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Numerical Study on Structural Optimization of Reinforced Concrete Cantilever Beam

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***Abstract*— In the design of structures, economy and safety are very important to structural designers. While there are clear guidelines and codes provisions for designing safety structures, the procedure for designing structures that satisfy both safety and cost is not fully understood. To address one of such problems, this study focused on the optimization of reinforced concrete cantilever beam subjected to concentrated load at the free end. Eighteen (18) finite element models of Reinforced Concrete (RC) cantilever beam is developed using general purpose finite element package ABAQUS. Twelve (12) of these models are used to investigate the effect of change in depth of section at the free end on the load and displacement response of the beam while three (3) models are used to study the effect of bottom reinforcement on the load carrying capacity of beam. Three different length of 1.5, 2 and 2.5m length is considered. The result shows that reducing the depth of the beam at the free end by 50% will result in optimal beam design without compromising the load capacity. Also, the bottom reinforcement area does not have effect on the load capacity of RC cantilever beam prior to the yield of reinforcement. During the post yield stage, increase in area of bottom reinforcement slightly increases the load capacity of beam. It is recommended that minimum area of reinforcement satisfying the code requirement should be used in areas where reinforcement is not required such as bottom section of RC cantilever beam because the function of such reinforcement is mainly to act as hanger bars, holding the shear reinforcement in place but does not contribute to the flexural capacity of the beam.**

Keywords—Reinforced Concrete; Structural Optimization; Reinforcement; Load capacity

I. INTRODUCTION

In the design of structural elements such as beam, slabs and columns, experience is needed in making important decisions aim at designing structures that are not only safe but economical. Two words which are of importance in structural engineering is safety and cost. Often times, emphasizes is placed on the former with little or no attention given to the later. Design codes for reinforced concrete, steel and composite structures have been developed to help in designing structural elements. These codes [1,2,3] outline procedures for designing structural elements that are safe but do not explicitly lead to structure that satisfy both economy and safety.

The need to design structures that satisfy both economy and safety has given rise to structural optimization. In the context of structural optimization, an optimal solution means choosing the most economical option without affecting the functionality of the structure [4]. An illustration of the concept of optimization in structural engineering design is shown in Figure 1. Generally, structural optimization consists of (1) size optimization (2), shape optimization and (3) topology optimization. Size optimization refers to the physical size of structural element within the structure, shape optimization refers to the geometric layout while topology optimization represent the internal configuration of members of a structure [5].

Key steps in optimization process includes; identifying the design variables, formulation of objective function, formulation of constraints and getting the solution. For a structural design problem, the objective function may be to minimise the weight, volume or cost. The design variables may include reinforcement size, cross sectional area or depth of the structural element while the constraint may be to achieve a predetermined load capacity or maximum displacement. The problem becomes more complex when the design variables are more than one [6]

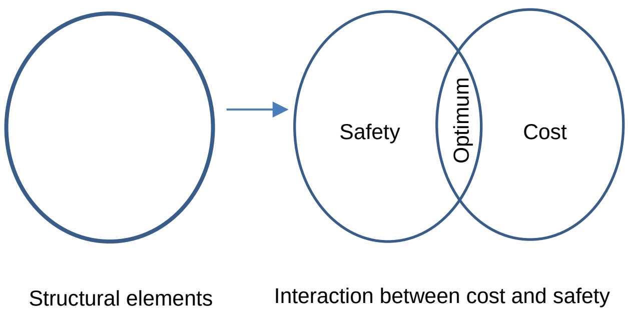

Figure 1: Interaction between cost and safety in structural design

A number of studies have been performed on the structural optimization of RC structures. Some of these studies focused on one area of optimisation i.e. size, shape or topology [7,8] while others considered two or more areas of optimization simultaneously [5,6].

