Effects On Velocity On Applying Variable Pressure Gradient To A Magnetohydrodynamic Fluid Flowing Between Plates With Inclined Magnetic Field

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***Abstract*—The effects of applying variable pressure gradient to a magnetohydrodynamic (MHD) fluid flowing between two plates in an inclined magnetic field has been investigated in this paper. The equations governing the flow are non-dimensionalised and solved numerically. The effects of Reynold's number, Hartmann number, Prandtl number are illustrated graphically. It was found out that the application of a variable pressure gradient in the presence of an inclined magnetic field resulted to an increase in velocity of the fluid.**

Keywords— Magnetohydrodynamic fluid flow, Suction number, Pressure Gradient.

NOMENCLATURE

b	Distance between plates
$\vec{B}$	Magnetic field vector
c_p	Specific heat capacity
$\vec{E}$	Electric field strength
e	Specific internal energy
Ec	Eckert parameter
F_x, F_y, F_z	Body force in x-, y-, z-axis
G	Volumetric coefficient
$\vec{H}$	Magnetic field strength
Ha	Hartmann number
$\vec{J}$	Electric current density
K	Thermal conductivity
L	Characteristic Length
P	Pressure
P^*	Dimensionless Pressure of fluid
Pr	Prandtl number
q_j	Heat
r	Specific enthalpy
Re	Reynold's number
s	Entropy
s_0	Suction parameter
T	Temperature
T_∞	Characteristic free stream temperature
T_w	Characteristic temperature on plate
u^*, v^*	Dimensionless velocity component
t	Time
U	Characteristic velocity
t^*	Dimensionless time
x^*, y^*	Dimensionless x, y Cartesian coordinates
$\vec{V}$	Velocity
u, v, w	Velocity components in x, y, z-axis
x, y, z	Cartesian coordinates
μ_e	Magnetic permeability
v_0	Suction velocity
ρ	Density of fluid
μ	Coefficient of viscosity
α	Angle of inclination between magnetic field and fluid flow direction
σ	Electrical conductivity
φ	Dissipation function
Δy	Distance interval
Δt	Time interval
∇^2	Laplacian operator
∇	Gradient operator
$\frac{\partial P}{\partial x}$	Pressure gradient

I. INTRODUCTION

When a line of magnetic force is imposed on moving electrical conductive fluid, it experiences a force acting on it and new currents are induced. The new induced currents in turn induce their own magnetic field which then affects the original magnetic field. These new currents generated have led to designing of MHD power generators and MHD pumps used in chemical energy technology. Attia [1] analyzed the effects of heat transmission between porous plates where exponential decaying pressure gradient was applied. Attia [2] investigated the outcomes of applying Hall current on temperature and velocity profiles of a couette flow. Chaturani [3] carried an investigation on two layered MHD model for parallel plate haemodialysis under the influence of uniform magnetic field that was applied perpendicularly. Ganesh [4] carried out the study where a viscous fluid flowed through two parallel porous plates with the fluid been drawn out on both walls of the plates at the same rate and the lines of magnetic force were applied transversely. Gunakala [5] carried out a study on unsteady couette flow in the presence of inclined magnetic field. Singh [6] conducted a study on couette flow when a constant pressure gradient was applied. Venkateswarlu [7] studied unsteady MHD fluid flowing pasta vertical porous plate. Mbugua [8] investigated MHD fluid flow between two parallel plates, the top plate being porous in transverse magnetic field. In this study they kept the pressure gradient constant. Singh [9] examined the outcomes of applying inclined magnetic field on unsteady flow. Manyonge [10] discussed MHD poiseuille flow passing through two plates where lower plate was porous in inclined magnetic field.

Despite investigations done on MHD flows past parallel porous plates subjected to inclined lines of magnetic force in the presence of variable pressure gradient where only upper plate is porous has got less attention and hence the motivation of this study. The objective of this study is to obtain the velocity profiles of the flow between the plates and establish the effects of varying, Hartmann number Prandtl number, Pressure gradient, suction parameter and Reynolds number on velocity profiles. This study is significant as it helps generate profiles that are helpful in designing of MHD generators, MHD pumps and dyeing industries

II. MATHEMATICAL ANALYSIS

In this study, a 2-dimensional fluid flow flowing through two parallel plates under a variable pressure gradient was considered. The plates used were situated at a distance of 2b apart. The upper plate was porous and had constant suction. The magnetic field applied was inclined at an angle α to the fluid flow. At time t=0 the upper and lower plate along with the fluid were stationary while at time t >0, the lower plate remained stationary as the upper plate started moving at a velocity u in the direction opposite to the fluid flow direction.

