The Impact Of Education On Coupled Risk-Awareness And Investor Sentiment In Multiplex Networks

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***Abstract*—As an important factor of investment behavior, investor sentiment has a direct impact on investors' investment income. And to some degree, the spread of investors' sentiment is similar to the spread of disease. In this paper, we model the coupled spreading dynamics of risk-awareness and investor sentiment. And then education is considered in this model. The upper layer represents the diffusion of the awareness, and the lower layer represents the sentiment diffusion, which is based on SIR model. Through the Monte Carlo methods and Microscopic Markov chain approach, the result shows that the rate of risk-awareness and the attenuation factor are also important factors affecting the model. In addition, we also find that it is not useful for the authority to strengthen the education at any time, and the authority should adjust the educational guidance strength in time according to the situation of the investors' own risk-awareness transformation. In addition, we find that the rate of risk-awareness and the attenuation factor are also important factors affecting the model.**

Keywords—Multiplex networks; Awareness diffusion; MMCA method; Investor sentiment

I. Introduction

Investment behavior is a manifestation of investors' psychological process, and investor sentiments can also affect stock prices. If investors are irrational, they will be likely to make irrational investment decisions. It is bad for the stock market. De et al. consider the investor sentiment in the stock price determination model for the first time, and then pointed out that if investor is emotional, investor sentiment will become a systemic risk affecting asset prices. Research also shows that stock prices are not only affected by the price mechanism, but also by individual factors[1,2]. Therefore, many scholars are interested in investor sentiment and began to do research on it.

After that, most of the literature begin to study the relationship between investor sentiment and stock prices in the stock market. For example, Shiller et al. explains why investors prefer a particular asset in the financial markets[4]. Due to herd behavior and imitative behavior in the stock market, Lux et al. interpret a stock market bubble as a self-organizing infection among traders[5]. In countries with immature markets, sentiment has a more pronounced effect on returns[6]. Investors can be divided into individual investors and institutional investors, which means different types of investors have different influences on the stock market. And in the Chinese stock market, individual investors make up the majority[7-10].

At the same time, some scholars study the relationship of financial assets from the perspective of complex network[11-13], but they seldom study the transmission process of investor sentiment. Investor sentiment is easily affected by external factors and in some ways, investor sentiment diffusion is similar to the spread of disease. People's awareness of the disease will affect the spread of the disease.If the disease risk awareness is considered, people will take measures to prevent the disease and reduce their susceptibility when the disease breaks out[14-16]. Granell et al. establish an SIS-UAU model in multiple networks to study the disease transmission and the awareness of prevention of disease transmission, and then consider the impact of mass media on the final disease size[17,18]. Since then, scholars improve the model[19-21]. Wang et al. improve the SIS model and study the SIR model in the multiplex networks. Study has shown that the epidemic threshold is related to the awareness diffusion and the epidemic network topology. Guo et al. propose an infectious disease model which is influenced by local awareness in multiple networks[22]. Considering individual heterogeneity, Pan et al. study the coupled dynamics of awareness diffusion and disease transmission in multiple networks, and divide the information in the model into contact information, local information and global prevalence information[23]. These researches are all about the coupled awareness-epidemic dynamics in the multiplex networks.

Similarly, when investors know some information, they will become irrational investors and make some irrational decisions. So, strengthen the risk-awareness of investors is crucial. Inspire these references, we study the impact of education on coupled risk-awareness and investor sentiment in multiplex networks. To strength the risk-awareness of investors, In addition to considering the impact of investors' own risk-awareness on the diffusion of sentiment, we also consider whether the authorities have any influence on the education of investors. In this model, individual heterogeneity is not considered, and we focus on the local risk-awareness.

The organization of this paper is as following: In Section 2, the UAU-SIR model with education is established. In Section 3, we use Microscopic Markov chain approach(MMCA) to analyze the model, and verify the accuracy of MMCA method through Monte Carlo(MC) simulation. The simulation results are presented in Section 4. At last, conclusions are provided in Section 5.