Reference [9] investigated the optimal design of reinforced concrete beams using Artificial Neural Network. The variables in their study included concrete compressive strength and yield strength of reinforcement. Their result showed that M40 grade of concrete and Fe 550 grade of reinforcing steel resulted in the most optimal design. In a related study, optimization of pretensioned concrete with consideration of material nonlinearity was investigated [10]. Illustrations on the effect of parameters like prestressing wires, geometrical discontinuities showed that the method could be used in optimization of prestressed concrete beam.

Despite the numerous optimization studies in the field of structural design, the effect of section depth at the free end of RC cantilever beam (tapered beam), on the global behavior of RC beam is not fully understood. Also, structural designer often uses higher area of reinforcement within the compression zone in a cantilever beam even when such reinforcement is not required. To address the aforementioned issues, this paper presents an optimization study using ABAQUS software.

II. METHODOLOGY

A. General Description

This study is performed using a general purpose software called ABAQUS. Concrete is modelled using a three-dimensional (3D) solid elements which is an 8-noded linear brick element (C3D8R) with reduced integration. Steel reinforcement is modelled using a two-noded linear beam element in space (B31). The material models for concrete and steel are described in subsequent sections.

B. Concrete Model

In ABAQUS, different material models exist for modelling the behavior of concrete. These include, smeared cracking model (SC), Concrete Damaged Plasticity model (CDP) and Brittle Cracking model. Of these material models, Concrete Damaged Plasticity model (CDP) is considered best in modelling the nonlinear behavior of concrete. It can model both static and dynamic behavior of concrete [11,12,13].

The CDP model was developed by Lubliner et al. [14]. In this model, the inelastic behavior of concrete is represented using the concepts of isotropic damage elasticity, isotropic tensile and compressive plasticity. Poison ratio for concrete is assumed to be 0.2 while the compressive strength of concrete (σ_{cu}) is assumed as 25N/mm^2. Since the post yield behavior of RC beam was not the focus of this study, concrete is modelled as elastic perfectly plastic in both tension and compression as shown in Figure 2. The tensile strength (σ_{tu}) is calculated using ACI code expression as shown in Equation 1. Damage parameters used in the Concrete Damaged Plasticity model (CDP) is shown in Table 1.

$$f_r = 0.62\sqrt{f_{cu}} \qquad (1)$$

TABLE 1: PARAMETER FOR CONCRETE DAMAGE PLASTICITY MODEL (CDP)

ψ	ε	f_{bo}/f_{co}	K_c	μ
36°	0.1	1.16	0.67	0

C. Steel reinforcing model

For reinforcing steel, an elastic perfectly plastic material model is chosen in this study. The yield strength of main reinforcing bars is assumed to be 500N/mm^2 while the yield strength of transverse reinforcement is taken as 250N/mm^2 representing mild steel which is commonly used for shear reinforcement in Nigeria. Elastic modulus is taken to be 200GPa while poison ratio is 0.3. The stress strain behavior of reinforcing steel is shown in Figure 2b.

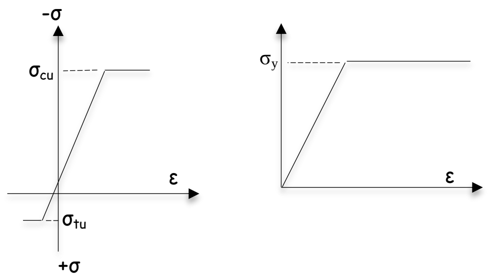

Figure 2: Stress-strain relationship of (a) concrete (b) reinforcing steel

D. Description of RC Beam Models

Three different length of beam (1.5, 2.0 and 2.5m) is considered in this study. For each length of beam, the depth of beam at fixed end is kept at 400mm while that of the free end is varied from 100mm to 400mm (increment of 100mm). In total, fifteen (15) models of RC beam is considered, twelve (12) of these models are used to investigate the effect of shape optimization on RC beam while 3 models are used to investigate the effect of cross sectional area of compression (bottom) reinforcement on the loading capacity of RC cantilever beam. The geometrical and reinforcement details of these models are shown in Table 2.