The following assumptions were made for this research study:

a) The fluid flow was of 2-dimensions.
b) The fluid was incompressible and unsteady.
c) The direction of fluid flow was along the x-direction.
d) Gravitational forces were taken to be negligible.
e) Darcy permeability and electrical conductivity are constants.
f) Both the upper and lower plates were of immeasurable length in x- and z-axis.
g) There was no potential difference applied externally.

The equations governing the flow are;

$$\frac{\partial \rho}{\partial t}+\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}\right)=0 \;\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial P}{\partial x}+\frac{\mu}{\rho}\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right)+\frac{F_x}{\rho}\ldots\ldots\ldots (2a)$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{1}{\rho}\frac{\partial P}{\partial y}+\frac{\mu}{\rho}\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}\right)+\frac{F_y}{\rho}\ldots\ldots\ldots (2b)$$

$$\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}=-\frac{1}{\rho}\frac{\partial P}{\partial y}+\frac{\mu}{\rho}\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 w}{\partial z^2}\right)+\frac{F_y}{\rho}\ldots\ldots (2c)$$

This flow is incompressible flow thus $\frac{\partial \rho}{\partial t}=0$ and the plates are of infinite length in x and z-directions thus $\frac{\partial u}{\partial x}=0$ and $\frac{\partial w}{\partial z}=0$.Equation (1) becomes

$$\frac{\partial v}{\partial y}=0 \;\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

Solving (3) we get

$$v=v_0\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

v_0 Is a constant and it is equivalent to suction velocity.

The flow is two-dimensional hence (2c) collapses.

Fluid is flowing along x-direction hence velocity profile along y-axis was equal to zero thus $v=0$ and equations (2a) and (2b) became

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial P}{\partial x}+\frac{\mu}{\rho}\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right)+\frac{F_x}{\rho}\ldots\ldots (5)$$

$$0=-\frac{1}{\rho}\frac{\partial P}{\partial y}+\frac{F_y}{\rho}\quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (6)$$

Gravitational forces are assumed to be negligible then

$$F_y=0\;\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (7)$$

Equation (7) was substituted on equation. (6) And this resulted to

$$0=-\frac{1}{\rho}\frac{\partial P}{\partial y}\;\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (8)$$

$$P=P(x)\;\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (9)$$

And thus equation (6) was eliminated.

Plates were of infinite length in x- and z-direction hence equation (2a) was simplified to

$$\frac{\partial u}{\partial t}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial P}{\partial x}+\frac{\mu}{\rho}\frac{\partial^2 u}{\partial y^2}+\frac{F_x}{\rho}\;\ldots\ldots\ldots\ldots\ldots\ldots\ldots (10)$$

On substituting equation (4) on (10), equation (10) became

$$\frac{\partial u}{\partial t}+v_o\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial P}{\partial x}+\frac{\mu}{\rho}\frac{\partial^2 u}{\partial y^2}+\frac{F_x}{\rho}\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (11)$$

In the above equation the last term F_x is a body force acting along the x-direction. In this research this body force was found out to be Lorentz force.

In this study, the lines of magnetic force were inclined at an angle α to the y-axis. Thus

$$\vec{B} = \vec{B}(0, Bsin\alpha, 0)\ldots\ldots\ldots\ldots\ldots \quad (12)$$

The moving plate was moving along x-axis with a velocity of u thus

$$\vec{V} = \vec{V}(u, 0,0)\ldots\ldots\ldots\ldots\ldots \quad (13)$$

$$\vec{J} = \sigma(\vec{V} \times \vec{B}) = \sigma \begin{vmatrix} \bar{\imath} & \bar{\jmath} & \bar{k} \\ u & 0 & 0 \\ 0 & Bsin\alpha & 0 \end{vmatrix} = \sigma uBsin\alpha\,\bar{k}\ldots\ldots\ldots (14)$$

On calculating Lorentz force we got

$$F = J \times B = \begin{vmatrix} \bar{\imath} & \bar{\jmath} & \bar{k} \\ 0 & 0 & u\sigma Bsin\alpha \\ 0 & Bsin\alpha & 0 \end{vmatrix}$$

$$= -u\sigma B^2 sin^2\alpha\bar{\imath} \ldots\ldots\ldots\ldots\ldots \quad (15)$$

When magnetic field is imposed on a substance, it penetrates through the substance to its inside. This phenomenon is called magnetic permeability and is expressed as

$$\mu_e = \frac{\vec{B}}{\vec{H}} \ldots\ldots\ldots\ldots\ldots \quad (16)$$

$$\vec{F} = -u\sigma\mu_e^2\vec{H}^2 sin^2\alpha\ldots\ldots\ldots\ldots\ldots \quad (17)$$

