an fuzzy open cover of $\tilde{B}_1$, $\tilde{B}_2$

so for each i=1,2 there exists a finite subset $\Lambda_i$of Λ

such that$\mu_{\tilde{B}_i}$ (X) ≤ sup $\{\mu_{\tilde{H}_\alpha}(X)\}$.

So, we have max {$\mu_{\tilde{B}_1}$(x) , $\mu_{\tilde{B}_2}$(x) } ≤ sup $\{\mu_{\tilde{H}_\alpha}(X)\}$, i=1,2

Then $\tilde{B}_1 \cup \tilde{B}_2$ is fuzzy δ-compact relative to (Ã,T̃).

Theorem (1.12):

If (Ã,T̃) is fuzzy Hausdorff space and Ã is fuzzy δ-compact relative to (Ã,T̃), then Ã is fuzzy closed .

Proof:

Let Ã is a fuzzy δ-compact set relative to fuzzy Hausdorff space (Ã,T̃) .

So Ã is fuzzy compact (by proposition (4.1.5))

Since (Ã,T̃) is a fuzzy Hausdorff space.

So Ã is a fuzzy closed .

Fuzzy δ-metacompact Space (2.0)

In this section we study a fuzzy δ-metacompact space and introduce some properties about this concepts. We also study the relationship between fuzzy δ-regular space and fuzzy δ-normal space under the conditions.

Definition 2.1 :

A fuzzy topological space (Ã,T̃) is said to be :

- **Fuzzy metacompact space** if every open cover has pairwise point finite parallel .
- **Fuzzy δ-metacompact space** if every δ-open cover has pairwise point finite parallel .

Proposition 2.2 :

If (Ã,T̃) is a fuzzy δ-regular space then the following statement are equivalent :

1. (Ã,T̃) is a fuzzy δ-metacompact space
2. Every fuzzy open cover of Ã has pairwise point finite parallel .
3. Every fuzzy open cover of Ã has pairwise point finite δ-closed parallel

Proof :

(1 ⟹ 2) let (Ã,T̃) be a fuzzy δ-metacompact space, then by definition (3.1.1) every fuzzy open cover has pairwise point finite parallel .

(2 ⟹ 3) let V =$\{\tilde{V}_\alpha : \alpha \in \Lambda\}$ be a fuzzy open covering of Ã and since (Ã,T̃) is a fuzzy δ-regular space, then for each $\mu_{x_r}(x) \leq \mu_{\tilde{V}_\alpha}(x)$ there exist fuzzy δ-open set $\tilde{U}_\lambda$ such that

$\mu_{x_r}(x) \leq \mu_{\tilde{U}_\lambda}(x) \leq \mu_{\delta cl(\tilde{U}_\lambda)}(x) \leq \mu_{\tilde{V}_\alpha}(x)$ for some $\alpha \in \Lambda$

and fuzzy δ-open covering $\{\tilde{U}_\lambda : \in \Lambda\}$ of Ã has pairwise point finite parallel $\{\tilde{D}_\beta : \beta \in \eta\}$

Since $\mu_{\delta cl(\tilde{D}_\beta)}(x) \leq \mu_{\delta cl(\tilde{U}_\lambda)}(x) \leq \mu_{\tilde{V}_\alpha}(x)$,

then { δcl($\tilde{D}_\beta$) : $\beta \in \eta$ }is a pairwise point finite δ-closed parallel of V

(3 ⟹ 1) let V =$\{\tilde{V}_\alpha : \alpha \in \Lambda\}$ be a fuzzy open covering of Ã

Then V has pairwise point finite δ-closed $\{\tilde{P}_\beta : \in \eta\}$

And V has a fuzzy locally finite δ-open refinement $\{\tilde{U}_\beta : \in \eta\}$.

