Application of MSEIR Model of Secondary Infection in Pest Control

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Abstract: **In this paper, the age-structure infection and vaccination is discussed.Under the assumption of a certain amount of insect pest and bollworm, using the theory and method of partial differential and ordinary differential equations, the expression of a reproductive MSEIR model with secondary is obtained $\Re(\varphi)$, At that time,if $\Re(\varphi)<1$ the equilibrium state was locally asymptotically stable. at that time, if $\Re(\varphi)>1$, the equilibrium state was unstable. At this time, there was an age equilibrium state, and it was proved that if $\Re(0)<1$ the age equilibrium state was globally asymptotically stable.**

Keywords—Helicoverpa armigera;MSEIR model;regeneration number;equilibrium state;age equilibrium state, stability

I. INTRODUCTION

In recent years, the application of mathematical models and computer combined means to study the problem of pest control has become one of the most concerned topics in many countries. Due to the help of electronic computers, various variables in the environment can be considered synthetically. The impact of the trend behavior, the density and spatial distribution of the host and the natural enemy on the system, and the MSEIR model as close as possible to the real host and natural enemy system of action, the African cotton bollworm is a major pest damage to cotton in recent years based on Considering the ecological characteristics of pests and cotton growth stages, environmental conditions and control measures at different stages of the insect state, the MSEIR model was established to determine when the number and age of cotton bollworm and the number of cotton reached a state of equilibrium The paper is organized as follows: In Section 1, the MSEIR model was established. In Section 2, analytical expressions for the number of regenerations $\Re(\varphi)$ and the number of basic regenerations $\Re_0$ were given and proved that the if $\Re(\varphi)<1$, the age equilibrium was locally stable at that time, if $\Re(\varphi)>1$ when the age equilibrium was unstable Section 3 discusses the global stability of the equilibrium states of age. Section 4 proves that $\Re(\varphi)>1$, there was an age equilibrium at the time.

II. MODEL

In order to establish a model with MSEIR, we first introduce some markers to indicate“a” representative the age of cotton bollworm, the“t” which means the growth time and growth cycle of cotton bollworm are divided into five parts: mating, spawning, larvae, pupae and adults Crossing, we use $M(a,t), S(a,t), E(a,t), I(a,t), R(a,t)$ said spawning, infestation of larvae, overwintering pupae, fecundity of adult and bollworm population density function for age A_1 and A_2 means that the total number of cotton bollworm at $\int_{A_1}^{A_2} S(a,t)da$,and above the age range is, expressed as the age-dependent birth rate and mortality , Here assuming and independent of the number, the age-dependent maternal mating cycle $b(a)$ and $u(a)$, and the age-dependent spawning cycle and larval engraftment cycle $(\alpha(a))^{-1}$ and $(\theta(a))^{-1}$, respectively. Using the cycle representing the wintering pupae, The disjunction in one cycle $\delta(a)$, $\alpha(a)$ and $\theta(a)$ thus leads to an appropriate reduction in the number, satisfaction, assumptions, and are positive continuous functions, non-negative continuous functions, assuming that the disjoint rate has the following form $\lambda(a,t) = k(a)\int_0^{\infty} h(\rho)I(\rho,t)d\rho$,Where and represents the morbidity and adult rate in the growth cycle of H. armigera. The MSEIR model is constructed under the following conditions. The MSEIR model can be described by the $h(a), k(a) \in C[0,+\infty)$, 且 $h(a), k(a) \geq 0$ 在 $[0,+\infty)$ initial boundary value problem of the following five partial differential equations:

$$\frac{\partial M(a,t)}{\partial t} + \frac{\partial M(a,t)}{\partial a} = -\left(u(a) + \delta(a)\right)M(a,t) \qquad (1.1a)$$

$$\frac{\partial S(a,t)}{\partial t} + \frac{\partial S(a,t)}{\partial a} = -\left(u(a) + \varphi(a)\right)S(a,t) + \delta(a) - \lambda(a,t)S(a,t) \qquad (1.1b)$$