To simulate the cantilever RC beam, the surface of one end of the beam is connected to a reference point using the COUPLING constraints option in ABAQUS. The motion of the connected surface is controlled by the motion of the reference point. The displacement and rotational degrees of freedom of the reference point (U_x, U_y, U_z, U_{rx}, U_{ry}, U_{rz}) is set to zero mimicking a fixed boundary condition. A loading plate is placed at the top side of the concrete near the free end. The loading plate is modelled using solid element with elastic properties to avoid damage of the loaded area. Interaction between the loading plate and concrete is defined using a SURFACE TO SURFACE based contact. The motion of surface of loading plate is constrained to a reference node which is push down in a displacement controlled manner until failure. The output of finite element models are very sensitive to the mesh size used. For this reason, a mesh sensitivity analysis is performed to obtain a mesh size that is not computationally costly but yields accurate result. The result of the analysis shows that a mesh size of 15mm for the solid and beam elements is adequate for this analysis. Figure 3 shows the mesh discretization for model A3 (1.5m RC beam tapered beam with depth of 300mm at the end).

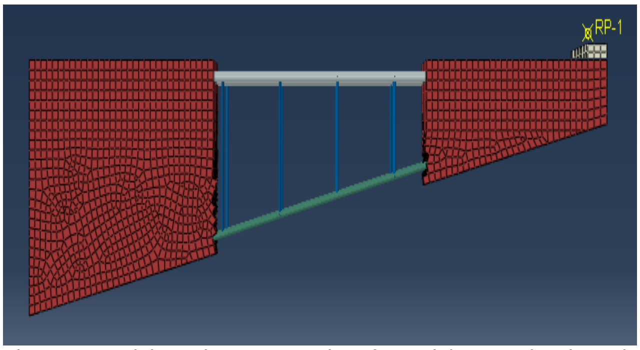

Figure 3: Finite element mesh of Model A3 showing the reinforcement and concrete

III. RESULT AND DISCUSSION

A. *Effect of RC depth at free end on load capacity*

The result of reinforced concrete beam models is considered in terms of global behavior (load versus displacement response) and local behavior (contour plot). Figure 4-6 shows the load displacement plot of models with 1.5, 2.0 and 2.5m length respectively. It can be seen from this plot that the load capacity of the beam decreases with decrease in the cross section of the beam at the free end. The decrease is more pronounced in models with depth of 100mm at free end. A critical look at this result indicates that the load capacity of models A2, B2, C2 and A3, B3 and C3 are almost the same with the result of models A1, B1 and C1 (models with constant depth) respectively but a noticeable difference is observed between load capacity of models A4, B4, C4 and A1, B1, C1 respectively. In terms of displacement, the models with the lowest depth (100mm) at the free end has the highest deflection at the yielding of reinforcement.

The result also shows that instead of using uniform beam depth over the entire length of the RC cantilever beam, a tapered beam can be used with the depth at the free end taken as 50% of the depth at fixed end. This will result in an optimized section with reduced self weight, reduced volume of concrete and formwork. For example, in a 2m RC cantilever beam, the volume of concrete for constant depth of 400mm (model B1) is 0.16m^3 whereas volume of concrete for the same length of beam but with 50 percent reduction in depth at free end (model B3) is 0.12m^3. This will result in a volumetric difference 0.04m^3 between model B1 and B3. Considering that the cost of 1m^3 of concrete is ₦70,000 ($184). Savings of ₦2,800 ($7) can be made without compromising the structural performance of the RC beam.

