Markov-based Comparative Analysis for Overtime Rule of the National Football League

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***Abstract*— This paper presents a Markov-based comparative analysis model to provide insight on whether the new overtime rule of the US National Football League is fairer than the sudden death format. In 2012, the National Football League instituted new rules governing overtime play in the event of a tie at the end of regulation. The presented model analyzes whether the new rules achieved their desired effect of creating more parity within the National Football League overtime system. Even though the game has evolved over the years, the desired effect of the new overtime rule was to minimize the advantage previously given to the receiving team to possess in the overtime period. This research also examined what effects a change to a 10-minute overtime period or a National Collegiate Athletic Association style system would have on game outcomes.**

Keywords—Markov chain; National Football League; comparative analysis; overtime rule

I. Introduction

Statistical analysis is present in a number of commonplace applications, such as sports, webpages on Google, and lots of recommended industrial systems. When it comes to sports, the analysis of teams depends highly on statistical methods to provide a value of the outcome to each team or individual in respect to the other alternatives. A particular team or individual's data feeds into an analytical model, which establishes the ratings and enables a prediction model to be created for an evaluation [1]. In a broad scope, most of sports analytics can be viewed as a pairwise comparison, in which a stronger alternative is always preferred to a weaker alternative [2]. This assumes a quality of absoluteness to each alternative's ranking. However, in reality, there are predictive uncertainties to the value of the outcome. Upsets within any sports Markov system are prone to occur because each game, match, and season have probabilities associated within the state of the team in any particular instance, which can also be considered a system [1]. These probabilities feed into the potential outcome of a game, but even the most devout sports followers and mathematicians cannot predict the outcome of games consistently enough to beat the odds. In the sports gambling world, bettors need to win 52.4% of their games to break even because the house takes 10% of the cut [3]. Making the correct bet half of the time does not sound very daunting, but even professionals struggle to make enough winning bets to generate income. In the National Football League (NFL), which is what this research will be examining, a field goal in an overtime possession could be the difference between losing money or beating the system. This allure of predicting the future is why discrete-time models are heavily utilized in sporting events.

II. Problem Statement

The problem being addressed is the 2012 NFL change in overtime rules. In the NFL, the overtime rule has always been controversial. Before 2012, the NFL overtime was in a sudden death format, with the first team to score automatically winning. In this format, the winner of the coin toss is significantly more likely to win the game than the loser of the coin toss. In fact, from 2000 – 2007, there were 124 overtime games, and the team who received the first kick-off (won the coin toss) won 60% of the games. Also, in these 124 overtime games, one team won on their first possession (without the other team touching the ball) 30% of the time [3]. The newer sudden death rule allows for each team to get at least one possession unless the team to receive the ball first scores a touchdown. This rule was implemented to make the NFL overtime fairer, but did the rule change achieve the desired effect?

In this short paper, therefore, we present a simple yet practical optimization approach, which employs the concept of Markov chain, to analyze the transition probabilities of an NFL overtime to determine whether the new format is more unbiased than the old format. Using a discrete-time Markov Chain, the likelihood of scoring on any given possession of a 15-minute overtime can be determined and analyzed between teams. The model will predict the outcome of the game and will look at the underlying advantage gained by winning the initial coin toss. A comparison will be made between both models to analyze the difference the new overtime model has made for both teams.

III. Model Formulation

A Markov chain is defined as a system of states that transitions from one state to another. In this model, we define four possible states for teams "Blue" and "Red." Team Blue is the team that possesses the ball first in the overtime period, which is normally the winner of the toss. Thus, two transient states are defined as the team that has possession of the ball [6]. The system can transition between these states if there is a turnover or if the team in possession punts. The other two states are defined as either team winning the game, and are therefore absorbing states. The probability of moving from state i to state j is denoted as P_{ij}. The probabilities P_{ij} are called transition probabilities, and they generate the probability distribution of the transition matrix. With this particular overtime example, the Markov chain is a discrete-time event in which each possession is a separate event $\{X_n, n=0,1,2,3\ldots\}$. For this model, we assume stationary transition probabilities for each offensive possession. If a Markov chain is regular, then the transition probability matrix takes the form of Equation (1) (see [7]):

$$\lim_{n\to\infty} P_{ij}^{(n)} = \begin{bmatrix} \pi_1 & \pi_2 & . & . & . & \pi_N \\ \pi_1 & \pi_2 & . & . & . & \pi_N \\ . & . & & & & . \\ . & . & & & & . \\ . & . & & & & . \\ \pi_1 & \pi_2 & . & . & . & \pi_N \end{bmatrix} = \pi_j \qquad (1)$$