II. Model

In this paper, we model the coupled spreading dynamics of awareness and investor sentiment based on ref.[17], which is also considered with education. The sketch of multiplex networks is shown in fig.1. As is shown, the upper layer represents the diffusion of risk-awareness, and the lower layer represents the physical contact layer, which means the spread of investor sentiment. All nodes in each layers are the same individuals, but the links are different in each of them. We all know investors can get their information through things like social media. On the upper layer, when investors get some information, they will have risk-awareness. So supposed that an unaware individual(U) can be informed by aware neighbor(A) with the probability α and become an aware individual, and the aware individual can forget the information and become the unaware node with the probability δ. The sentiment spreading layer is assumed to be similar to the classical SIR model, we divided the investor into three types by sentiments: susceptible(S), infected(I), recovered(R). Susceptible individual means investor have no idea, infected individual means investor is irrational and excited in some information, and recovered individual means investor has calm down.That is to say, a susceptible node can be infected by infected neighbor with the probability β and the infected node can be recovered with the probability μ. There are many means of government intervention, such as education. Besides, to strength the risk-awareness of investor, authority take some measures to education investors. The authority educates the investor and the unaware individual turn to the aware individual with the probability m.

Besides, if an individual is aware in the upper layer and is susceptible in the lower layer, it reduces its own infectivity by a factor γ. To distinguish the state in the upper layer, we assume that the infection rate of the original unaware node is β^U and the subsequent infectivity after being aware is β^A, which satisfies $\beta^A = \gamma\beta^U$. If the particular case of γ, the aware individuals are completely immune to the infection.

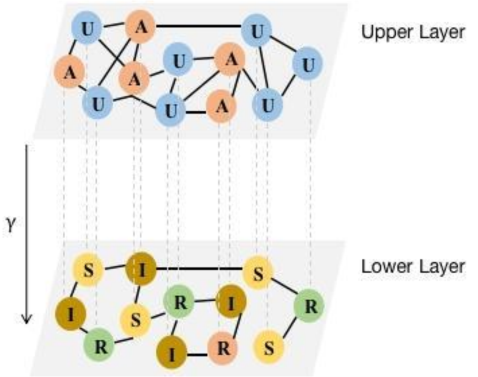

Fig. 1 Model description for UAU-SIR dynamic in multiplex networks. The upper layer represents the diffusion of the risk-awareness, in which nodes are unaware(U) or aware(A). And the lower layer represents the sentiment diffusion, in which nodes are susceptible(S), infected(I), recovered(R).

Here, we supposed if the unaware individual can be infected by the infected individual, he/she will realize the risk and become aware individual. Hence, an individual can be in five different states: unaware and susceptible (US), unaware and recovered (UR), aware and susceptible (AS), aware and infected (AI) or aware and recovered (AR). To reveal the possible states of nodes and their transitions clearly, we draw the transition probability trees, which is illustrated in Fig.2. According to the Fig.2, the microscopic Markov chain approach (MMCA)evolution equations can also be obtained.

III. Theoretical analysis

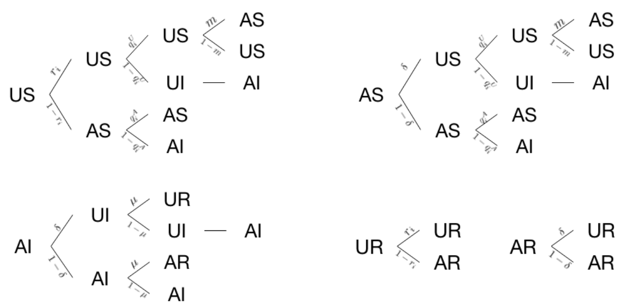

Fig. 2 The transition probability trees for five different states(US,UR,AS,AI,AR). The root of each tree represents the state at time t and the leaves of each tree represents the state at time t+1. The education of the authority is considered in this process.

In this section, denoting a_{ij} and b_{ij} be the adjacency matrices that support the UAU and SIR processes, respectively. Each node i in five different states at time t has a certain probability, denoted by $p_i^{US}(t), p_i^{AS}(t), p_i^{AI}(t), p_i^{UR}(t)$ and $p_i^{AR}(t)$, respectively, where $p_i^{US}(t) + p_i^{AS}(t) + p_i^{AI}(t) + p_i^{UR}(t) + p_i^{AR}(t) = 1$. According to the absence of dynamical correlations[17], the probability of node i not being informed by any neighbors is $r_i(t)$, and the probability of unaware (aware) susceptible node i not being infected is ${q_i}^U({q_i}^A)$. They are expressed as:

$$r_i(t) = \prod_j [1 - a_{ji} p_j^A(t) \alpha] \quad (1)$$

$$q_i^A(t) = \prod_j [1 - b_{ji} p_j^I(t) \beta^A] \quad (2)$$

$$q_i^U(t) = \prod_j [1 - b_{ji} p_j^I(t) \beta^U] \quad (3)$$

For each state at time t, the five transition probability trees have been illustrated in Fig.2. According to the method of MMCA, the probability of five states of the node i at time $t+1$ is:

$$p_i^{US}(t+1) = p_i^{US}(t) r_i(t) q_i^U(t)(1-m) + p_i^{AS}(t) \delta q_i^U(t)(1-m) \quad (4)$$

$$p_i^{UR}(t+1) = p_i^{UR}(t) r_i(t) + p_i^{AI}(t) \delta \mu + p_i^{AR}(t) \delta \quad (5)$$

$$\begin{aligned} p_i^{AS}(t+1) = & p_i^{US}(t)\left[r_i(t) q_i^U(t) m + (1-r_i(t)) q_i^A(t)\right] \\ & + p_i^{AS}\left[\delta q_i^U(t) m + (1-\delta) q_i^A(t)\right] \end{aligned} \quad (6)$$

$$\begin{aligned} p_i^{AI}(t+1) = & p_i^{US}(t)\left[r_i(t)(1-q_i^U(t)) + (1-r_i(t))(1-q_i^A(t))\right] \\ & + p_i^{AS}(t)\left[(1-\delta)(1-q_i^A(t)) + \delta(1-q_i^U(t))\right] + p_i^{AI}(t)(1-\mu) \end{aligned} \quad (7)$$

$$p_i^{AR}(t+1) = p_i^{UR}(t)(1-r_i(t)) + p_i^{AR}(t)(1-\delta) + p_i^{AI}(t)\mu \quad (8)$$

Denoting the sentiment transmission threshold be β_c. Since the sentiment transmission threshold determines whether the sentiment can outbreak or die out, it is important to analyze the effects of the different parameters on the sentiment transmission threshold β_c. Near the sentiment transmission threshold, the possibility of each node i being infected when $t \to +\infty$ is supposed to be infinitely small, i.e., $p_i^{AI} = \varepsilon_i \ll 1$. Consequently,

$$q_i^A(t) \approx 1 - \beta^A \sum_j b_{ji} \varepsilon_j \quad (9)$$

$$q_i^U(t) \approx 1 - \beta^U \sum_j b_{ji} \varepsilon_j \quad (10)$$

To simplify the calculation, let $\alpha^A = \beta^A \sum_j b_{ji} \varepsilon_j$ and inserting Eq.(9) in Eq.(7), we have the equation:

$$\begin{gathered} \varepsilon_i = \varepsilon_i(1-\mu) + p_i^{US}\left[r_i(t)\alpha^U + (1-r_i(t))\alpha^A\right] + p_i^{AS}\left[(1-\delta)\alpha^A + \delta\alpha^U\right] \\ \mu\varepsilon_i = \left[r_i(t) p_i^{US} + \delta p_i^{AS}\right]\beta^U \sum_j b_{ji}\varepsilon_j + \left\{[1-r_i(t)] p_i^{US} + (1-\delta) p_i^{AS}\right\}\beta^A \sum_j b_{ji}\varepsilon_j \\ \left\{[(1-\gamma) r_i(t) + \gamma] p_i^{US} + [(1-\gamma)\delta + \gamma] p_i^{AS}\right\}\beta^U \sum_j b_{ji}\varepsilon_j - \mu\varepsilon_i = 0 \end{gathered} \quad (11)$$

Noting that $p_i^A(t) = p_i^{AS}(t) + p_i^{AI}(t) + p_i^{AR}(t)$ and $p_i^U(t) + p_i^A(t) = 1$, near the threshold, the probability of a node to be either positive or negative is close to zero, so $p_i^A(t) \approx p_i^{AS}(t)$, $p_i^U(t) \approx p_i^{US}(t)$ and $p_i^{US}(t) = 1 - p_i^{AS}(t)$.

Therefore,

$$\sum_j \left\{\left[(1-\gamma)(\delta - r_i(t)) p_i^A + r_i(t) + (1-r_i(t))\gamma\right] b_{ji} - \frac{\mu}{\beta^U} t_{ji}\right\}\varepsilon_i = 0, \quad (12)$$

where t_{ji} is the element of the identity matrix.