$$\frac{\partial E(a,t)}{\partial t} + \frac{\partial E(a,t)}{\partial a} = -\left(u(a) + \delta(a)\right)E(a,t) + \lambda(a,t)\left[S(a,t) + \sigma R(a,t)\right] \qquad (1.1c)$$

$$\frac{\partial I(a,t)}{\partial t} + \frac{\partial I(a,t)}{\partial a} = -\left(u(a) + \theta(a)\right)I(a,t) + \alpha(a)E(a,t) \qquad (1.1d)$$

$$\frac{\partial R(a,t)}{\partial t} + \frac{\partial R(a,t)}{\partial a} = -u(a)R(a,t) + \theta(a)I(a,t) + \varphi(a)S(a,t)$$

$$-\sigma\lambda(a,t)R(a,t) \qquad (1.1e)$$

among them

$$M(0,t)=\int_0^{\infty} b(a)\left[M(a,t)+S(a,t)+E(a,t)+I(a,t)+R(a,t)\right]da \quad (1.1f)$$

$$S(0,t)=E(0,t)=I(0,t)=R(0,t)=0 \quad (1.1g)$$

$$M(a,0)=M_0(a),S(a,0)=S_0(a),E(a,0)=E_0(a),I(a,0)=I_0(a)$$

$$R(a,0)=R_0(a) \quad (1.1h)$$

all of the equations together to get the following about age density function

$P(a,t)=M(a,t)+S(a,t)+E(a,t)+I(a,t)+R(a,t)$ equation

$$\frac{\partial P(a,t)}{\partial t}+\frac{\partial P(a,t)}{\partial a}=-u(a)P(a,t) \quad (1.2a)$$

$$P(0,t)=\int_0^{\infty} b(a)P(a,t)da \quad (1.2b)$$

$$P(a,0)=P_0(a) \quad (1.2c)$$

Assumptions below $b(a)\in L^{\infty}[0,+\infty),b(a)\geq 0,\forall a\in[0,+\infty)$

$$u(a)\in C[0,+\infty),u(a)\geq 0,\forall a\in[0,+\infty),\int_0^{+\infty} u(a)da=+\infty$$

And assuming that $b(a)$ and $u(a)$ the value is 0 when the"a" maximum value of a certain age is exceeded, it is further assumed that the number of the bollworm is in a steady state, that is, the following expression

$$\int_0^{+\infty} b(a)\exp\left\{-\int_0^{a} u(\tau)d\tau\right\}da=1 \quad (1.3)$$

$$P(a,t)=P_{\infty}(a)=b_0\exp\left\{-\int_0^{a} u(\tau)d\tau\right\} \quad (1.4)$$

According to the practical significance of the model, the number distribution of cotton bollworm in each cycle is satisfied

$$M_0(a)\geq 0,S_0(a)\geq 0,E_0(a)\geq 0,I_0(a)\geq 0,R_0(a)\geq 0$$

$$M_0(a)+S_0(a)+E_0(a)+I_0(a)+R_0(a)=P_{\infty}(a) \quad (1.5)$$

The following relation can be derived from the (1.4) and (1.5)

$$b_0=\int_0^{+\infty}\left[M_0(a)+S_0(a)+E_0(a)+I_0(a)+R_0(a)\right]da\cdot\left(\int_0^{+\infty} e^{-\int_0^{a} u(\tau)d\tau}da\right)^{-1} \quad (1.6)$$

Make the following transformation

$$m(a,t)=\frac{M(a,t)}{P_\infty(a)},s(a,t)=\frac{S(a,t)}{P_\infty(a)},e(a,t)=\frac{E(a,t)}{P_\infty(a)}$$

$$i(a,t)=\frac{I(a,t)}{P_\infty(a)},r(a,t)=\frac{R(a,t)}{P_\infty(a)} \qquad (1.7)$$