Hence (Ã ,Ť) is a fuzzy δ-metacompact space ∎

Proposition 2.3 :

If (Ã,Ť) is a fuzzy δ^*-regular space then the following statement are equivalent :

1. (Ã,Ť) is a fuzzy δ-metacompact space
2. Every fuzzy open cover of Ã has pairwise point finite parallel .
3. Every fuzzy open cover of Ã has pairwise point finite δ-closed parallel

Proof : Obvious

Proposition 2.4 :

If (Ã,Ť) is a fuzzy δ-metacompact space, then every fuzzy δ-regular space is a fuzzy δ-normal space.

Proof :

Suppose that (Ã,Ť) is a fuzzy δ-metacompact space and fuzzy δ-regular space.

Let $\tilde{F}_1$ and $\tilde{F}_2$ be two distinct fuzzy closed sets of (Ã, Ť)

since (Ã ,Ť) is a fuzzy δ-regular space, then the finite fuzzy open cover

D = { $\tilde{R}_1{}^c$, $\tilde{R}_2{}^c$ } of Ã has a fuzzy locally finite fuzzy δ-closed refinement **H** = { $\tilde{E}_1$, $\tilde{E}_2$ } such that $\mu_{\tilde{E}_1}(x) \le \mu_{\tilde{R}_1{}^c}(x)$ and $\mu_{\tilde{E}_2}(x) \le \mu_{\tilde{R}_2{}^c}(x)$, hence $\mu_{\tilde{R}_1}(x) \le \mu_{\tilde{E}_1{}^c}(x)$ and $\mu_{\tilde{R}_2}(x) \le \mu_{\tilde{E}_2{}^c}(x)$, since **H** is a fuzzy pairwise point finite parallel , then $\tilde{E}_1{}^c \,\tilde{\bar{q}}\, \tilde{E}_2{}^c$ therefore (Ã, Ť) is a fuzzy δ- normal space ∎

Proposition 2.5 :

If (Ã ,Ť) is a fuzzy δ-metacompact space then every fuzzy δ^*-regular space is a fuzzy δ-normal space.

Proof : Obvious

Proposition 2.6 :

A fuzzy δ^*-regular space (Ã,Ť) is a fuzzy δ-metacompact space and fuzzy δ-normal space if and only if for each fuzzy open cover of Ã has pairwise point finite δ-closed parallel .

Proof :

(⟹) Let (Ã,Ť) be a fuzzy δ-metacompact space and fuzzy δ-normal space and **D** $=\{\tilde{D}_\alpha : \alpha \in \Lambda\}$ be a fuzzy open covering of fuzzy δ^*-regular space (Ã ,Ť), then for each $\mu_{x_r}(x) \le \mu_{\tilde{D}_\alpha}(x)$ there exist fuzzy δ-open set $\tilde{U}_\lambda$ such that

$\mu_{x_r}(x) \le \mu_{\tilde{U}_\lambda}(x) \le \mu_{\delta cl(\tilde{U}_\lambda)}(x) \le \mu_{\tilde{D}_\alpha}(x)$ for some $\alpha \in \Lambda$

and fuzzy δ-open covering $\{\tilde{U}_\lambda : \in \Lambda\}$ of Ã has pairwise point finite δ-open parallel $\{\tilde{B}_\beta : \beta \in \eta\}$

since $\mu_{\delta cl(\tilde{B}_\beta)}(x) \le \mu_{\delta cl(\tilde{U}_\lambda)}(x) \le \mu_{\tilde{D}_\alpha}(x)$,

then $\{\delta cl(\tilde{B}_\beta) : \beta \in \eta\}$ has pairwise point finite δ-closed parallel D

(⟸) let **D** $=\{\tilde{D}_\alpha : \alpha \in \Lambda\}$ be a fuzzy open covering of fuzzy δ^*-regular space (Ã ,Ť), then **D** has pairwise point finite δ-closed parallel $\{\tilde{F}_\beta : \in \eta\}$,and **D** has pairwise point finite δ-open parallel $\{\tilde{U}_\beta : \in \eta\}$, hence (Ã,Ť) is a fuzzy δ-metacompact space and by proposition(4.1.5) we get (Ã,Ť) is a fuzzy δ-normal space ∎

Proposition 2.7 :

