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Stability Analysis Of Investor Sentiment Spreading Model Considering Personality In Heterogeneous Networks

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***Abstract*—In the stock market, investor sentiment has a significant influence on the decision making of investors. In this paper, to investigate the effects of personality in the dynamics of investor sentiment spreading, we propose a 2S2IR model while assuming that spreading rate of each individual depends on his/her personality. We divide the personality of individuals into active and passive. The active person has the great ability of spreading and receiving sentiment. We calculate the basic reproduction number based on the next generation matrix method and prove that the sentiment free equilibrium is global asymptotically stable and the positive equilibrium is global attractivity. Furthermore, some numerical simulations are performed, the results show that the influence of different personality of individuals.**

Keywords—sentiment; heterogeneous network; stability; basic reproduction number

I. INTRODUCTION

Investor sentiment reflect their emotions, attitudes and values when making decisions. It is very important research objects in the stock market and it also play a vital role in the dynamic behavior of investors. The traditional epidemic spreading model studies the spread of infectious diseases by classifying people. It is a classic model in the field of mathematical research. We optimize and improve the classic epidemic model to give it a new background for studying investor sentiment spreading process.

Scholars have conducted a lot of research on the relationship between investor sentiment and financial markets, confirming that investor sentiment has a considerable impact on the stock market [1][2][3]. It is reasonable to apply the epidemic model to investigate the spreading of investor sentiment [4]. In the field of epidemic model researching, the classic epidemic models such as SI [5], SIS [6], SIR [7] is very representative. Some people improved the classic model like Pan et al. investigate the coupled awareness-epidemic dynamics in multiplex networks considering individual heterogeneity [8]. Zang et al. propose a global awareness controlled spreading model (GACS) to explore the interplay between the coupled dynamical processes [9]. Chang et al. present a stochastic SIRS epidemic model with two different nonlinear incidence rates and double epidemic asymmetrical hypothesis [10]. In the field of rumor spreading investigation, Daley and Kendall first introduced the DK model of rumor spreading [11][12], then Maki and Thomson proposed a MK rumor spreading model [13]. A series of improvements have been made. In the recent years, hesitation mechanism [14], counterattack and self-resistance mechanisms [15], the different attitudes [16] are introduced in the rumor spreading models. Wang et al. presented a new SIR model by considering the trust mechanism between ignorants and spreaders [17], Huo et al. investigated the general rumor spreading model with psychological effect [18]. Zhao [19][20], Nekovee [21] and Gu [22] et al. studied the effects of forgetting and remembering mechanism on the rumors spreading from different perspectives. Afassinou extend the classical SIR model by considering the impact of education rates in rumor propagation dynamics [23]. Song et al. discussed the impact of scientific knowledge [24]. Zhao et al. established a rumor spreading model by considering the prevalence of new media [25]. Some scholars also improve the transmission rate to be more practical. Fu et al. proposed a piecewise linear function to characterized the infectivity [26], Cheng et al. considering nonlinear spreading rate to establish the model's mean field equations [27]. Some scholars found that different nodes show the great difference in their activity [27][28][29] the active person are more likely to accept ang spread rumors. In addition, Jie, Zan et al. studied the process of double-rumor propagation in homogeneous complex networks [30], and complex networks [31].

In this paper, we define investor sentiment as the investor's emotional attitude and value towards the relevant events. And make the following reasonable assumptions: the stock market as a heterogeneous network, the investors as nodes in the network. New entrants to the stock market are unaware of the relevant events and therefore no sentiment. When they come into contact with the sentiment spreaders, the emotional attitude of the sentiment spreaders will influence them to become the corresponding spreaders or choose not to spread sentiment. In order to be more practical, this paper considers the different personality of individuals, and divides the population into two types: active and passive. Individuals with different personality have different ability to transmit and receive sentiment. The purpose of this paper is to provide a theoretical basis for controlling investors' decision-making behavior in the stock market by studying and analyzing the sentiment spreading process of investors.

The rest of this paper is organized as follows. In section 2, we establish the model of investor sentiment propagation and obtain the corresponding mean-field equations. In the third section, we discuss and prove the stability of the sentiment-free equilibrium, and prove the existence and global attractivity of the positive equilibrium in the heterogeneous network. In section 4, related numerical simulations are performed, and the important factor which affects the sentiment spreading process are discussed. Finally, some conclusions and discussions are given in section 5.

II. INVESTOR SENTIMENT MODEL

Assume that each investor in the stock market is in one of the following five states: active susceptible S_1, who are new to the stock market, whose attitudes to events in the stock market will be affected by others and their personalities are active; passive susceptible S_2, their sentiment acceptance is relatively weak; active spreader I_1, active individuals who spread sentiment to their contacts; passive spreader I_2, compared with active spreaders, their ability to spread sentiment is relatively weak; stifler R, who do not accept or transmit sentiment to others, and therefore do not distinguish between active and passive types. Among them, S_1 and S_2 receive sentiment, I_1 and I_2 spread sentiment, and R do not spread or receive sentiment.

Assuming that the stock market has a coming rate of e, an leaving rate of q, an active individual as a percentage of new entrants of p, the process of investor sentiment spreading in the stock market is shown in Fig. 1, where β_1, β_2 represents the spreading ability of I_1, I_2, so $\beta_1 > \beta_2$, λ_1, λ_2 stands for the sentiment receiving ability of S_1, S_2, so $\lambda_1 > \lambda_2$, α_1, α_2 stands for the probability that the spreaders transform into stiflers.