The system (1.1) will be systematized as a simple system

$$\frac{\partial m(a,t)}{\partial t}+\frac{\partial m(a,t)}{\partial a}=-\delta(a)m(a,t) \qquad (1.8a)$$

$$\frac{\partial s(a,t)}{\partial t}+\frac{\partial s(a,t)}{\partial a}=\delta(a)m(a,t)-\varphi(a)s(a,t)-\lambda(a,t)s(a,t) \qquad (1.8b)$$

$$\frac{\partial e(a,t)}{\partial t}+\frac{\partial e(a,t)}{\partial a}=-\alpha(a)e(a,t)+\lambda(a,t)\left[s(a,t)+\sigma r(a,t)\right] \qquad (1.8c)$$

$$\frac{\partial i(a,t)}{\partial t}+\frac{\partial i(a,t)}{\partial a}=-\theta(a)i(a,t)+\alpha(a)e(a,t) \qquad (1.8d)$$

$$\frac{\partial r(a,t)}{\partial t}+\frac{\partial r(a,t)}{\partial a}=\theta(a)i(a,t)+\varphi(a)s(a,t)-\sigma\lambda(a,t)r(a,t) \qquad (1.8e)$$

$$m(0,t)=1,s(0,t)=e(0,t)=i(0,t)=r(0,t)=0 \qquad (1.8f)$$

$$m(a,0)=m_0(a),s(a,0)=s_0(a),e(a,0)=e_0(a),$$

$$i(a,0)=i_0(a),r(a,0)=r_0(a), \qquad (1.8g)$$

$$m(a,t)+s(a,t)+e(a,t)+i(a,t)+r(a,t)=1 \qquad (1.8h)$$

$$\lambda(a,t)=k(a)\int_0^\infty h(\rho)P_\infty(\rho)i(\rho,t)d\rho \qquad (1.8i)$$

In the next section, we derive $\Re(\psi)$ expressions, generally called $\Re(\psi)$ egenerated numbers

III Expression of Regeneration Number and Local Stability of Age Equilibrium

First, find the age equilibrium point of the model (1.8), that is, to satisfy the solution of the following equation

$$\left(\bar{m}(a),\bar{s}(a),0,0,\bar{r}(a)\right)$$

$$\frac{d\bar{m}(a)}{da}=-\delta(a)\bar{m}(a) \qquad (1.9a)$$

$$\frac{d\bar{s}(a)}{da} = \delta(a)\bar{m}(a) - \varphi(a)\delta(a)\bar{s}(a) \qquad (1.9b)$$

$$\frac{d\bar{e}(a)}{da} = -\alpha(a)\bar{e}(a) \qquad (1.9c)$$

$$\frac{d\bar{i}(a)}{da} = -\theta(a)\bar{i}(a) + \alpha(a)\bar{e}(a) \qquad (1.9d)$$

$$\frac{d\bar{r}(a)}{da} = \theta(a)\bar{i}(a) + \varphi(a)\bar{s}(a) \qquad (1.9e)$$

$$\bar{m}(0) = 1, \bar{s}(0) = \bar{e}(0) = \bar{i}(0) = \bar{r}(0) = 0 \qquad (1.9f)$$

$$\bar{m}(a) + \bar{s}(a) + \bar{e}(a) + \bar{i}(a) + \bar{r}(a) = 1 \qquad (1.9g)$$

It is easy to understand the (1.8) system there is the following balance

$$\bar{m}(a) = \exp\left(-\int_0^a \delta(\tau)d\tau\right) \qquad (2.1a)$$

$$\bar{s}(a) = \int_0^a \delta(\tau)\exp\left\{-\int_0^\tau \delta(v)dv\right\} \cdot \exp\left\{-\int_\tau^a \varphi(v)dv\right\}d\tau$$

$$= \exp\left\{-\int_0^a \varphi(\tau)d\tau\right\} - \exp\left\{-\int_0^a \delta(\tau)d\tau\right\}$$

$$+\int_0^a \varphi(\tau)\exp\left\{-\int_0^\tau \delta(v)dv\right\} \cdot \exp\left\{-\int_\tau^a \varphi(v)dv\right\}d\tau \qquad (2.1b)$$

$$\bar{e}(a) = \bar{i}(a) = 0 \qquad (2.1c)$$

$$\bar{e}(a) = \bar{i}(a) = 0, \bar{r}(a) = 1 - e^{-\int_0^a \varphi(\tau)d\tau} - \int_0^a \varphi(\tau) \cdot e^{-\int_0^\tau \delta(v)dv} \cdot e^{-\int_\tau^a \varphi(v)dv} d\tau \qquad (2.1d)$$