If (Ã,Ť) is a fuzzy regular space then the following statements are equivalent:

1) (Ã,Ť) is a fuzzy δ-metacompact space
2) Every fuzzy open cover of Ã has pairwise point finite parallel .
3) Every fuzzy open cover of Ã has pairwise point finite δ-closed parallel
4) (Ã,Ť) is a fuzzy metacompact space

Proof :

(1 ⟹ 2) Obvious

(2 ⟹ 3) Obvious

(3 ⟹ 4) let **D** $=\{\tilde{D}_\alpha : \alpha \in \Lambda\}$ be a fuzzy open covering of Ã,

then **D** has pairwise point finite δ-closed parallel$\{\tilde{F}_\beta : \in \eta\}$

and **D** has pairwise point finite δ-open parallel$\{\tilde{U}_\beta : \in \eta\}$.

Hence (Ã,Ť) is a fuzzy metacompact space

(4 ⟹ 3) let **D** $=\{\tilde{D}_\alpha : \alpha \in \Lambda\}$ be a fuzzy open covering of Ã,

then **D** has pairwise point finite δ-open parallel$\{\tilde{U}_\beta : \in \eta\}$

By proposition (1.2.6) we get **D** has pairwise point finite δ-open parallel $\{\tilde{U}_\beta : \in \eta\}$,

hence (Ã ,Ť) is a fuzzy δ-metacompact space ∎

Proposition 2.8 :

If every fuzzy δ-open set in fuzzy δ-metacompact space (Ã ,Ť) is fuzzy δ-metacompact space, then every fuzzy subspace $(\tilde{B}, \tilde{T}_B)$ is fuzzy δ-metacompact space .

Proof :

Let $\tilde{C}$ be a fuzzy subset of a fuzzy topological space $(\tilde{B}, \tilde{T}_B)$,

then $\mu_{\tilde{C}}(x) = \min\{ \mu_{\tilde{C}}(x) , \mu_{\tilde{A}}(x) : \tilde{A} \in \tilde{T} \}$

By hypothesis it is clear that $\max\{\mu_{\tilde{A}}(x)\}$ is fuzzy δ-open and so δ-metacompact

Let **D** $=\{\tilde{D}_\alpha : \alpha \in \Lambda\}$ be a fuzzy open covering of $\max\{\mu_{\tilde{A}}(x)\}$, then min{ $\mu_{\tilde{C}}(x)$, $\{\tilde{D}_\alpha : \alpha \in \Lambda\}$} be a fuzzy open covering of $\mu_{\tilde{C}}(x)$ since max{$\mu_{\tilde{A}}(x)$} is fuzzy δ-metacompact, then **D** =$\{\tilde{D}_\alpha : \alpha \in \Lambda\}$ has pairwise point finite δ-open parallel $\{\tilde{U}_\beta : \beta \in \eta\}$,and this implies that

min{$\mu_{\tilde{C}}(x)$, $\{\tilde{U}_\beta : \in \eta\}$} is pairwise point finite δ-open parallel of min{ $\mu_{\tilde{C}}(x)$, $\{\tilde{D}_\alpha : \alpha \in \Lambda\}$},

hence ($\tilde{B}$, $\tilde{T}_B$) is fuzzy δ-metacompact space ■

Proposition 2.9 :

A fuzzy topological space (Ã,T̃) is a fuzzy δ-metacompact space if every fuzzy regular closed set of Ã is fuzzy δ-metacompact space .