The solution of the sentiment transmission threshold turn to an eigenvalue problem for the matrix H, whose elements are

$$h_{ji} = \left[(1-\gamma)(\delta - r_i(t)) p_i^A + r_i(t) + (1-r_i(t))\gamma\right] b_{ji} \quad , \quad (13)$$

The onset of the sentiment diffusion is given by the largest eigenvalue of H, the critical point is written as

$$\beta_c^U = \frac{\mu}{\Lambda_{max}(H)}. \quad (14)$$

Obviously, the sentiment transmission threshold depends on the diffusion of risk-awareness and the parameter γ.

IV. NUMERICAL SIMULATIONS

In the last two sections, we have got the evolution equations for each investor using the MMCA, and we analyze that the sentiment transmission threshold β_c is related to the diffusion of risk-awareness and the parameter γ. Here, we will verify the accuracy of MMCA method through MC simulation, and then use MMCA method to predict the final size of the recovered individual. All simulation results are the average value after 20 runs.

To make the network more like a real investor network, we build a scale-free network with $N = 2000$, $P(k) \sim k^{-2.5}$. The upper layer is almost the same as the lower layer, except that the upper layer has 800 extra random edges than the lower layer. The initial condition is set to be that 5% of nodes are infected in the diffusion of investor sentiment. After the initial condition are given, the probability that the investors are in different states at each time can be expressed by iteration. When the system tends to be stable, the proportion of aware and recovered investor in the total population can be expressed as $\rho^A = \sum_i p_i{}^A / N$, $\rho^R = \sum_i p_i{}^R / N$.

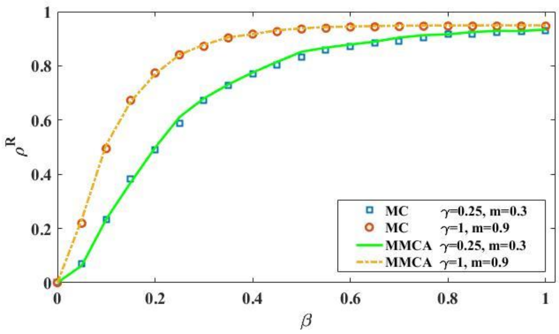

Fig. 3 ρ^R as a function of β obtained using the MMCA method and MC simulation in BA networks. Parameter values is set as follows: $\alpha = 0.25, \delta = 0.3, \mu = 0.4$.

First, in order to verify the accuracy of MMCA method, the results of MMCA method and MC simulation are compared. The infection rate β and final size of recovered investor ρ^R between the two methods is shown in the fig.3. It is proved that the results obtained by MMCA method are in good agreement with MC simulation results. So the MMCA method is helpful to analyze the threshold and the final outbreak size of the model. In this paper, we just study the effect of parameters α, γ, m, so let assign a fixed value to the parameter $\delta = 0.3, \mu = 0.4$.

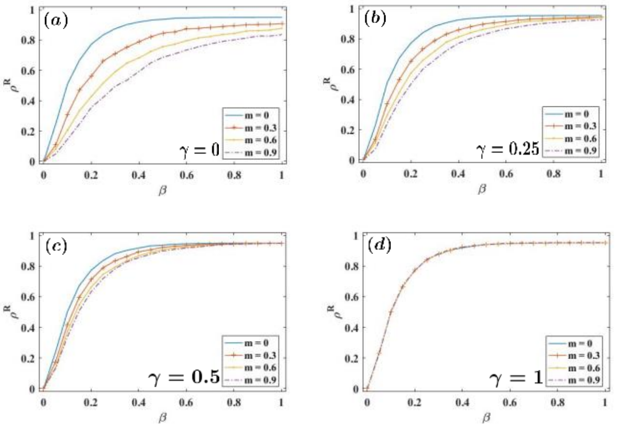

Fig. 4 ρ^R as a function of β under different values of m at different γ. Parameter values is set as follows: $(a). \alpha = 0.1, \delta = 0.3, \mu = 0.4$; $(b). \alpha = 0.1, \delta = 0.3, \mu = 0.4$; $(c). \alpha = 0.1, \delta = 0.3, \mu = 0.4$; $(d). \alpha = 0.1, \delta = 0.3, \mu = 0.4$.