Substituting (17) on (11) we have

$$\frac{\partial u}{\partial t} + v_o\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial P}{\partial x} + \frac{\mu}{\rho}\frac{\partial^2 u}{\partial y^2} - \frac{u\sigma\mu_e^2 H^2 sin^2\alpha}{\rho} \ldots\ldots\ldots \quad (18)$$

Using the following non-dimensional quantities

$$x^* = \frac{x}{L}, y^* = \frac{y}{L}, P^* = \frac{P}{\rho U^2}, u^* = \frac{u}{U}, \quad t^* = \frac{tU}{L}, s_0 = \frac{v_0}{U},$$

$$Re = \frac{\rho UL}{\mu}, Ha = L\mu_e H\sqrt{\frac{\sigma}{\mu}}$$

Equation (18) became

$$\frac{\partial u^*}{\partial t^*} + \frac{v_o}{U}\frac{\partial u^*}{\partial y^*} = -\frac{\partial P^*}{\partial x^*} + \frac{\mu}{\rho LU}\frac{\partial^2 u^*}{\partial y^{*2}} - \frac{\sigma\mu_e^2 H^2\,\mathrm{L}\sin^2\alpha}{\rho U^2}u^* \ldots\ldots\ldots \quad (19)$$

The initial and boundary conditions are

$t = 0, u = 0 \quad \mathrm{at} -\mathrm{b} \le \mathrm{y} \le \mathrm{b}$

$t > 0,\ u = 0 \quad at\ y = -b$

(3.6.19)

$t > 0\ , u = U \quad at\ y = b$

The non-dimensionalised conditions of the equations became

$t^* = 0 \quad u^* = 0 \quad \mathrm{at} \quad -1 \le y^* \le 1$

$t^* > 0 \quad u^* = 0 \quad \mathrm{at}\ \ y^* = -1$

$t^* > 0 \quad u^* = 0 \quad \mathrm{at}\ \ y^* = 1$

III. METHOD OF SOLUTION

In this study we will solve the equation numerically using finite difference technique.

The finite difference forms of U and T are

$$\frac{\partial u^*}{\partial y^*} = \frac{U_j^{k+1} - U_{j-1}^{k+1} + U_j^k - U_{j-1}^k}{2\Delta y} \ldots\ldots\ldots\ldots\ldots \quad (20)$$

$$\frac{\partial^2 u^*}{\partial y^{*2}} = \frac{U_{j+1}^{k+1} - 2U_j^{k+1} + U_{j-1}^{k+1} + U_{j+1}^k - 2U_j^k + U_{j-1}^k}{2(\Delta y)^2} \ldots\ldots\ldots \quad (21)$$

$$\frac{\partial u^*}{\partial t^*} = \frac{U_j^{k+1} - U_j^k}{\Delta t} \ldots\ldots\ldots\ldots\ldots \quad (22)$$

We substituted these forms on (19) and then made U_j^{k+1} the subject

$$U_j^{k+1} = \left\{U_j^k - \Delta t\frac{\partial P^*}{\partial x^*} - s_0\Delta t\left(\frac{U_j^k - U_{j-1}^{k+1} - U_{j-1}^k}{2\Delta y}\right) + \frac{\Delta t}{Re}\left(\frac{U_{j+1}^{k+1} + U_{j-1}^{k+1} + U_{j+1}^k - 2U_j^k + U_{j-1}^k}{2(\Delta y)^2}\right) - \frac{Ha^2}{Re}\Delta t\sin^2\alpha U_j^k\right\} \div \left(1 + \frac{s_0\Delta t}{2\Delta y} + \frac{\Delta t}{Re(\Delta y)^2}\right) \ldots\ldots\ldots \quad (23)$$

This was the final equation and it was solved using Matlab software.

IV. RESULTS

Using (23) we analyzed the effects of various parameters on velocity. The analysis is illustrated using graphs below;