In the heterogeneous networks, the densities of five groups with degree k at time t are $S_{1k}(t)$, $S_{2k}(t)$, $I_{1k}(t)$, $I_{2k}(t)$, $R_k(t)$ based on above diagram, the dynamic mean-field equations is presented by

$$\begin{cases} \frac{dS_{1k}}{dt} = pe - kS_{1k}(\beta_1\theta_1 + \beta_2\theta_2) - qS_{1k} \\ \frac{dS_{2k}}{dt} = (1-p)e - kS_{2k}(\beta_1\theta_1 + \beta_2\theta_2) - qS_{2k} \\ \frac{dI_{1k}}{dt} = k\lambda_1 S_{1k}(\beta_1\theta_1 + \beta_2\theta_2) - (\alpha_1 + q)I_{1k} \\ \frac{dI_{2k}}{dt} = k\lambda_2 S_{2k}(\beta_1\theta_1 + \beta_2\theta_2) - (\alpha_2 + q)I_{2k} \\ \frac{dR_k}{dt} = k[(1-\lambda_1)S_{1k} + (1-\lambda_2)S_{2k}](\beta_1\theta_1 + \beta_2\theta_2) \\ \qquad +\alpha_1 I_{1k} + \alpha_2 I_{2k} - qR_k \end{cases}$$

(1)

where $\theta_1 = \frac{1}{<k>}\sum kp(k)I_{1k}(t)$, $\theta_2 = \frac{1}{<k>}\sum kp(k)I_{2k}(t)$ and $S_{1k}(t) + S_{2k}(t) + I_{1k}(t) + I_{2k}(t) + R_k(t) = 1$。

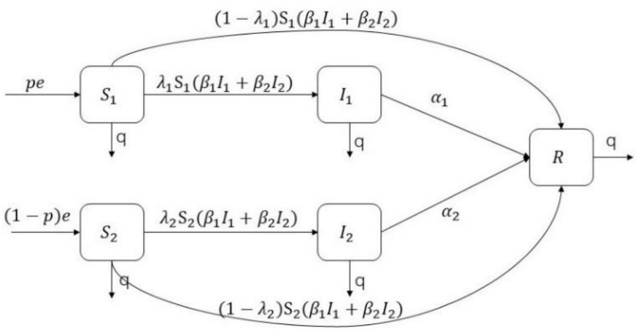


Fig. 1. *The 2S2IR sentiment spreading process*

III. MODEL STABILITY ANALYSIS

A. *The Basic Reproduction Number* $\mathfrak{R}_0$

When there is no sentiment spreading in the system, $I_{1k} = I_{2k} = R_k = 0$, when $t \to \infty$, $\theta \to 0$, by $\begin{cases} \frac{dS_{1k}}{dt} = pe - kS_{1k}(\beta_1\theta_1 + \beta_2\theta_2) - qS_{1k} \\ \frac{dS_{2k}}{dt} = (1-p)e - kS_{2k}(\beta_1\theta_1 + \beta_2\theta_2) - qS_{2k} \end{cases}$, we have $S_{1k}(t) \to p, S_{2k}(t) \to 1-p, (t \to \infty)$. Let the sentiment-free equilibrium $E_0 = \{p, 1-p, 0,0,0\}_k$, so $S_{1k}:S_{2k} = p:(1-p)$, $(t \to \infty)$. Make $\begin{cases} S_{1k} = p(1 - I_{1k} - I_{2k} - R_k) \\ S_{2k} = (1-p)(1 - I_{1k} - I_{2k} - R_k) \end{cases}$, system (1) can be written as

$$\begin{cases} \frac{dI_{1k}}{dt} = k\lambda_1 p(1 - I_{1k} - I_{2k} - R_k)\theta - (\alpha_1 + q)I_{1k} \\ \frac{dI_{2k}}{dt} = k\lambda_2(1-p)(1 - I_{1k} - I_{2k} - R_k)\theta - (\alpha_2 \\ \qquad +q)I_{2k} \\ \frac{dR_k}{dt} = k[(1-\lambda_1)p + (1-\lambda_2)(1-p)](1 - I_{1k} \\ \qquad -I_{2k} - R_k)\theta + \alpha_1 I_{1k} + \alpha_2 I_{2k} - qR_k \end{cases}$$

(2)

Applying the next generation matrix method [32], where the rate of new infections is $\mathcal{F}(x) =$

$$\begin{pmatrix} k\lambda_1 p(1 - I_{1k} - I_{2k} - R_k)\theta \\ k\lambda_2(1-p)(1 - I_{1k} - I_{2k} - R_k)\theta \\ k[(1-\lambda_1)p + (1-\lambda_2)(1-p)](1 - I_{1k} - I_{2k} - R_k)\theta \end{pmatrix}.$$

And the transfer rate of individuals out of compartments is $\mathcal{V}(x) = \begin{pmatrix} (\alpha_1 + q)I_{1k} \\ (\alpha_2 + q)I_{2k} \\ -\alpha_1 I_{1k} - \alpha_2 I_{2k} + qR_k \end{pmatrix}$, the Jacobian matrices of $\mathcal{F}(x)$ and $\mathcal{V}(x)$, at the sentiment-free equilibrium E_0 are as showing

$$F = D\mathcal{F}(E_0) = \begin{pmatrix} F_{11} & F_{12} & 0 \\ F_{21} & F_{22} & 0 \\ F_{31} & F_{32} & 0 \end{pmatrix},$$

$$V = D\mathcal{V}(E_0) = \begin{pmatrix} (\alpha_1 + q)E & 0 & 0 \\ 0 & (\alpha_2 + q)E & 0 \\ -\alpha_1 E & -\alpha_2 E & qE \end{pmatrix}.$$

Where

$$F_{11} = \frac{\lambda_1 p \beta_1}{<k>} \begin{pmatrix} 1\cdot 1\cdot p(1) & 1\cdot 2\cdot p(2) & \cdots & 1\cdot n\cdot p(n) \\ 2\cdot 1\cdot p(1) & 2\cdot 2\cdot p(2) & \cdots & 2\cdot n\cdot p(n) \\ \vdots & \vdots & \ddots & \vdots \\ n\cdot 1\cdot p(1) & n\cdot 2\cdot p(2) & \cdots & n\cdot n\cdot p(n) \end{pmatrix},$$

$F_{12} = \frac{\beta_2}{\beta_1}F_{11}$, $F_{21} = \frac{(1-p)\lambda_2}{p\lambda_1}F_{11}$, $F_{22} = \frac{\beta_2}{\beta_1}F_{21}$, $F_{31} = \frac{[(1-\lambda_1)p+(1-\lambda_2)(1-p)]}{p\lambda_1}F_{11}$, $F_{32} = \frac{\beta_2}{\beta_1}F_{31}$. And E is an identity matrix, 0 indicates zero matrix. It is clear that V is a nonsingular matrix, and F is a nonnegative matrix. According to the concept of next generation matrix, we can get that the basic reproduction number of system (1) equals to $\Re_0 = \rho(FV^{-1}) = \frac{<k^2>[p\beta_1\lambda_1(\alpha_2+q)+(1-p)\beta_2\lambda_2(\alpha_1+q)]}{<k>(\alpha_1+q)(\alpha_2+q)}$, where $<k^2>=\sum k^2 p(k)$. According to [32], we obtain the following theorem.