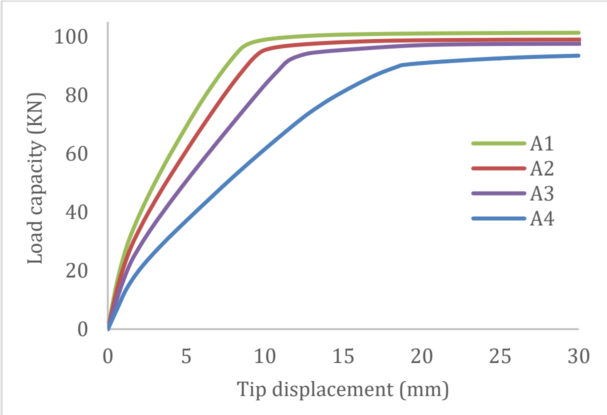

Figure 4: Load displacement response for RC beam Models of 1.5m length and different depth at free end

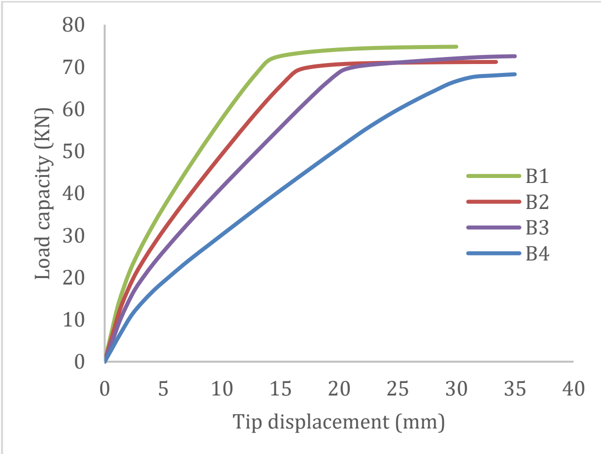

Figure 5: Load displacement response for RC beam Models of 2m length and different depth at free end

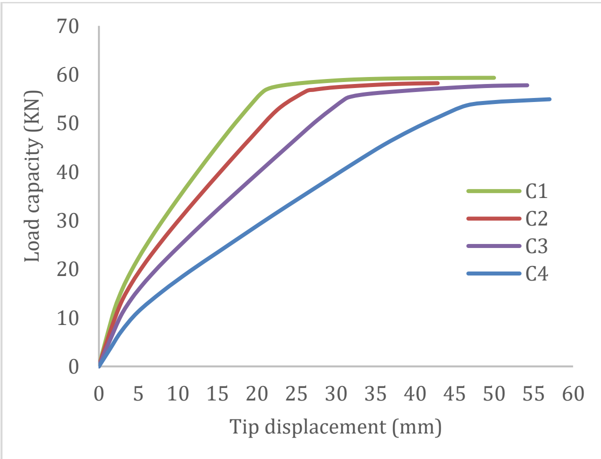

Figure 6: Load displacement response for RC beam Models of 2.5m length and different depth at free end

Table 4 shows the load at yield and the corresponding displacement. Also presented in the table is the ratio of load capacity and corresponding displacement of models with variable depth to models with uniform depth. Comparing the result with RC beam of constant depth, the load capacity decreases by 1.5, 3.5 and 6.95% for models A1, B1 and C1 respectively. Similar results are observed for models A2, B2, C2 and C2, C3, C4.

Comparing RC beam models with uniform depth and that of variable depth at free end, the displacement at yield increases by between 12-16%, 38-44% and 107-121% in models with depth of 100, 200 and 300mm respectively. Although the model with free end depth of 100mm result in the most economical beam, the deflection of the beam which is twice that of model with uniform depth may not satisfy the serviceability condition. Since design must satisfy economy and safety, models with section depth of 200mm can be selected to fulfil both the economy and safety since deflection is only 40% higher than model with uniform depth.

The contour plot at the yield of tension reinforcement is shown in Figure 7-10 for RC beam with span length of 2m. The behavior of other beams (1.5 and 2.5m) is similar to the one described here. It can be observed that concrete compression zone over the length of the beam decreases with decrease in the depth of section at the free end for a given beam length. This could be due to the reduced weight of beam leading to a lower rate of cracks as the depth of section decreases.