The limiting (or steady-state) probabilities must also satisfy the following long-run behavior conditions:

$$\sum_j \pi_j = 1 \qquad (2)$$

$$\pi_j = \sum_i \pi_i P_{ij} \qquad (3)$$

$$\pi_j > 0 \qquad (4)$$

Using this format, the outcome of any given possession in overtime can be denoted by its own transition probability matrix. However, since the nature of a Markov chain means its present state is independent of the past events, the model formulation for this overtime analysis does not include the momentum a team may carry going into an overtime or the change in decision-making by the coach to a more conservative style. With sudden death, a coach would be more apt to settle for a field goal over a touchdown because they're weighed the same, which requires less yardage to be gained than driving it up the whole field for a touchdown. It also does not take into account the skill level of the teams entering overtime, meaning one team could be undefeated and the other could have zero wins on the season [6]. This model will simply be an unbiased examination of the old and current overtime format and the potential advantage of winning the initial coin toss and receiving the first possession. Below in Equation (5), the basis for the transition matrix is shown for each potential overtime possession [7]. To find the steady-state probabilities, or probability of victory after "n" possessions, the matrix is evaluated as it approaches infinity. However, in a traditional 15 minute overtime with the average length of possession being about 2.5 minutes, a maximum of three possessions per team, or six possessions total, can be assumed for the purpose of the transition matrices [10].

$$P^n = \begin{bmatrix} P_{00}^n & P_{01}^n & . & . & . & P_{0M}^n \\ P_{10}^n & P_{11}^n & . & . & . & P_{1M}^n \\ . & . & & & & . \\ . & . & & & & . \\ . & . & & & & . \\ P_{M0}^n & P_{M1}^n & . & . & . & P_{MM}^n \end{bmatrix}$$

For the old overtime model, the transition matrix will be comprised of data from the 2007-2011 NFL seasons, which was the five years leading up to the new overtime model being implemented in 2012. During this stretch, there was an average of 5,786 offensive drives per year. There was also an average of 1,132 offensive touchdowns, either by run or pass, and 805 field goals being scored [6]. If these offensive scoring possessions are added together and factored into the average number of possessions per season, we find that 33.4% of offensive possessions end in a scoring drive. In the old sudden death overtime format, any of these scoring drives means the end of the game. This 33.4% is close to the 30% value found with one possession overtime scores, and the difference can be accounted by the conservative nature of overtime decisions.

There are other aspects of the game that need to be evaluated in the model formulation process. The non-scoring ones include average number of punts, turnover-on-downs (i.e., getting a stop on fourth down), turnovers that do not end in a scoring play (fumbles lost or an interception), and missed field goals. Of the 5,786 offensive drives, 2,402 ended in a punt, 254 ended in a turnover-on-downs, 168 resulted in missed field goals, and 769 ended in a non-scoring turnover during that five-year timeframe [8]. The defensive scoring aspects to incorporate are fumbles returned for a touchdown, interceptions returned for a touchdown, and safeties (getting tackled in the offense's own end zone). From 2007-2011, there was an average of 81 scoring plays from a fumble or interception and 17 from a safety [8]. The only other way to score is found within special team play, otherwise known as kick and put returns. The scoring and defensive plays being evaluated result in the following transition probabilities:

$$\textit{Change of Possession (\%)} = \frac{2,402+254+168+769}{5,786} = 62.1\%$$

$$\textit{Defense Winning with a Turnover (\%)} = \frac{81+17}{5,786} = 1.70\%$$

These transition possibilities allow the old overtime system to be analyzed in a Team A and Team B possession matrix. Fig. 1 shows a transition diagram for the possible states of this overtime system. Since the probability of returning and punt or kickoff is not being evaluated, those probabilities will be absorbed in the percentage of plays that end in a scoring possession. This will increase the probability of scoring from 33.4% to 36.2%