Th1. If the basic reproduction number $\Re_0 < 1$, the sentiment-free equilibrium E_0 of system (1) is locally asymptotically stable, while if $\Re_0 > 1$,it is unstable.

B. *Global Stability of Sentiment-free Equilibrium*

Th2. If the basic reproduction number $\Re_0 < 1$, the sentiment-free equilibrium E_0 of system (1) is globally asymptotically stable.

Proof. The system (2) can be written as $\frac{dx}{dt} = Ax + z(x)$, where Ax is the linear part, $z(x)$ is the nonlinear part, and

$$z(x) = -\begin{pmatrix} 1\lambda_1 p(I_{11}+I_{21}+R_1)\theta \\ \vdots \\ n\lambda_1 p(I_{1n}+I_{2n}+R_n)\theta \\ 1\lambda_2(1-p)(I_{11}+I_{21}+R_1)\theta \\ \vdots \\ n\lambda_2(1-p)(I_{1n}+I_{2n}+R_n)\theta \\ 1[(1-\lambda_1)p+(1-\lambda_2)(1-p)](I_{11}+I_{21}+R_1)\theta \\ \vdots \\ n[(1-\lambda_1)p+(1-\lambda_2)(1-p)](I_{1n}+I_{2n}+R_n)\theta \end{pmatrix},$$

$A = F - V$. Due to $z(x)$ is negative, thus $\frac{dx}{dt} \le Ax$, the linear system $\frac{dx}{dt} = Ax$ only exist one equilibrium $E_0 = (0,\cdots,0)_{3n}$, from [32], we can see that the eigenvalues of the matrix A all have negative real part when $\Re_0 < 1$, so the linear system is stable, i.e. $I_{1k} \to 0, I_{2k} \to 0, R_k \to 0 (t \to \infty), k = 1,2,\cdots,n$. From the comparison theorem in [33], when $\Re_0 < 1$,the sentiment-free equilibrium E_0 is globally asymptotically stable.

C. *Existence of Positive Equilibrium*

To calculate the positive solution, let

$$\begin{cases} pe - kS_{1k}(\beta_1\theta_1+\beta_2\theta_2) - qS_{1k} = 0 \\ (1-p)e - kS_{2k}(\beta_1\theta_1+\beta_2\theta_2) - qS_{2k} = 0 \\ k\lambda_1 S_{1k}(\beta_1\theta_1+\beta_2\theta_2) - (\alpha_1+q)I_{1k} = 0 \\ k\lambda_2 S_{2k}(\beta_1\theta_1+\beta_2\theta_2) - (\alpha_2+q)I_{2k} = 0 \\ k[(1-\lambda_1)p+(1-\lambda_2)(1-p)](S_{1k}+S_{2k}) \cdot \\ \quad (\beta_1\theta_1+\beta_2\theta_2) + \alpha_1 I_{1k} + \alpha_2 I_{2k} - qR_k = 0 \end{cases},$$

we can get the positive equilibrium point $E^* = (S^*_{1k}, S^*_{2k}, I^*_{1k}, I^*_{2k}, R^*_k)$, where

$$\begin{cases} S^*_{1k} = \frac{pe}{k\theta+q} \\ S^*_{2k} = \frac{(1-p)e}{k\theta+q} \\ I^*_{1k} = \frac{pek\lambda_1\theta}{(\alpha_1+q)(k\theta+q)} \\ I^*_{2k} = \frac{(1-p)ek\lambda_2\theta}{(\alpha_2+q)(k\theta+q)} \\ R^*_k = \frac{k\theta[(\alpha_1+q)(\alpha_2+q)-ep\lambda_1(\alpha_2+q)-e(1-p)\lambda_2(\alpha_1+q)]}{(\alpha_1+q)(\alpha_2+q)(k\theta+q)} \end{cases} \quad (3)$$

Th3. If the basic reproduction number $\Re_0 > 1$,the system (1) exists a unique positive equilibrium $E^* = (S^*_{1k}, S^*_{2k}, I^*_{1k}, I^*_{2k}, R^*_k)$, where the definition of E^* is given by (3).

Proof. Let $\theta = \beta_1\theta_1 + \beta_2\theta_2$, then we have

$$\begin{aligned} \theta = \beta_1\theta_1 + \beta_2\theta_2 &= \frac{1}{<k>}\sum kp(k)(\beta_1 I_{1k} + \beta_2 I_{2k}) \\ &= \frac{1}{<k>}\sum kp(k)\Big(\frac{\beta_1 pek\lambda_1\theta}{(\alpha_1+q)(k\theta+q)} \\ &\quad + \frac{\beta_2(1-p)ek\lambda_2\theta}{(\alpha_2+q)(k\theta+q)}\Big) \\ &= \frac{e\theta}{<k>}\Big[\frac{\beta_1 p\lambda_1}{(\alpha_1+q)} \\ &\quad + \frac{\beta_2(1-p)\lambda_2}{(\alpha_2+q)}\Big]\sum\frac{k^2 p(k)}{(k\theta+q)} = f(\theta). \end{aligned}$$

Derivative of $f(\theta)$ when $\theta = 0$, since $\frac{df(\theta)}{d\theta}|_{\theta=0} = \frac{<k^2>[p\beta_1\lambda_1(\alpha_2+q)+(1-p)\beta_2\lambda_2(\alpha_1+q)]}{<k>(\alpha_1+q)(\alpha_2+q)} = \Re_0$, and $f(\beta_1+\beta_2) \le \beta_1+\beta_2$, so when $\Re_0 > 1$, the system (1) exist a unique positive equilibrium.

D. *The Global Attractivity of Positive Equilibrium*

Lemma 1. When $\Re_0 > 1$,there exist a positive constant $\xi > 0$, such that $\lim_{t\to\infty} inf\{S_k(t), I_{1k}(t), I_{2k}(t), R_{1k}(t), R_{2k}(t)\} \ge \xi$ [34].