Proof :

Let **D=**{ $\tilde{H}_\alpha : \alpha \in \Lambda$ } be a fuzzy open cover of Ã

And $\mu_{cl(\tilde{H}_\alpha)}(x) \neq \mu_{\tilde{A}}(x)$, then cl(int($\tilde{H}_\alpha{}^c$)) is a fuzzy regular closed set of Ã, {min{$\mu_{\mathrm{cl}(\mathrm{int}(\tilde{H}_\alpha{}^c))}(x)$, $\mu_{\tilde{H}_\lambda}(x)$} : $\lambda \in \Lambda$, $\lambda \neq \alpha$ } is a fuzzy open covering of cl(int($\tilde{H}_\alpha{}^c$)), then there exist family of fuzzy δ-open $\{\tilde{G}_\beta : \beta \in \eta\}$ in Ã such that {min{$\mu_{\mathrm{cl}(\mathrm{int}(\tilde{H}_\alpha{}^c))}(x)$, $\mu_{\tilde{G}_\beta}(x)$} : $\beta \in \eta$ } pairwise point finite δ-open parallel of {min{$\mu_{\mathrm{cl}(\mathrm{int}(\tilde{H}_\alpha{}^c))}(x)$, $\mu_{\tilde{H}_\lambda}(x)$} : $\lambda \in \Lambda$, $\lambda \neq \alpha$ } and covers cl(int($\tilde{H}_\alpha{}^c$)).

Hence max{$\mu_{\tilde{H}_\alpha}(x)$, {min{$\mu_{\mathrm{cl}(\mathrm{int}(\tilde{H}_\alpha{}^c))}(x)$, $\mu_{\tilde{G}_\beta}(x)$} : $\beta \in \eta$, $\beta \neq \alpha$ }

Is a pairwise point finite δ-open parallel of **D** and covering Ã

therefore (Ã ,T̃) is a fuzzy δ-metacompact space ■

Definition 2.10 :

A fuzzy topological space (Ã,T̃) is said to be :

- **Fuzzy P-metacompact (S-metacompact) space** if every P-open (S-open) cover has pairwise point finite parallel .
- **Fuzzy δP-metacompact (δS-metacompact) space** if every δP-open (δS-open) cover has pairwise point finite parallel .
- **Fuzzy P-Lindlöf (S-Lindlöf) space** if every P-open (S-open) cover has countable sub-cover .
- **Fuzzy δP-Lindlöf (δS-Lindlöf) space** if every δP-open (δS-open) cover has countable sub-cover .

Proposition 2.11 :

A fuzzy topological space (Ã ,$\tilde{T}$) is fuzzy δS-metacompact space if and only if it is fuzzy δ-metacompact space and fuzzy δP-metacompact space .

Proof :

Assume that (Ã ,$\tilde{T}$) is fuzzy δS-metacompact space , and

Let $\tilde{U}$={U_α: $\alpha \in \Delta$ } is δP-open cover of $\tilde{A}$, Then $\tilde{U}$ is open cover of fuzzy topological space $\tilde{A}$

Since , $\tilde{A}$ is fuzzy δS-metacompact space and $\tilde{U}$ has pairwise point finite parallel refinement .

Hence , $\tilde{A}$ is fuzzy δP-metacompact space .

So, $\tilde{A}$ is fuzzy δ-metacompact space .

Now , Let $\tilde{A}$ is fuzzy δ-metacompact space and fuzzy δP-metacompact space

Also , Let $\tilde{U}$={U_α: $\alpha \in \Delta$ } is δ-open cover of $\tilde{A}$

If $\tilde{U}$ is δP-open cover Then the result follows ,

If $\tilde{U}$ is not δP-open coverThen (Ã ,$\tilde{T}$) is fuzzy δ-metacompact space.

So $\tilde{U}$ has pairwise point finite parallel refinement ,

Then (Ã ,$\tilde{T}$) is fuzzy δS-metacompact space .

Proposition 2.12 :

If a fuzzy topology (Ã ,$\tilde{T}$) is hereditary fuzzy δ-metacompact space then it is fuzzy δS-metacompact space .

Proof :

Let $\tilde{U}$={U_α: $\alpha \in \Delta$ } is δ-open cover of Ã

Then , $\tilde{U}$=∪{U_α: $\alpha \in \Delta$ } is fuzzy δ-metacompact space

It has a point finite open parallel refinement of $\tilde{U}$

Then $\tilde{U}$ is a point finite open parallel refinement of $\tilde{U}$.