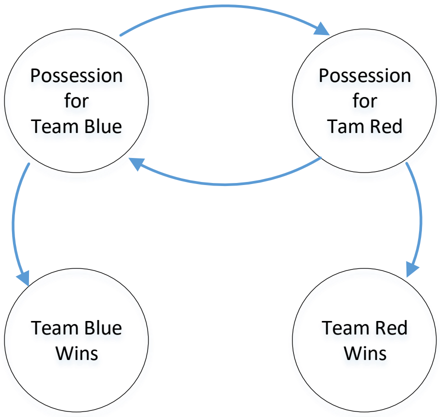

Fig. 1: Transition diagram

Table 1. Transition Probabilities for Old Overtime

	Possession for Team Blue	Possession for Team Red	Team Blue Wins	Team Red Wins
Possession for Team Blue	0	0.621	0.362	0.0170
Possession for Team Red	0.621	0	0.0170	0.362
Team Blue Wins	0	0	1	0
Team Red Wins	0	0	0	1

Regarding the transition probabilities for the new overtime model, the overtime rules implemented in 2012 changed the format from sudden death to guaranteeing each team an offensive possession. The only exception to this is if a touchdown is scored on the first possession. The ability for each team to have a possession means coaches are forced to consider less conservative offensive drives, since a field goal does not guarantee the team a win. This potential change in coaching strategy, as well as the natural evolution of the game, likely means different transition probabilities, so data from the 2012-2016 season are analyzed in this model formulation. On the offensive end, there were 5,971 offensive possessions, 1,199 passing or running touchdowns scored, and 846 field goals scored. During this period on the defensive end, 2,431 drives ended in a forced punt, 238 ended in a turnover-on-downs, 154 resulted in missed field goals, and 681 ended in either an interception or fumble [8]. (6) The numbers for this period not only indicate an increase in offensive productivity, but they show more of an inclination towards scoring a touchdown over a (7) field goal. This less conservative approach will be reflected in the new model's transition probability matrix. The new matrix will undergo a same model formulation process in its initial steps, but since teams behave differently when they need more than a field go to win, different probabilities will be reflected in the matrix. There will also be the consideration that if a team scores a field goal on their possession, the conservative approach of the sudden death overtime format will be a part of the coaching strategy. The probabilities of a touchdown, field, goal, and a change of possession are featured below:

$$\textit{Change of Possession (\%)} = \frac{2,431+238+154+681}{5,786} = 58.7\%$$

$$\textit{Defense Winning with a Turnover (\%)} = \frac{82+18}{5,971} = 1.70\%$$

$$\textit{Field Goal (\%)} = \frac{846}{5,971} = 14.2\%$$

$$\textit{Touchdown (\%)} = \frac{1,199}{5,971} = 20.1\%$$

The possible states of the new overtime format include the probabilities listed above, but it is not limited to just these possibilities. The transition probabilities from T_2 must be used to estimate the probability of a score of any kind and a change of possession after one offensive possession. This is applicable because both teams could be guaranteed at least one possession.

IV. Results and Discussion

The matrix for both overtime models assumes that the game will not end on a kickoff or a punt return, so the probabilities listed are not reflective of those outcomes. To begin the comparative analysis of the old overtime's model and the new one, the probabilities found in Table 1 are converted into the 1-stage transition probability matrix, T_1. It is the initial transition matrix for Old Overtime and the first row of the matrix refers to the probability of being in any given situation if Team Blue starts with the ball. To find the probabilities after the *n* possession, take matrix T_1 to the n^{th} power and continue to read along the top row.