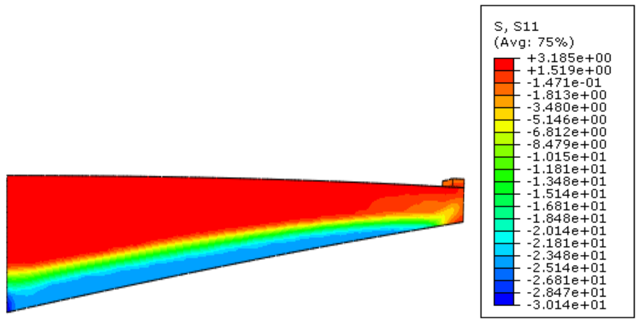

Figure 7: Contour plot of concrete stress distribution in beam with 100mm depth at free end

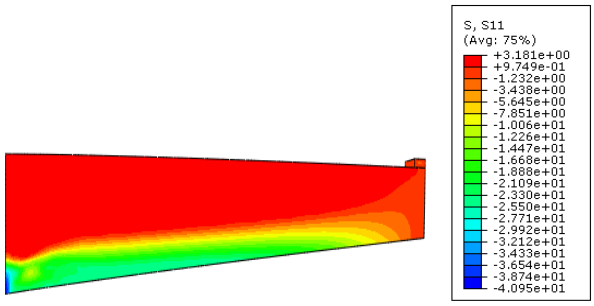

Figure 8: Contour plot of concrete stress distribution in beam with 200mm depth at free end

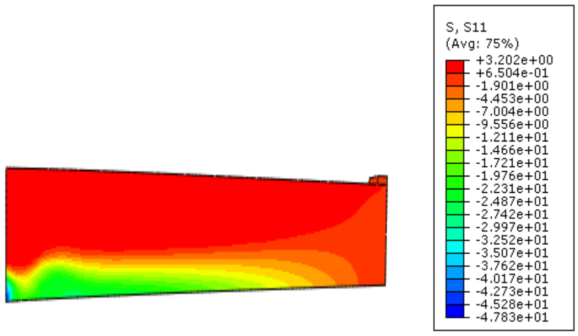

Figure 9: Contour plot of concrete stress distribution in beam with 300mm depth at free end

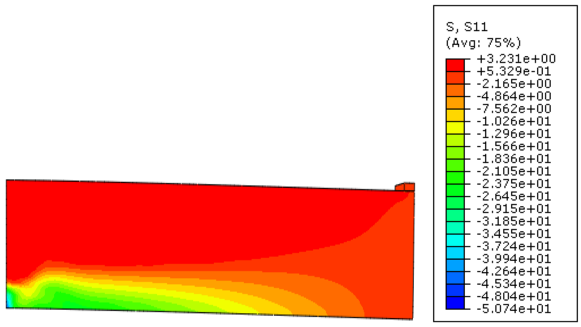

Figure 10: Contour plot of concrete stress distribution in beam constant depth (control beam)

B. *Effect of compression reinforcement size on load capacity of beam*

The load-displacement curve for models D1, D2 and D3 representing RC cantilever beam with bottom reinforcement of 2Y10, 2Y12 and 2Y16 respectively is shown in Figure 11. It can be seen that prior to yielding of reinforcement, the three models behave in a similar manner. After yielding, the load capacity slightly increases as the size of reinforcement (or cross sectional area) increases. This increase in load capacity is very small compared to the difference in cost of reinforcement.

In Nigeria for instance, the market price of standard 12m length of 16mm diameter reinforcement is ₦3900 ($10) while price of same length of 10mm diameter is ₦1700 ($4). For every standard length of 12m, cost savings of ₦2,200 ($6) can be made in using 10mm diameter instead of 16mm diameter reinforcement without compromising the structural integrity of beam.