Th5. Suppose that $(S_k, I_{1k}, I_{2k}, R_{1k}, R_{2k})$ is a solution of system (1), if $\Re_0 > 1$, then $\lim_{t\to\infty} inf\{S_k(t), I_{1k}(t), I_{2k}(t), R_{1k}(t), R_{2k}(t)\} = (S^*_k, I^*_{1k}, I^*_{2k}, R^*_{1k}, R^*_{2k})$, where $E^*(S^*_k, I^*_{1k}, I^*_{2k}, R^*_{1k}, R^*_{2k})$ satisfying (3) for $k = 1,2,\cdots$.

Proof: According to Th4, there exist a positive constant $0 < \xi < \frac{1}{4}$ and a large enough constant $T > 0$ so that $I_{1k}(t) \ge \xi, I_{2k}(t) \ge \xi, (t > T)$, Therefore, we have $\beta_1+\beta_2 \ge \theta > (\beta_1+\beta_2)\xi, (t > T)$, Substituting it into the first equation of system (1), we get

$$\dot{S_{1k}} \le pe - kS_{1k}(\beta_1+\beta_2)\xi - qS_{1k}, (t > T)$$

By the comparison principle of differential equation, we get $\forall 0 < \xi_1 < \frac{k(\beta_1+\beta_2)\xi+(1-p)e}{2(k(\beta_1+\beta_2)\xi+q)}, \exists t_1 > T$

s.t. $S_k \le X^{(1)}_k - \xi_1, (t > t_1)$,

where $X^{(1)}_k = \frac{pe}{k(\beta_1+\beta_2)\xi+q} + 2\xi_1 < 1$.

From the second equation of system (1), we have

$$\dot{S_{2k}} \le (1-p)e - kS_{2k}(\beta_1+\beta_2)\xi - qS_{2k}, (t > t_1)$$

$\therefore \forall 0 < \xi_2 < min\{\frac{1}{2}, \xi_1, \frac{k(\beta_1+\beta_2)\xi+pe}{2(k(\beta_1+\beta_2)\xi+q)}\}, \exists t_2 > t_1$

s.t. $S_{2k} \le Y^{(1)}_k - \xi_2, (t > t_2)$,

where $Y^{(1)}_k = \frac{(1-p)e}{k(\beta_1+\beta_2)\xi+q} + 2\xi_2 < 1$.

From the third equation of system (1), we have

$$\dot{I_{1k}} \le k\lambda_1(\beta_1+\beta_2)(1-I_{1k}) - (\alpha_1+q)I_{1k}, (t > t_2)$$

$\therefore \forall 0<\xi_3<min\left\{\frac{1}{3},\xi_2,\frac{\alpha_1+q}{2(\lambda_1 k(\beta_1+\beta_2)+\alpha_1+q)}\right\}$,$\exists t_3>t_2$

s.t. $I_{1k}\le Z_k^{(1)}-\xi_3,(t>t_3)$,

where $Z_k^{(1)}=\frac{\lambda_1 k(\beta_1+\beta_2)}{\lambda_1 k(\beta_1+\beta_2)+\alpha_1+q}+2\xi_3<1$.

From the fourth equation of system (1), we have

$$\dot{I_{2k}}\le k\lambda_2(\beta_1+\beta_2)(1-I_{2k})-(\alpha_2+q)I_{2k},(t>t_3)$$

$\therefore \forall 0<\xi_4<min\left\{\frac{1}{4},\xi_3,\frac{\alpha_2+q}{2(\lambda_2 k(\beta_1+\beta_2)+\alpha_2+q)}\right\}$,$\exists t_4>t_3$

s.t. $I_{2k}\le W_k^{(1)}-\xi_4,(t>t_4)$,

where $W_k^{(1)}=\frac{\lambda_2 k(\beta_1+\beta_2)}{\lambda_2 k(\beta_1+\beta_2)+\alpha_2+q}+2\xi_4<1$.

On the other hand, substituting $\theta\le\beta_1+\beta_2$ into the first equation of system (1), we get

$\dot{S_{1k}}\ge pe-k(\beta_1+\beta_2)S_{1k}-qS_{1k},(t>T)$

$\therefore \forall 0<\xi_5<min\left\{\frac{1}{5},\xi_4,\frac{pe}{2(k(\beta_1+\beta_2)+q)}\right\}$,$\exists t_5>t_4$

s.t. $S_{1k}\ge x_k^{(1)}+\xi_5,(t>t_5)$,

where $x_k^{(1)}=\frac{pe}{k(\beta_1+\beta_2)+q}-2\xi_5>0$.

Similarly, from the second equation of system (1), we have $\dot{S_{2k}}\ge(1-p)e-k(\beta_1+\beta_2)S_{2k}-qS_{2k},(t>\mathrm{T})$

$\therefore \forall 0<\xi_6<min\left\{\frac{1}{6},\xi_5,\frac{(1-p)e}{2(k(\beta_1+\beta_2)+q)}\right\}$,$\exists t_6>t_5$

s.t. $S_{2k}\ge y_k^{(1)}+\xi_6,(t>t_6)$,

where $y_k^{(1)}=\frac{(1-p)e}{k(\beta_1+\beta_2)+q}-2\xi_6>0$.

From the third equation of system (1), we have

$$\dot{I_{1k}}\ge\lambda_1 kx_k^{(1)}(\beta_1+\beta_2)\xi-(\alpha_1+q)I_{1k},(t>t_6)$$

$\therefore \forall 0<\xi_7<min\left\{\frac{1}{7},\xi_6,\frac{\lambda_1 kx_k^{(1)}(\beta_1+\beta_2)\xi}{2(\alpha_1+q)}\right\}$,$\exists t_7>t_6$

s.t. $I_{1k}\ge z_k^{(1)}+\xi_7,(t>t_7)$,

where $z_k^{(1)}=\frac{\lambda kx_k^{(1)}(\beta_1+\beta_2)\xi}{\alpha+q}-2\xi_7>0$.

From the fourth equation of system (1), we have

$$\dot{I_{2k}}\ge\lambda_2 ky_k^{(1)}(\beta_1+\beta_2)\xi-(\alpha_2+q)I_{2k},(t>t_7)$$

$\therefore \forall 0<\xi_8<min\left\{\frac{1}{8},\xi_7,\frac{\lambda_2 ky_k^{(1)}(\beta_1+\beta_2)\xi}{2(\alpha_2+q)}\right\}$,$\exists t_8>t_7$

s.t. $I_{2k}\ge w_k^{(1)}+\xi_8,(t>t_8)$,

where $w_k^{(1)}=\frac{\lambda_2 ky_k^{(1)}(\beta_1+\beta_2)\xi}{\alpha_2+q}-2\xi_8>0$.