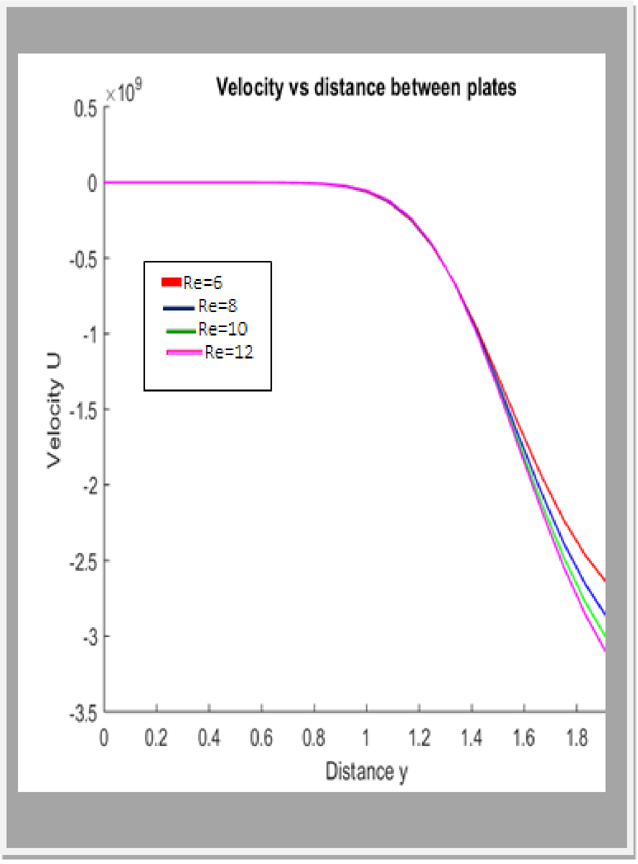

Fig. 1. Velocity profiles for different values of Reynold's number (Re)