In order to study the local stability of the age equilibrium, we linearize the (2.1) equation at the equilibrium (1.8) point

Make $m(a,t) = m^*(a,t) + \bar{m}(a), s(a,t) = s^*(a,t) + \bar{s}(a), e(a,t) = e^*(a,t) + \bar{e}(a)$

$$i(a,t) = i^*(a,t) + \bar{i}(a), r(a,t) = r^*(a,t) + \bar{r}(a) \qquad (2.2)$$

(2.2) will be brought into $(1.8a - 1.8e)$ and out of higher order terms, and the linearized equation of (1.8) system at the age equilibrium (2.1) point is obtained

$$\frac{\partial m^*(a,t)}{\partial t} + \frac{\partial m^*(a,t)}{\partial a} = -\delta(a)m^*(a,t) \qquad (2.3a)$$

$$\frac{\partial s^*(a,t)}{\partial t}+\frac{\partial s*(a,t)}{\partial a}=\delta(a)m^*(a,t)-\varphi(a)s^*(a,t)-\lambda^*(a,t)\bar{s}(a) \qquad (2.3b)$$

$$\frac{\partial e^*(a,t)}{\partial t}+\frac{\partial e^*(a,t)}{\partial a}=-\alpha(a)e^*(a,t)+\lambda^*(a,t)\left[\bar{s}(a)+\sigma\bar{r}(a)\right] \qquad (2.3c)$$

$$\frac{\partial i^*(a,t)}{\partial t}+\frac{\partial i^*(a,t)}{\partial a}=-\theta(a)i^*(a,t)+\alpha(a)e^*(a,t) \qquad (2.3d)$$

$$\frac{\partial r^*(a,t)}{\partial t}+\frac{\partial r^*(a,t)}{\partial a}=\theta(a)i^*(a,t)+\varphi(a)s^*(a,t)-\sigma\lambda^*(a,t)\bar{r}(a) \qquad (2.3e)$$

Here $\lambda^*(a,t)=k(a)\int_0^\infty h(\rho)P_\infty(\rho)i^*(\rho,t)d\rho \qquad (2.3f)$

It is assumed that (2.3) the solution has the following form

$$\left(m^*(a,t),s^*(a,t),e^*(a,t),i^*(a,t),r^*(a,t)\right)=\left(\bar{M}(a),\bar{S}(a),\bar{E}(a),\bar{I}(a),\bar{R}(a)\right)e^{\lambda t}$$

Take it into the $(2.3c)$ and $(2.3d)$ equation get the new equation

$$\frac{d\bar{E}(a)}{da}=-(\lambda+\alpha(a))\bar{E}(a)+k(a)Q_\varphi(a)V_0 \qquad (2.4a)$$

$$\frac{d\bar{I}(a)}{da}=-(\lambda+\theta(a))\bar{I}(a)+\alpha(a)\bar{E}(a) \qquad (2.4b)$$

$$\bar{E}(0)=\bar{I}(0)=0 \qquad (2.4c)$$

Here $V_0=\int_0^\infty h(a)P_\infty(\rho)\bar{I}(a)da \qquad (2.5)$

$$Q_\varphi(a)=\bar{s}(a)+\sigma\bar{r}(a)=\sigma-\exp\left\{-\int_0^a\delta(\tau)d\tau\right\}$$

$$+(1-\sigma)\left[\exp\left\{-\int_r^a\varphi(\tau)d\tau\right\}+\int_0^a\varphi(\tau)\exp\left\{-\int_0^\tau\delta(s)ds\right\}\exp\left\{-\int_r^a\varphi(s)ds\right\}d\tau\right] \quad (2.6)$$