Since ξ is a constant small enough, obviously : $0<x_k^{(1)}<X_k^{(1)}<1$, $0<y_k^{(1)}<Y_k^{(1)}<1$, $0<z_k^{(1)}<Z_k^{(1)}<1$, $0<w_k^{(1)}<W_k^{(1)}<1$.

Let $m^{(j)}=\frac{1}{\langle k\rangle}\sum kp(k)\left(\beta_1 z_k^{(j)}+\beta_2 w_k^{(j)}\right)$,

$$M^{(j)}=\frac{1}{\langle k\rangle}\sum kp(k)(\beta_1 Z_k^{(j)}+\beta_2 W_k^{(j)}),\mathrm{j}=1,2,\cdots$$

It is clear that : $0<m^{(j)}\le\theta\le M^{(j)}<\beta_1+\beta_2,\mathrm{t}>t_8$.

So, we get $\dot{S_{1k}}\le pe-kS_{1k}m^{(1)}-qS_{1k},(t>t_8)$

By the comparison principle of differential equation, we get $\forall 0<\xi_9<min\left\{\frac{1}{9},\xi_8\right\}$,$\exists t_9>t_8$

s.t. $S_{1k}\le X_k^{(2)}\triangleq min\left\{X_k^{(1)}-\xi_1,\frac{pe}{km^{(1)}+q}+\xi_9\right\},(t>t_9)$.

Similarly, we have

$$\dot{S_{2k}}\le(1-p)e-kS_{2k}m^{(1)}-qS_{2k},(t>t_9)$$

$\therefore \forall 0<\xi_{10}<min\left\{\frac{1}{10},\xi_9\right\}$,$\exists t_{10}>t_9$

s.t. $S_{2k}\le Y_k^{(2)}\triangleq min\left\{Y_k^{(1)}-\xi_2,\frac{(1-p)e}{km^{(1)}+q}+\xi_{10}\right\},(t>t_{10})$.

And then $\dot{I_{1k}}\le\lambda_1 kX_k^{(2)}M^{(1)}-(\alpha_1+q)I_{1k},(t>t_{10})$

$\therefore \forall 0<\xi_{11}<min\left\{\frac{1}{11},\xi_{10}\right\}$,$\exists t_{11}>t_{10}$

s.t. $I_{1k}\le Z_k^{(2)}\triangleq min\left\{Z_k^{(1)}-\xi_3,\frac{\lambda_1 kX_k^{(2)}M^{(1)}}{\alpha_1+q}+\xi_{11}\right\},(t>t_{11})$. From the fourth equation of system (1), we have

$$\dot{I_{2k}}\le\lambda_2 kY_k^{(2)}M^{(1)}-(\alpha_2+q)I_{2k},(t>t_{11})$$

$\therefore \forall 0<\xi_{12}<min\left\{\frac{1}{12},\xi_{11}\right\}$,$\exists t_{12}>t_{11}$

s.t. $I_{2k}\le W_k^{(2)}\triangleq min\left\{W_k^{(1)}-\xi_4,\frac{\lambda_2 kY_k^{(2)}M^{(1)}}{\alpha_2+q}+\xi_{12}\right\},(t>t_{12})$. Turning back to system (1), we can obtain

$$\dot{S_{1k}}\ge pe-kS_{1k}M^{(2)}-qS_{1k},(t>t_{12})$$

$\therefore \forall 0<\xi_{13}<min\left\{\frac{1}{13},\xi_{12},\frac{pe}{2(kM^{(2)}+q)}\right\}$,$\exists t_{13}>t_{12}$

s.t. $S_{1k}\ge x_k^{(2)}+\xi_{13},(t>t_{13})$,

where $x_k^{(2)}=max\left\{x_k^{(1)}+\xi_5,\frac{pe}{kM^{(2)}+q}-2\xi_{13}\right\}$.

It follows that

$$\dot{S_{2k}}\ge(1-p)e-kS_{2k}M^{(2)}-qS_{2k},(t>t_{13})$$

$\therefore \forall 0<\xi_{14}<min\left\{\frac{1}{14},\xi_{13},\frac{(1-p)e}{2(kM^{(2)}+q)}\right\}$,$\exists t_{14}>t_{13}$

s.t. $S_{2k}\ge y_k^{(2)}+\xi_{14},(t>t_{14})$,

where $y_k^{(2)}=max\left\{y_k^{(1)}+\xi_6,\frac{(1-p)e}{kM^{(2)}+q}-2\xi_{14}\right\}$.

From the third equation

$$\dot{I_{1k}}\ge\lambda_1 kx_k^{(2)}m^{(1)}-(\alpha_1+q)I_{1k},(t>t_{14})$$

$\therefore \forall 0<\xi_{15}<min\left\{\frac{1}{15},\xi_{14},\frac{\lambda_1 kx_k^{(2)}m^{(1)}}{2(\alpha_1+q)}\right\}$,$\exists t_{15}>t_{14}$

s.t. $I_{1k}\ge z_k^{(2)}+\xi_{15},(t>t_{15})$,

where $z_k^{(2)}=max\left\{z_k^{(1)}+\xi_7,\frac{\lambda_1 kx_k^{(2)}m^{(1)}}{\alpha_1+q}-2\xi_{15}\right\}$.

Then $\dot{I_{2k}}\ge\lambda_2 ky_k^{(2)}m^{(1)}-(\alpha_2+q)I_{2k},(t>t_{15})$

$\therefore \forall 0<\xi_{16}<min\left\{\frac{1}{16},\xi_{16},\frac{\lambda_2 ky_k^{(2)}m^{(1)}}{2(\alpha_2+q)}\right\}$,$\exists t_{16}>t_{15}$

s.t. $I_{2k}\ge w_k^{(2)}+\xi_{16},(t>t_{16})$,

where $w_k^{(2)}=max\left\{w_k^{(1)}+\xi_8,\frac{\lambda_2 ky_k^{(2)}m^{(1)}}{\alpha_2+q}-2\xi_{16}\right\}$.