$$T_1^1 = \begin{bmatrix} 0 & 0.621 & 0.362 & 0.170 \\ 0.621 & 0 & 0.170 & 0.362 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{12}$$

Equation (13) highlights that after three possessions, the probability of Team Blue (first receiving team) having won is 51.2%, while the probability of Team Red (first defending team) having won is 24.8%. However, it is also shown that there is a 24.0% probability the game is not won within these first three alternating possessions and Team Red is about ready to start the fourth overtime possession. If the sudden death game is still tied after three alternating possessions, odds are the game will still have three more before the overtime period ends. Through a potential maximum of six possessions, Equation (14) shows that Team Blue has a 57.2% chance of winning and Team Red has a 37.1% chance of winning. There is also a 5.70% chance of the game ending in a tie, which is near the 3.6% of actual overtime games to end in a tie from 1974-2011 under the sudden death format.

$$T_1^3 = \begin{bmatrix} 0 & 0.240 & 0.512 & 0.248 \\ 0.240 & 0 & 0.248 & 0.512 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{13}$$

$$T_1^6 = \begin{bmatrix} 0.0570 & 0 & 0.572 & 0.371 \\ 0 & 0.0570 & 0.371 & 0.572 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{14}$$

Now, to be able to compare the legacy overtime model to the overtime model implemented in 2012, the available states will be expanded to reflect the guaranteed possession of the kicking team if the first receiving team does not score a touchdown. This includes the potential to score a touchdown on the first offensive possession of the overtime, go back and forth with field goals in the first two drives, and not scoring until later in the overtime. Again, scoring off of a kickoff and punt return is not being evaluated as its own separate event, so it will be added to the probability of scoring a touchdown. This probability goes from 20.1% to 25.4%. Another assumption will be made to integrate the probability of a tie. This probability will be absorbed in the change of the other team evening up the score, which means the value of 57.2% will go up to 61.2%. This more accurately reflects the real probabilities found in overtime games from 2000-2007, with the receiving team winning 60% of the time [3]. Below, Equation (16) depicts the potential change in overtime scenarios and the different coaching styles reflective of the new overtime.

Table 2. Transition Probabilities for New Overtime

	Blue 0-0	Red 0-0	Blue 3-0	Red 3-0	Blue 0-3	Red 0-3	Blue 3-3	Red 3-3	Blue Wins	Red Wins
Blue 0-0	0	0.587	0	0.142	0	0	0	0	0.254	0.0170
Red 0-0	0.587	0	0	0	0.142	0	0	0	0.0170	0.254
Blue 3-0	0	0	0	0.612	0	0	0	0	0.371	0.0170
Red 3-0	0	0	0.587	0	0	0	0.142	0	0.0170	0.254
Blue 0-3	0	0	0	0	0	0.587	0	0.142	0.254	0.0170
Red 0-3	0	0	0	0	0.612	0	0	0	0.0170	0.371
Blue 3-3	0	0	0	0	0	0	0	0.612	0.371	0.0170
Red 3-3	0	0	0	0	0	0	0.612	0	0.0170	0.371
Blue Wins	0	0	0	0	0	0	0	0	1	0
Red Wins	0	0	0	0	0	0	0	0	0	1

Table 2 reads similarly to the figure reflected in the old overtime, with the only difference being in the aspects of the game (touchdowns, field goals, change of possession) that impact the new overtime format on an individual basis. Putting this table in a Markov chain and raising it to the sixth power allows for the comparison of the two overtime models upon the completion of overtime. In Equation (15), the top row highlights that the probability Team Blue (receiving team) has won after six possessions is 50.1%, as compared to the 57.2% value found in the old overtime model. Team Red's (first defending team) chances of winning after six possessions has also increased slightly from 37.1% to 37.3%. The decreased advantage for Team Blue can be attributed to the guarantee of two possessions if no touchdown has been scored. The ratio between the receiving and defending team has become more favorable, which was the goal as well. In the old overtime model, there was a 20.1% difference favoring the receiving team, but in the solution to the new overtime model, there is only a 12.8% difference (still favoring the receiving team). This model is also within 2% of predicting the likelihood of there being a tie, which is approximately 6% of overtime games since 2012 and is represented by the 4.1% value in the model [9]. The two subsets of data used for the separate models, 2007-2011 and 2012-2016, could have also affected the model outcome because of the evolving nature of the game and coaching styles.