Second, Fig.4 shows the density of recovered individual ρ^R as a function of β under different values of m when γ takes four different values. Obviously, no matter how the parameter changes, ρ^R always increases as β increases. In each subgraph, ρ^R decreases as m increases. When $\gamma \to 1$, m has little effect on the final size of recovered investor. When $\gamma = 0$, the risk-aware investor is immune to infected investor. That is to say, strengthening the education of investors can enhance the risk awareness of investors, but it will reduce the final size of recovered investor instead. At this moment, controlling susceptibility is also important.

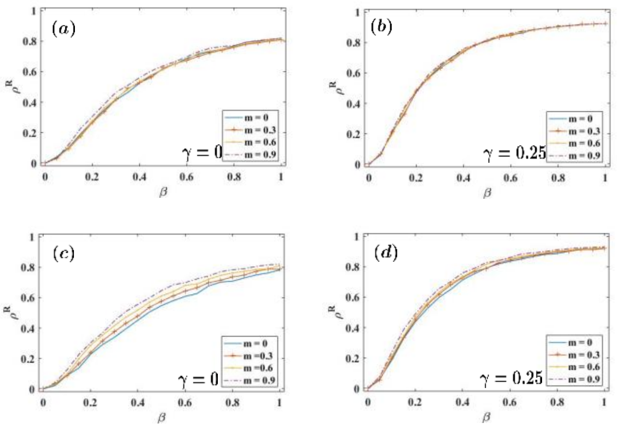

Fig. 5 ρ^R as a function of β under different values of m. at different α and γ. Parameter values is set as follows: $(a). \alpha = 0.5, \delta = 0.3, \mu = 0.4$; $(b). \alpha = 0.5, \delta = 0.3, \mu = 0.4$; $(c). \alpha = 1, \delta = 0.3, \mu = 0.4$; $(d). \alpha = 1, \delta = 0.3, \mu = 0.4$.

Compared to Fig.4, there are some differences in Fig.5. The biggest difference is that he influence of different values m on ρ^R has obvious changes. In Fig.5(c), ρ^R increases as m increases. This phenomenon is the opposite of Fig.4. So, when $\alpha < 0.5$, ρ^R decrease as m increases; When $\alpha > 0.5$, ρ^R increases as m increases. The result shows that the final size of recovered investors is affected by the education of the authority. According to Fig4(b), Fig.5(b)(d), ρ^R decreases as α increases.

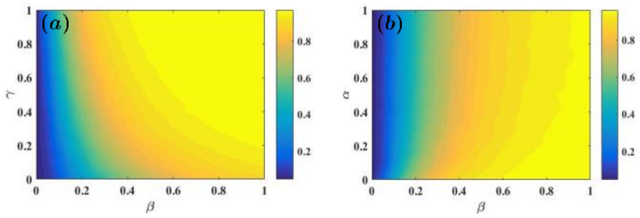

Fig. 6 The full-phase diagram $\beta - \gamma$ and $\beta - \alpha$ with fixed values $m = 0.3, \delta = 0.3, \mu = 0.4$. The parameter is set as $(a). \alpha = 0.25, (b). \gamma = 0.25$.

Finally, Fig,6(a) and Fig,6(b) present the full-phase diagram $\beta - \gamma$ and $\beta - \alpha$ of UAU-SIR dynamics for two layer BA networks, respectively. The results show an inverse change of β_c with γ and the increasing the rate of diffusion of risk-awareness α can increase the threshold and decrease the final size of recovered investors.

V. CONCLUSION

In this paper, we have established the the equation by using MMCA method and derived the propagation threshold. The results of a large number of MC simulations are compared with the MMCA method, and the propagation threshold and the final size of the model are analyzed.

The results show that MMCA method is in good agreement with MC simulation, and MMCA method can effectively predict the results; The coupling relationship between the transmission of risk-awareness and the investors' sentiment transmission influences the transmission threshold and the final size of the model. When $\gamma = 0$, the education of the authority has the most significant effect on the final size. At last, when $\alpha < 0.5$, strengthening the education can have an inhibiting effect on the final size, but when $\alpha > 0.5$, it can have a promoting effect. Hence, the authority should adjust the educational guidance strength in time according to the situation of the investors' own risk-awareness transformation.

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