Similarly, we carry out step $\mathrm{h}(\mathrm{h}\ge 3)$ of the calculation and get ten sequences: $\left\{X_k^{(h)}\right\}$, $\left\{Y_k^{(h)}\right\}$, $\left\{Z_k^{(h)}\right\}$, $\left\{W_k^{(h)}\right\}$, $\left\{x_k^{(h)}\right\}$, $\left\{y_k^{(h)}\right\}$, $\left\{z_k^{(h)}\right\}$, $\left\{w_k^{(h)}\right\}$. Obviously, the first four sequences are monotone decreasing and the last four sequences are monotone increasing. Thus, there exist a large enough positive integer G, s.t. when $\mathrm{h}\ge\mathrm{G}$ we have $X_k^{(h)}=\frac{pe}{km^{(h-1)}+q}+\xi_{8h-7}$, $Y_k^{(h)}=\frac{(1-p)e}{km^{(h-1)}+q}+\xi_{8h-6}$, $Z_k^{(h)}=\frac{\lambda_1 kX_k^{(h)}M^{(h-1)}}{\alpha_1+q}+\xi_{8h-5}$, $W_k^{(h)}=\frac{\lambda_2 kY_k^{(h)}M^{(h-1)}}{\alpha_2+q}+\xi_{8h-4}$, $x_k^{(h)}=\frac{pe}{kM^{(h)}+q}-2\xi_{8h-3}$, $y_k^{(h)}=\frac{(1-p)e}{kM^{(h)}+q}-2\xi_{8h-2}$, $z_k^{(h)}=\frac{\lambda_1 kx_k^{(h)}m^{(h-1)}}{\alpha_1+q}-2\xi_{8h-1}$, $w_k^{(h)}=\frac{\lambda_2 ky_k^{(h)}m^{(h-1)}}{\alpha_2+q}-2\xi_{8h}$.

Obviously, $x_k^{(h)}\le S_{1k}\le X_k^{(h)}$, $y_k^{(h)}\le S_{2k}\le Y_k^{(h)}$, $z_k^{(h)}\le I_{1k}\le Z_k^{(h)}$, $w_k^{(h)}\le I_{2k}\le W_k^{(h)}$, and the limit of the above sequence exists, let $\lim_{\mathrm{h}\to\infty}\Omega_k^{(h)}=\Omega_k$,where $\Omega_k^{(h)}=\left\{X_k^{(h)},Y_k^{(h)},Z_k^{(h)},W_k^{(h)},x_k^{(h)},y_k^{(h)},z_k^{(h)},w_k^{(h)}\right\}$, $\Omega_k=\{X_k,Y_k,Z_k,W_k,x_k,y_k,z_k,w_k\}$. For $0<\xi_h<\frac{1}{h}$, when $\mathrm{h}\to\infty,\xi_h\to 0$.Thinking $\mathrm{h}\to\infty$ by directly calculating, we get $X_k=\frac{pe}{km+q}$, $Y_k=\frac{(1-p)e}{km+q}$, $Z_k=\frac{\lambda_1 k X_k M}{\alpha_1+q}$, $W_k=\frac{\lambda_2 k Y_k M}{\alpha_2+q}$, $x_k=\frac{pe}{kM+q}$, $y_k=\frac{(1-p)e}{kM+q}$, $z_k=\frac{\lambda_1 k x_k m}{\alpha_1+q}$, $w_k=\frac{\lambda_2 k y_k m}{\alpha_2+q}$. Let $\mathrm{M}=\frac{1}{\langle k\rangle}\sum kp(k)\left(\beta_1 Z_k+\beta_2 W_k\right)$, $\mathrm{m}=\frac{1}{\langle k\rangle}\sum kp(k)\left(\beta_1 z_k+\beta_2 w_k\right)$.

So $\mathrm{M}=\frac{1}{\langle k\rangle}\sum kp(k)\left(\beta_1\frac{\lambda_1 kM}{\alpha_1+q}\frac{pe}{km+q}+\beta_2\frac{\lambda_2 kM}{\alpha_2+q}\frac{(1-p)e}{km+q}\right)$, i.e. $1=\frac{1}{\langle k\rangle}\sum kp(k)\frac{ke}{(km+q)}\frac{\beta_1\lambda_1 p(\alpha_2+q)+\beta_2\lambda_2(1-p))(\alpha_1+q)}{(\alpha_1+q)(\alpha_2+q)}$

(4)

and $\mathrm{m}=\frac{1}{\langle k\rangle}\sum kp(k)\left(\beta_1\frac{k\lambda_1 m}{\alpha_1+q}\frac{pe}{kM+q}+\beta_2\frac{k\lambda_2 m}{\alpha_2+q}\frac{(1-p)e}{kM+q}\right)$, so $1=\frac{1}{\langle k\rangle}\sum kp(k)\frac{ke}{(kM+q)}\frac{\beta_1\lambda_1 p(\alpha_2+q)+\beta_2\lambda_2(1-p))(\alpha_1+q)}{(\alpha_1+q)(\alpha_2+q)}$

(5)

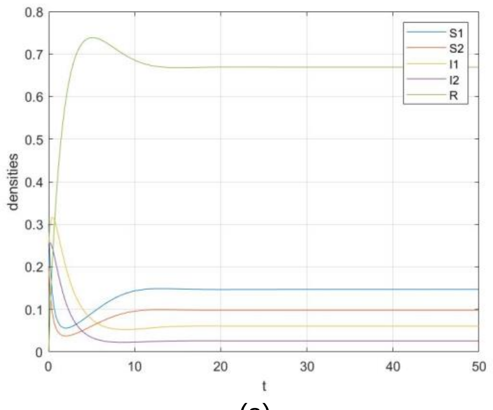
By (4)-(5), we obtain

$$0=(\mathrm{M}-\mathrm{m})\frac{\beta_1\lambda_1 p(\alpha_2+q)+\beta_2\lambda_2(1-p))(\alpha_1+q)}{(\alpha_1+q)(\alpha_2+q)}\frac{e}{\langle k\rangle}\cdot\sum k^3 p(k)\frac{1}{(kM+q)(km+q)}.$$