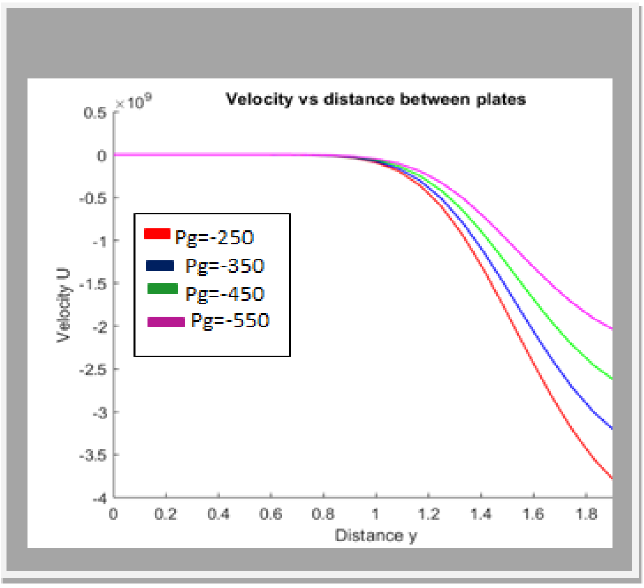

Fig. 2. Velocity profiles for different values of Pressure gradient

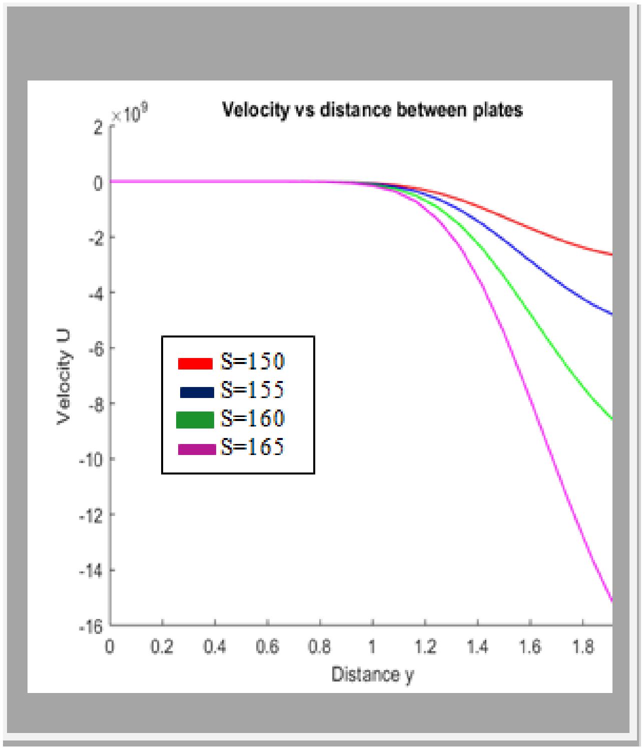

Fig. 3. Velocity profies for different values of suction parameter

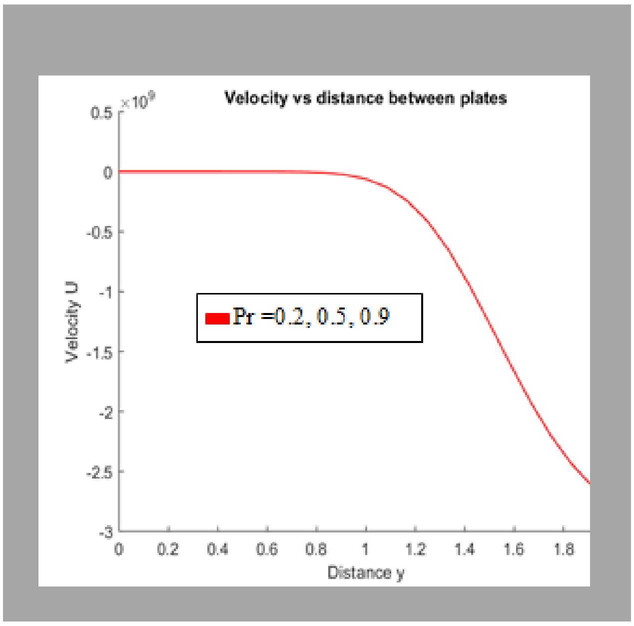

Fig. 4. Velocity profiles for different values of Prandtl numbers

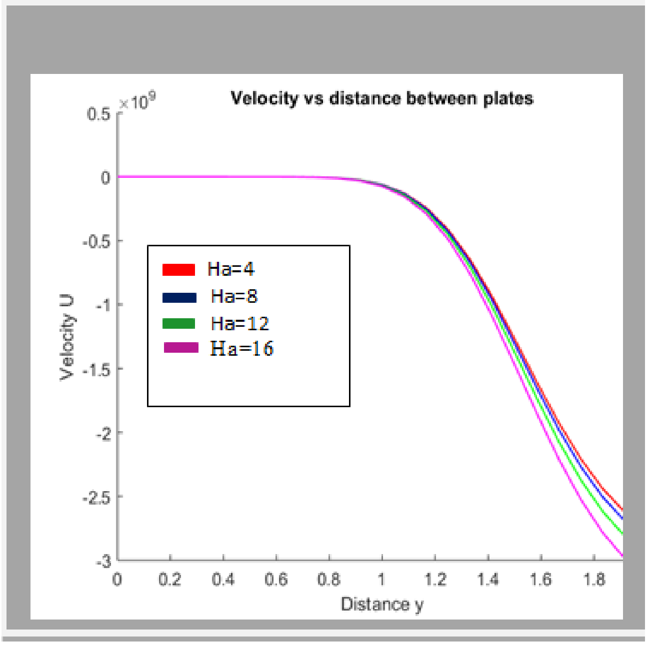

Fig. 5. Velocity profiles for different values of Hartmann number

V. DISCUSSIONS

Figure 1 illustrates the effects Reynold's number on velocity profiles. It was observed that an increase in Reynold's number led to a decrease in velocity. This is because of the dominance of viscous forces over inertial force. An increase in Reynold's number for instance from Re=8 to Re =10 means the viscous force in Re=10 is greater compared to the one in Re=8. Viscous force is the force that opposes the forward movement of a fluid. An increase in this force results to a decrease in velocity profile of the fluid as the resistance to the fluid flow has increased.

In figure 2, the pressure gradient is negative because it acts in the same direction as the fluid flow. It was observed that increasing the pressure gradient on the flow led to an increase in velocity. This is because the pressure gradient aided the velocity by pushing the fluid. It was observed also that at the stationary plate the flow assumed the velocity of the plate.

Figure 3 shows the effect of suction parameter on velocity. We observe that an increase in the suction parameter yields a decrease in velocity profile. This is due to pressure. When the suction parameter was increased, pressure of the fluid flowing between the plates diminished. This resulted to the velocity of the fluid to slow down and hence decrease in velocity.

From figure 4 it was observed there was no effect on velocity when the Prandtl number was varied with velocity profile. This is attributed to the fact that Prandtl number focuses mainly on heat exchange between two substances and hence its effect is felt only on temperature of the fluid. However when it comes to velocity, it has no effect on it.

From figure 5 it was observed that an increase in Hartmann number results to a decrease in velocity. This is because of Lorentz force. The presence of magnetic field in an electrically conducting fluid introduces Lorentz force which opposes the fluid motion. When the Hartmann number is increased it implies increasing the electromagnetic forces and consequently the Lorentz force increases. The increased resistance to the flow by the increased Lorentz force results to the decrease in velocity.

VI. CONCLUSION

From this research study the following conclusions were made;

a) The application of variable pressure gradient has significant effects on the velocity profiles of a fluid.
b) The velocity profiles of a fluid can be increased by increasing the pressure gradient.
c) The fluid velocity can be decreased by increasing the suction parameter, Hartmann number or Reynold's number.

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