Proposition 2.13 :

A fuzzy δP-metacompact space is fuzzy δP-Lindelöf space .

Proof :

Let $\tilde{U}$={U_α: $\alpha \in \Delta$ } be δP-open cover of Ã

Assume that $\tilde{U}$ has not countable subcover of $\tilde{A}$,

Let $\tilde{V}$={V_β: $\beta \in \Delta$ } be a point finite parallel refinement of $\tilde{U}$

Let **D** be countable dense subset of $\tilde{A}$, Then $V_\beta \cap \boldsymbol{D} \neq \emptyset , \forall \beta \in \Gamma$

Thus **D** is an uncountable , this is contradiction

Hence , the result .

Definition 2.14 :

A fuzzy topological space (Ã,T̃) is said to be :

- **Fuzzy countably P-metacompact (S-metacompact) space** if every countable P-open (S-open) cover has pairwise point finite parallel refinement.
- **Fuzzy countably δP-metacompact (δS-metacompact) space** if every countable δP-open (δS-open) cover has pairwise point finite parallel refinement.

- **Fuzzy countably P-metalindelöf space** if every countable P-open cover has pairwise countable parallel refinement.
- **Fuzzy countably δP-metalindelöf space** if every countable δP-open cover has pairwise countable parallel refinement.

Proposition 2.15 :

Every fuzzy δP-metalindelöf countably δP-metacompact space is fuzzy δP-metacompact space .

Proof :

Let $\widetilde{U}$={U_α: α ∈ Δ } be δP-open cover of Ã , Since Ã is fuzzy δP-metalindelöf space ,

Then $\widetilde{U}$ has a point countably parallel refinement $\tilde{V} = \{V_{\alpha_i}\}_{i=1}^{\infty}$

Which is also δP-open cover of $\tilde{A}$

Since , $\tilde{A}$ a fuzzy countably δP-metacompact space

So , $\tilde{V}$ has a point finite parallel refinement $\widetilde{W}$ of $\tilde{V}$

Hence , Ã is fuzzy δP-metacompact space .

Definition 2.16:

A fuzzy topological space (Ã,T̃) is said to be :

- **Fuzzy P-compact (S-compact) space** if every P-open (S-open) cover has finite sub-cover .
- **Fuzzy δP-compact space** if every δP-open cover has finite sub-cover .
- **Fuzzy P-countably compact (S-countably compact) space** if every countable P-open (countable S-open) cover of a fuzzy set has a finite sub-cover .
- **Fuzzy δP-countably compact (δS-countably compact) space** if every countable δP-open (countable δS-open) cover of a fuzzy set has a finite sub-cover .

Proposition 2.17 :

Every fuzzy δP-countably compact δP-metacompact space is fuzzy δP-compact space .

Proof:

Let (Ã , $\tilde{T}$) is fuzzy topological space ,

It is sufficient to show that fuzzy topological space (Ã , $\tilde{T}$) is fuzzy δP-compact space .

Let $\widetilde{U}$={U_α: α ∈ Δ } be δP-open cover of a fuzzy δP-countably compact δP-metacompact space ,

So , there exist an irreducible point finite open refinement

$\tilde{V}$={V_α: α ∈ Δ } of the cover $\widetilde{U}$

The cover $\tilde{V}$ being irreducible , ∀$\alpha\epsilon\Delta$

∃ a point $x_\alpha \in V_\alpha / \cup_{\alpha\neq\alpha^*} V_\alpha$

Since , the sets V_α cover the fuzzy set $\tilde{A}$

Every point $x_r \in \tilde{A}$ has a neighborhood which contains exactly one point of the set K={x_α: α ∈ Δ}

Hence , the derived set of K denoted by $K^d = \emptyset$

Since , the fuzzy set (Ã , $\tilde{T}$) is δP-countably compact space

The set K is finite , also the set Δ is also finite and the open cover $\tilde{V}$ is a finite open refinement of $\widetilde{U}$ for (Ã , $\tilde{T}$)

So , (Ã , $\tilde{T}$) is fuzzy δP-compact space .

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