Solve the $(2.4a)$ and $(2.4b)$ equation and get

$$\bar{E}(a)=V_0\int_0^a k(\rho)Q_\varphi(a)\exp\left\{-\int_\rho^a(\lambda+\alpha(\tau))d\tau\right\}d\rho \qquad (2.7a)$$

$$\bar{I}(a)=\int_0^a\alpha(\xi)\bar{E}(\xi)\exp\left\{-\int_\xi^a(\lambda+\theta(\tau))d\tau\right\}d\xi \qquad (2.7b)$$

Use $(2.7a)$ substitute $(2.7b)$ and exchange the order of points to get the expression of $\bar{I}(a)$

$$\bar{I}(a)=V_0\int_0^\infty k(\rho)Q_\varphi(a)\int_\rho^a\alpha(\xi)\cdot e^{-\int_\rho^\xi(\lambda+\alpha(\tau))d\tau}\cdot e^{-\int_\xi^a(\lambda+\theta(\tau))d\tau}d\xi d\rho \qquad (2.8)$$

Use (2.8) substitute (2.5) and exchange the order of points to get the expression

$$V_0 = V_0\int_0^{+\infty} k(\rho)Q_\varphi(a)\int_\rho^{+\infty} h(a)P_\infty(a)\int_\rho^a \alpha(\xi)\cdot$$
$$exp\left\{-\int_\rho^a(\lambda+\alpha(\tau))d\tau\right\}\cdot \exp\left\{-\int_\xi^a(\lambda+\theta(\tau))d\tau\right\}d\xi da d\rho \qquad (2.9)$$

The formula of (2.9) equations on both sides divided by V_0 $(V_0 \neq 0)$, get the following characteristic equation

$$1 = \int_0^{+\infty} k(\rho)Q_\varphi(a)\int_\rho^{+\infty} h(a)P_\infty(a)\cdot e^{-(a-\rho)\lambda}\int_\rho^a \alpha(\xi)$$
$$\cdot exp\left\{-\int_\rho^a \alpha(\tau)d\tau\right\}\cdot \exp\left\{-\int_\xi^a \theta(\tau)d\tau\right\}d\xi da d\rho = T(\lambda) \qquad (2.10)$$

Define the number of regenerations $\Re(\varphi) = T(0)$, which is

$$\Re(\varphi) = \int_0^{+\infty} k(\rho)Q_\varphi(a)\int_\rho^{+\infty} h(a)P_\infty(a)\int_\rho^a \alpha(\xi)\cdot e^{-\int_\rho^a \alpha(\tau)d\tau}\cdot e^{-\int_\xi^a \theta(\tau)d\tau} d\xi da d\rho \qquad (2.11)$$

Can get the following result

A Theorem1:If $\Re(\varphi) < 1$, the age equilibrium (2.1) is gradual and steady; If $\Re(\varphi) > 1$, the age equilibrium (2.1) is unstable .

Prove: By (2.10) type, when $\lambda \in R$, We have $T'(\lambda) < 0, \lim_{\lambda\to+\infty} T(\lambda) = 0$, $\lim_{\lambda\to-\infty} T(\lambda) = +\infty$, So function $T(\lambda)$ in $(-\infty,+\infty)$ Is a strictly monotonically decreasing function if and only if $\Re(\psi) > 1, (\Re(\psi) = 1)$, The (2.10) equation has a unique positive real root, to prove λ^* Is the real root of the real part of the $T(\lambda) = 1$ equation. Assume $\lambda = x + yi$ $(x, y \in R)$ Is an arbitrary complex root of the (2.10) equation, noting $1 = T(\lambda) = |T(x+iy)| \le T(x)$ So there $x \le \lambda^*$。即 $\operatorname{Re}\lambda \le \lambda^*$

It can be seen, If $\Re(\varphi) < 1$, the age equilibrium (2.1) is gradual and steady; If $\Re(\varphi) > 1$, the age equilibrium (2.1) is unstable. Definition $\Re_0 = \Re(0)$,which is

$$\Re_0 = \int_0^{+\infty} k(\rho)\left[1 - e^{-\int_0^\rho \delta(\tau)d\tau}\right]\cdot\int_\rho^{+\infty} h(a)P_\infty(a)\int_\rho^a \alpha(\xi)e^{-\int_\rho^\xi \alpha(\tau)d\tau}\cdot e^{-\int_\xi^a \theta(\tau)d\tau} d\xi da d\rho \qquad (2.12)$$,Noticed