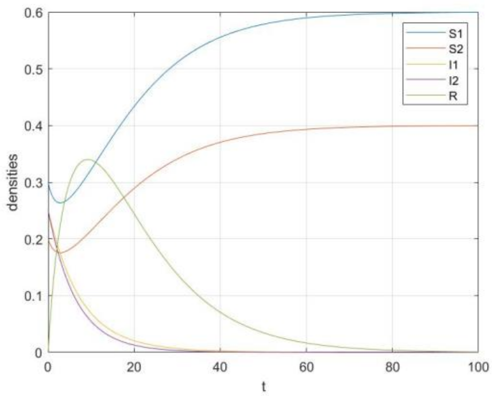
So, we get that $\mathrm{M}=\mathrm{m}$, i.e. $z_k=Z_k$, $w_k=W_k.\mathrm{k}=1,2,\cdots$. So $\lim_{t\to\infty}S_{1k}(t)=X_k=x_k$, $\lim_{t\to\infty}S_{2k}(t)=Y_k=y_k$, $\lim_{t\to\infty}I_{1k}(t)=Z_k=z_k$, $\lim_{t\to\infty}I_{2k}(t)=W_k=w_k$. Finally $X_k=S_{1k}^*, Y_k=S_{2k}^*, Z_k=I_{1k}^*, W_k=I_{2k}^*$. Due to $R_k=1-S_{1k}-S_{2k}-I_{1k}-I_{2k}$, therefore $\lim_{t\to\infty}R_k(t)=R_k^*$. so $E^*(S_{1k}^*,S_{2k}^*,I_{1k}^*,I_{2k}^*,R_k^*)$ is global attractivity.

IV. NUMBERICAL SIMULATIONS

In this section, some numerical simulations were performed to analyze the process of investor sentiment spreading and verify the theory presented in the previous section. We first establish a heterogeneous network and set the relevant parameters as: $p=0.6, e=q=0.1, \beta_1=0.8, \beta_2=0.6, \lambda_1=0.8, \lambda_2=0.6, \alpha_1=0.5, \alpha_2=0.6$, under this circumstances, the basic reproduction number $\Re_0>1$, the density with time of the five groups is shown in Fig. 2 (a). It can be seen from the figure that when $\Re_0>1$, in the final state, the sentiment spreader persists and maintains a relatively stable state. In Fig. 2 (b) shows the parameters are set as: $p=0.6, e=q=0.1, \beta_1=0.1, \beta_2=0.05, \lambda_1=0.1, \lambda_2=0.05, \alpha_1=0.05, \alpha_2=0.06$, the $\Re_0<1$, in the final state, the sentiment spreader and stifler disappear, and only the susceptible exists, that is, there is no sentiment spreading in the system.



(a)



(b)

Fig. 2. *The densities of five groups with time when $\Re_0>1$ (a) and $\Re_0>1$ (b)*

From the Fig. 2 (a), we can see that when $\Re_0>1$, in the initial state S_1, S_2 exist in the ratio of $p:1-p$, the densities of sentiment susceptible firstly decrease sharply and then rise to a steady value, and in the process of sentiment spreading, the density of active and passive susceptible is always $p:1-p$. The densities of two kinds of spreaders rapidly sour to the peak and then fall a steady value, the density of active spreaders is greater than the passive spreaders under steady state. This is due to the new arrivals of active accounted for a larger, and in the process of sentiment spreading the active one is strong in the spreading and acceptance of sentiment. The density of stiflers increased rapidly and reached the peak, then decreased gradually and became stable. When $\Re_0<1$, because the values of the propagation parameter and the acceptance parameter are small, the densities of susceptible decreases first and then increases gradually, finally exists in the system with the density of p and $1-p$, the spreader's density decreases gradually and finally equals to zero, the density of stiflers increases to a peak, then decreases, and finally disappears into the system. As we all know, the greater the capacity to spread and receive sentiment, the greater the chance that a susceptible will transfer to a spreader, so as the parameters $\beta_1,\beta_2,\lambda_1,\lambda_2$ increase, the final proportion of spreaders in the system will increase, as the parameters α_1,α_2 increase, the final density of stiflers will increase. Therefore, external intervention to suppress or encourage the corresponding sentiment spreading can make the system in a positive sentiment prevalence and negative sentiment disappeared.

Fig. 3 shows the densities curve of five groups in the system as the degree of nodes k changes. It can be seen from the figure that as the degree k increases, the density of the two types of susceptible gradually decreases, and the peak value of two types of spreaders gradually increases, and the final density increases very small. Therefore, it is believed that the degree k has basically no effect on the density of the sentiment spreaders, and the greater the degree, the susceptible more easily to accept sentiment and transfer into spreaders. The greater the degree k, the greater the proportion of stiflers in the final state, that is, the more people in the system do not spread or accept sentiment.

Next, we will discuss the influence of the personality parameter p on the spreading process. As shown in Fig. 4, as p changes, the ratio of S_1 and S_2 is always $p:1-p$, but the ratio of I_1 and I_2 is changing during the spreading process, With the increase of p, the density of passive spreaders changes small, while the active spreaders gradually increases. And there is also a slight increase in the density of stiflers at final state. Therefore, the larger the proportion of active individuals among the new entrants, the greater the density of active sentiment spreaders in the final state, and the lower the density of passive spreaders. The proportion of active individuals increases can conducive to the further spreading of sentiment.