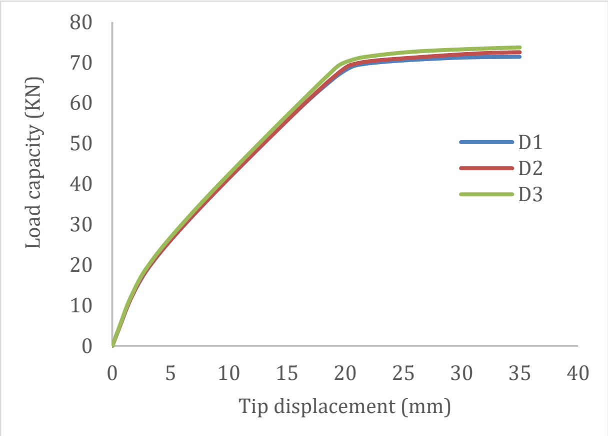

Figure 11: Load displacement response of RC beams with different area of compression reinforcement

For singly reinforced section, tensile force in the tension zone is counter balance by compressive force provided by concrete with compression reinforcement playing no significant role in the process. The only function of compression reinforcement is to act as hanger bars by holding the shear reinforcement in position. This implies that reducing the area of reinforcement will result in reduction in cost of reinforcement. As a guide, the area of reinforcement in the compression zone can be chosen based on minimum area of reinforcement specify by the code which for BS 8110 is 0.13bh%.

Table 2: Detail of beam models used in shape optimization study

Model ID	Length (m)	Width of beam (mm)	Depth at fixed end (mm)	Depth at free end (mm)	Top reinforcement	Bottom reinforcement
A1	1.5	200	400	400	3Y16	2Y12
A2	1.5	200	400	300	3Y16	2Y12
A3	1.5	200	400	200	3Y16	2Y12
A4	1.5	200	400	100	3Y16	2Y12
B1	2.0	200	400	400	3Y16	2Y12
B2	2.0	200	400	300	3Y16	2Y12
B3	2.0	200	400	200	3Y16	2Y12
B4	2.0	200	400	100	3Y16	2Y12
C1	2.5	200	400	400	3Y16	2Y12
C2	2.5	200	400	300	3Y16	2Y12
C3	2.5	200	400	200	3Y16	2Y12
C4	2.5	200	400	100	3Y16	2Y12

Table 3: Detail of beam models used to study the effect of bottom reinforcement on load carrying capacity

Model ID	Length (m)	Width of beam (mm)	Depth at fixed end (mm)	Depth at free end (mm)	Top reinforcement	Bottom reinforcement
D1	2.0	200	400	200	3Y16	2Y16
D2	2.0	200	400	200	3Y16	2Y12
D3	2.0	200	400	200	3Y16	2Y10

Table 4: Result of load and displacement at yield for different models

Model ID	Load at yield (KN)	Displacement at yield (mm)	Load ratio of models to model with constant depth	Yield displacement ratio of models to model with constant depth
A1	97.34	8.89	1.000	1.000
A2	95.88	10.33	0.985	1.162
A3	94.02	12.60	0.965	1.417
A4	90.60	19.33	0.931	2.174
B1	72.39	14.77	1.000	1.000
B2	69.77	17.23	0.964	1.167
B3	69.21	20.34	0.956	1.377
B4	67.56	31.39	0.933	2.125
C1	57.46	22.25	1.000	1.000
C2	56.84	27.12	0.989	1.218
C3	55.48	32.02	0.966	1.439
C4	53.64	46.53	0.933	2.091

III. Conclusion and Recommendation

This study focused on the optimization of RC cantilever beam. Two main issues; effect of tapered beam and area of bottom reinforcement on the global behavior of RC cantilever beam is investigated. From the result, the following conclusions are made;

(1) Load capacity of beam decreases whereas displacement at the tip increases with decrease in the depth of the beam at free end.
(2) The area of reinforcement in compression zone does not influence the global behavior of the beam within the elastic region. However, after the yield of reinforcement, load capacity slightly increases with increase in the size and number of bottom reinforcement
(3) The behavior of RC cantilever beam can be accurately modelled using ABAQUS Finite Element package

Based on the conclusion above, it is recommended that depth of section at free end should be 50% of depth at the fixed end. This will result in cost saving from concrete. Furthermore, the area of bottom reinforcement in RC cantilever beam should be selected based on the minimum reinforcement area according to code provision provided that the section is singly reinforced and does not require compression reinforcement.

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