$Q_\varphi(a) = \bar{s}(a) + \sigma\bar{r}(a) \le 1 - e^{-\int_0^a \delta(\tau)d\tau}$ For all $a > 0$ set up , so we have $\Re(\varphi) \le \Re_0$

IV Global stability of age equilibrium states

The second section to prove f $\Re(\varphi)<1$, the age equilibrium (2.1) is gradual and steady; This section will prove

if $\Re_0<1$, The equilibrium state of age is globally asymptotically stable

B Theorem 2:if $\Re_0<1$, The system of (1.8) equilibrium state of age is globally asymptotically stable.

Prove: Integrate the equation $(1.8a)$ along the characteristic line

$$m(a,t)=m(0,t-a)\cdot\exp\left\{-\int_0^a\delta(\tau)d\tau\right\},a<t \qquad (3.1)$$

Get from $(1.8f)$,when $a<t$, $m(0,t-a)=1$, then

$$m(a,t)=\exp\left\{-\int_0^a\delta(\tau)d\tau\right\}=\overline{m}(a),a<t \qquad (3.2)$$

We can get $\lim\limits_{t\to+\infty}m(a,t)=\overline{m}(a)$ $\qquad (3.3)$

Make $f(a,t)$ for the t moment of the a age of the adult cotton bollworm emergence ratio,when $a<t$,

$$f(a,t)=\lambda(a,t)\left[s(a,t)+\sigma r(a,t)\right]\le k(a)\left[1-e^{-\int_0^a\delta(\tau)d\tau}\right]\cdot V(t) \qquad (3.4)$$

Here $\lambda(a,t)=k(a)V(t)$, $V(t)=\int_0^{+\infty}h(a)P_\infty(a)i(a,t)da$ (3.5)

Here we use $\sigma\le1$, By $(1.8h)$ and (3.2) to get the real of

$$s(a,t)+\sigma r(a,t)\le1-e^{-\int_0^a\delta(\tau)d\tau}$$

Integrate the $(1.8c)$ equation along the characteristic line

$$e(a,t)=\int_0^a f(a-\xi,t-\xi)\cdot\exp\left\{-\int_{a-\xi}^a\alpha(\tau)d\tau\right\}d\xi,a<t \qquad (3.6)$$

The same can be obtained

$$i(a,t)=\int_0^a\alpha(a-\eta)e(a-\eta,t-\eta)\cdot\exp\left\{-\int_{a-\eta}^a\theta(\tau)d\tau\right\}d\eta,a<t \qquad (3.7)$$

Use the (3.6) equation into (3.7), when $a<t$,we can get

$$\begin{aligned}i(a,t)&=\int_0^a\alpha(a-\eta)\int_0^{a-\eta}f(a-\eta-\xi,t-\eta-\xi)\\&\cdot\exp\left\{-\int_{a-\eta-\xi}^{a-\eta}\alpha(\tau)d\tau\right\}d\xi\cdot\exp\left\{-\int_{a-\eta}^a\theta(\tau)d\tau\right\}d\eta\end{aligned} \qquad (3.8)$$

Make changes in (3.8) and change the order of points

$$i(a,t)=\int_0^a f(\rho,t-a+\rho)\int_\rho^a\alpha(\xi)\exp\left\{-\int_\rho^\xi\alpha(\tau)d\tau\right\}\cdot\exp\left\{-\int_\xi^a\theta(\tau)d\tau\right\}d\xi d\rho \ (3.9)$$

Use the (3.5) and (3.9) into (3.4) to get

$$f(a,t) \le k(a)\left(1-e^{-\int_0^a \delta(\tau)d\tau}\right)\int_0^{+\infty} h(a)P_\infty(a)\int_0^a f(\rho,t-a+\rho)$$

$$\cdot\int_\rho^a \alpha(\xi)\exp\left\{-\int_\rho^\xi \alpha(\tau)d\tau\right\}\exp\left\{-\int_\xi^a \theta(\tau)d\tau\right\}d\xi d\rho da \qquad (3.10)$$