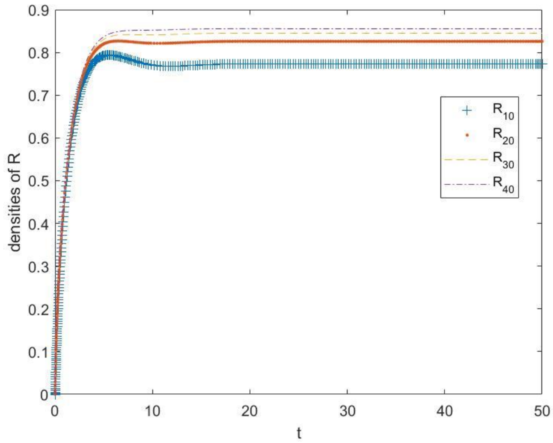
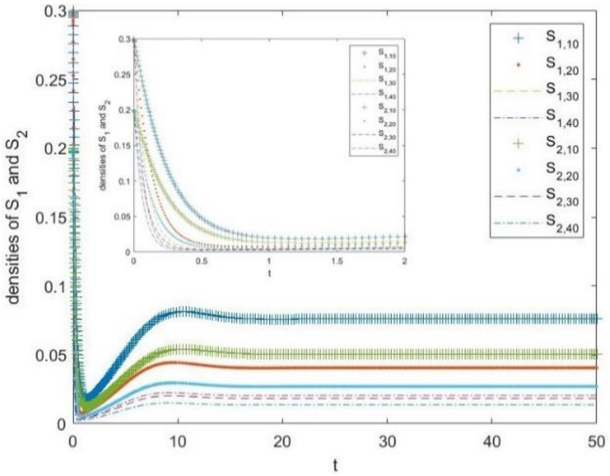
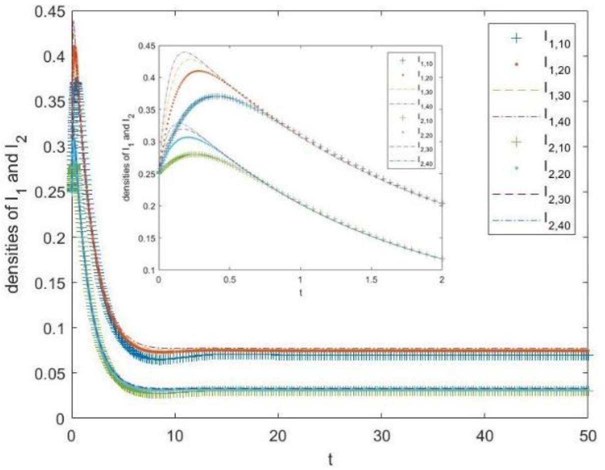
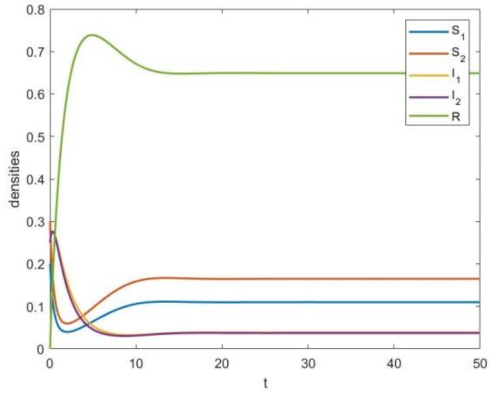


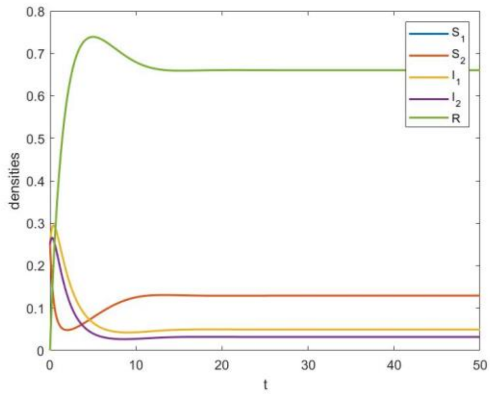
Fig. 3. *The densities if five groups with different degrees*



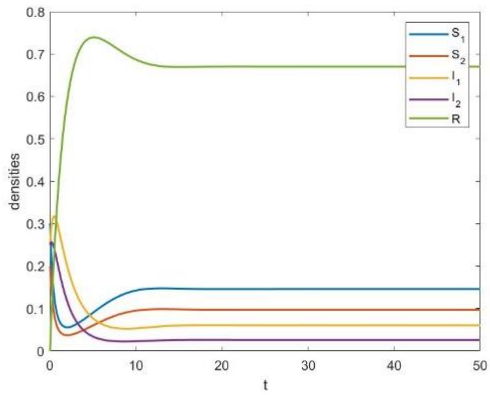




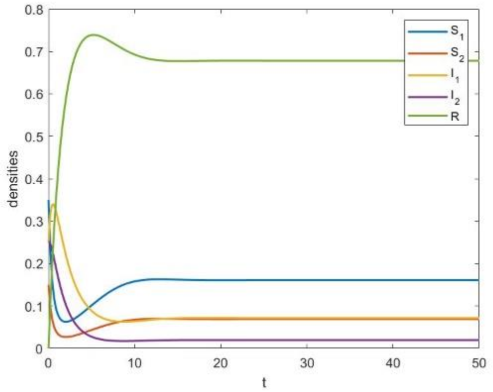
$p=0.4$



$p=0.5$



$p=0.6$



$p = 0.7$

Fig. 4. *The densities of five groups with different personality parameter* p

V. CONCLUSION

Sentiment spreading is a complicated process. In this paper, we have improved the classic epidemic model and considered individual differences in personality established an investor sentiment 2S2IR model. Dividing individuals in the stock market into five categories, which have different abilities to accept or spread emotions.

Through numerical analysis and proof, we conclude that when the system's basic reproduction number $\Re_0 < 1$, there is no sentiment spreaders in the system, that is, there is no sentiment spreading in the system, and the sentiment-free equilibrium is globally stable. When $\Re_0 > 1$, the system exists a unique positive equilibrium point, that is, the sentiment spreaders continue to exist in the steady state, and the positive equilibrium point is globally attractive.

In this paper, a heterogeneous network is used for numerical simulation. The results confirm that the basic reproduction number is the threshold for sentiment spreading, that is, when $\Re_0 < 1$ the sentiment will disappear in the system, when $\Re_0 > 1$ the sentiment will be broken out. The following conclusions are also obtained: 1. Increasing the spreading parameters and acceptance parameters is conducive to the transmission of sentiment, so through the relevant external interventions, the sentiment spreading and acceptance parameters can be adjusted to achieve the purpose of encouraging or inhibiting sentiment transmission. 2. The greater the degree, the fewer the susceptible in the stable state, and the greater the number of spreaders and stiflers. Therefore, the proportion of five groups in the stable state is controlled by the behaviors and concepts of the people with greater influence. 3. Personality parameters have a great influence on the process of sentiment spreading. The increase in personality parameter p can increase the number of active sentiment spreaders. Therefore, increasing the proportion of active individuals among new entrants can speed up the transmission of sentiment. Conversely, it inhibits.

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