$$T_2^6 = \begin{bmatrix} 0.041 & 0 & 0.031 & 0 & 0.031 & 0 & 0.023 & 0 & 0.501 & 0.373 \\ 0 & 0.041 & 0 & 0.031 & 0 & 0.031 & 0 & 0.023 & 0.373 & 0.501 \\ 0 & 0 & 0.046 & 0 & 0 & 0 & 0.035 & 0 & 0.625 & 0.293 \\ 0 & 0 & 0 & 0.046 & 0 & 0 & 0 & 0.035 & 0.465 & 0.454 \\ 0 & 0 & 0 & 0 & 0.030 & 0 & 0.035 & 0 & 0.454 & 0.465 \\ 0 & 0 & 0 & 0 & 0 & 0.046 & 0 & 0.035 & 0.293 & 0.625 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.052 & 0 & 0.578 & 0.370 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.052 & 0.370 & 0.578 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Another exampled embedded within this problem is the evaluation of the playoff game overtime system for professional football, college football, and any rule changes within those overtime systems. For the college football playoff, each team is guaranteed a possession that starts on 25 yard line, the order of which is decided by a coin toss before the overtime period. If the teams remain tied after individually having two overtime possessions, they have to go for two point conversions instead of an extra point after a touchdown. Each of these coaching scenarios, as well as each individual red zone drive (a drive that gets within 20 yards of the goal, has its own associated probabilities and can be analyzed using a Markov chain technique being utilized in this research. The college football overtime format has been considered in the NFL game as well because it supports a more aggressive and exciting play. If the first offensive possession ends in a touchdown, then the other team is forced to coach away from settling for a field goal. This alone would generate an entirely different transition probabilities matrix and is considered to be fairer than the NFL overtime. Since the basis for the professional model is already on-hand, the transition matrix for the new overtime model will be used to examine the probabilities of the playoff format. The difference between the playoff and regular season format is that in the playoffs, there is no time limit in overtime because there needs to be a winning team that advances to the next round. Therefore, the limit to this matrix will approach infinity and will be evaluated at its steady-state. It will show the overall advantage the receiving team has after winning the coin toss in overtime. It is unlikely a team will even approach six possessions, let alone come close to the steady-state model, but there is still an advantage in receiving the coin toss. On the following page, Figure 9 depicts the new overtimes' mode in its steady-state format.

$$\lim_{n\to\infty}(T_3)^n = \begin{bmatrix} 0&0&0&0&0&0&0&0&0.575&0.425 \\ 0&0&0&0&0&0&0&0&0.425&0.575 \\ 0&0&0&0&0&0&0&0&0.678&0.322 \\ 0&0&0&0&0&0&0&0&0.502&0.498 \\ 0&0&0&0&0&0&0&0&0.498&0.502 \\ 0&0&0&0&0&0&0&0&0.322&0.678 \\ 0&0&0&0&0&0&0&0&0.610&0.390 \\ 0&0&0&0&0&0&0&0&0.390&0.610 \\ 0&0&0&0&0&0&0&0&1&0 \\ 0&0&0&0&0&0&0&0&0&1 \end{bmatrix} \quad (16)$$

As show in Equation (16), the advantage to the receiving team is 57.5% - 42.5%. This is comparable to the 57.2% advantage found in Equation (14) when any possession will win. In reality, there has been 87 game since the NFL has instituted this new overtime rule, and 54.8% have been ties [9].

Another rule change the NFL has been considering for its overtime format is shortening the length of overtime from 15 minutes to 10 minutes [9]. In order to analyze this example, the number of offensive possessions completed must be reduced from six total to four, otherwise known as two per team. To highlight the probabilities associated with this change, the Markov chain in Table 2 must be considered at the fourth power that is shown in Equation (17).

$$T_2^4 = \begin{bmatrix} 0.119&0&0.059&0&0.059&0&0.029&0&0.420&0.315 \\ 0&0.119&0&0.059&0&0.059&0&0.029&0.315&0.420 \\ 0&0&0.129&0&0&0&0.064&0&0.552&0.259 \\ 0&0&0&0.129&0&0&0&0.064&0.412&0.395 \\ 0&0&0&0&0.129&0&0.064&0&0.395&0.412 \\ 0&0&0&0&0&0.129&0&0.064&0.256&0.552 \\ 0&0&0&0&0&0&0.140&0&0.524&0.336 \\ 0&0&0&0&0&0&0&0.140&0.336&0.524 \\ 0&0&0&0&0&0&0&0&1&0 \\ 0&0&0&0&0&0&0&0&0&1 \end{bmatrix}$$

In the 10 minute overtime format, there is a 42.0% chance of the receiving team winning the game on their first offensive possession and a 31.5% chance of the defending team getting the ball back to win. There is also an 11.9% chance the game will end of a tie, which increases by each offensive possession. This is double any average the league has ever experienced and would be a detrimental aspect of the 10 minute overtime format. Shortening or lengthening the game would definitely have an impact on the probability distribution of a tie, but how much it impacts this distribution is shown in Table 3.

Table 3. Probability of Game Completion

Number of Possessions	Game Completion (%)
2	46.9
4	73.5
6	87.4
8	94.2
10	97.4
12	98.9

V. Conclusion and Future Research

The goal of this research is to conduct a comparative analysis of two different NFL overtime formats and gain insight into understanding on whether the new overtime rule in 2012 is fairer than the sudden death format. An emphasis was placed on the probability of victory for the receiving team, known as either Team Blue or Red, and it became clear that winning the coin toss is advantageous for a favorable outcome. If a team loses the initial coin toss in the sudden death format, they are automatically an underdog. They are a still a statistical underdog in the new overtime format, but after six possessions, the margin between the team on offense and defense is not as wide as the old model. The sudden death overtime format was more advantageous for the receiving team by 7.2%, so the NFL transitioning to a two possession format had the intended results.

The NFL overtime model still needs modifications to achieve more parity, but it is headed in the right statistical direction. This parity may be achieved by switching to the college football overtime format, but it will not be reached shortening the length of the overtime period. If the overtime period is lengthened to guarantee an outcome, such as in the postseason overtime model, games have the potential to last up to an extra half of play. An extra half of play means a higher risk of injury, a slower recovery rate, and an extension of the viewer's commitment level. If shortening the overtime period is the next step in the perceived solution, the probability of game completion drops dramatically. The potential to tie does not reflect well on the game because it is harder to evaluate in the standings and is not well received by fans who expect to leave knowing which team was better. In fact, a way to improve the presented model is to integrate a Markov ranking method to weight the stronger and weaker teams as they enter overtime. It can be argued that true parity will never be achieved in overtime because one team is always more skillful than the other, so it is important to integrate that into the model as the potential outcomes are being evaluated.

Using a model similar to the one in this study, further research can be done to evaluate probabilities in all sporting environments. College football would be the easiest sport to integrate this into, especially with a comparative analysis on their overtime system and the new NFL overtime system [1]. In fact, fans like the college system because it is all about the strength of each unit. Both teams start off with the exact same circumstances, so the offensive and defensive units who are truly more skillful generally prevail. Another sporting that the presented model can be applied to is the National Collegiate Athletic Association (NCAA) March Madness tournament [10]. In this college basketball tournament, the teams are already paired and weighted by their rankings, so a model can be formulated on the probabilities of advancing to the next round (next state). Unfortunately, the model would not be sensitive to upsets, but the tops seeds generally advance in the way they're predicted. What can be gathered from this research, is that the presented model can be utilized in any sporting environment that utilizes in-game statistics to generate its probability distribution. Given that sports play such an important role in society and generate a significant cash flow, it is important to invest in studies such as these in an effort to pursuit parity and a fair system of rules.

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