Make $Y(a) = \limsup_{t\to+\infty} f(a,t)$

On both sides of (3.10) take the upper limit $t \to +\infty$, using Fatou lemma to get

$$Y(a) \le k(a)\cdot\left(1-\exp\left\{-\int_0^a \delta(\tau)d\tau\right\}\right)\int_0^{+\infty} h(a)P_\infty(a)\int_0^a Y(p)$$

$$\cdot\int_\rho^a \alpha(\xi)\exp\left\{-\int_\rho^\xi \alpha(\tau)d\tau\right\}\exp\left\{-\int_\xi^a \theta(\tau)d\tau\right\}d\xi d\rho da \qquad (3.12)$$

Use C said the following constants

$$C = \int_0^{+\infty} h(a)P_\infty(a)\int_0^a Y(p)\cdot\int_\rho^a \alpha(\xi)e^{-\int_\rho^\xi \alpha(\tau)d\tau}\cdot e^{-\int_\xi^a \theta(\tau)d\tau}d\xi d\rho da \qquad (3.13)$$

The inequality can be rewritten as $0 \le Y(a) \le Ck(a)\left(1-e^{-\int_0^a \delta(\tau)d\tau}\right) \qquad (3.14)$

By (3.13) and (3.14) that we can get

$$C \le \int_0^{+\infty} h(a)P_\infty(a)\int_0^a Ck(\rho)\left(1-e^{-\int_0^\rho \delta(\tau)d\tau}\right)\cdot\int_\rho^a \alpha(\xi)e^{-\int_\rho^\xi \alpha(\tau)d\tau}e^{-\int_\xi^a \theta(\tau)d\tau}d\xi d\rho da$$

$$= C\int_0^{+\infty} k(\rho)\left(1-e^{-\int_0^\rho \delta(\tau)d\tau}\right)\int_\rho^{+\infty} h(a)P_\infty(a)\int_\rho^a \alpha(\xi)e^{-\int_\rho^\xi \alpha(\tau)d\tau}e^{-\int_\xi^a \theta(\tau)d\tau}d\xi d\rho da$$

$$= C\Re_0 \qquad (3.15)$$

By (3.15) we knowing that if $\Re_0 < 1$, then $C = 0$, therefore, if $\Re_0 < 1$, then

$Y(a) = 0$, which is $\limsup_{t\to+\infty} f(a,t) = 0$

On both sides (3.6) of (3.7) take the limit $t \to +\infty$, we can get

$$\lim_{t\to+\infty} e(a,t) = \bar{e}(a) = 0, \lim_{t\to+\infty} i(a,t) = \bar{i}(a) = 0, \lim_{t\to+\infty} V(t) = 0 \qquad (3.16)$$

Integrate the $(1.8b)$ equation along the characteristic line

$$s(a,t) = \int_0^a \delta(\tau)m(\tau,t-a+\tau)\exp\left\{-\int_\tau^a \left[\varphi(v)+k(v)V(t-a+v)\right]dv\right\}d\tau, a < t \quad (3.17)$$

On both sides of (3.17) take the limit $t \to +\infty$, Then apply (3.2), (3.16) and $(2,1b)$,we can get

$$\lim_{t\to+\infty} s(a,t) = \int_0^a \delta(\tau)\exp\left\{-\int_0^\tau \delta(v)dv\right\}\cdot\exp\left\{-\int_\tau^a \varphi(v)dv\right\}d\tau = \bar{s}(a) \qquad (3.18)$$

Get from $(1.8h)$ and $(1.9g)$

$$\lim_{t\to+\infty} r(a,t) = \lim_{t\to+\infty}\left[1-m(a,t)-s(a,t)-e(a,t)-i(a,t)\right]$$
$$= \lim_{t\to+\infty}\left[1-\bar{m}(a)-\bar{s}(a)-\bar{e}(a)-\bar{i}(a)\right] = \bar{r}(a) \qquad (3.19)$$

By (2.2), (3.3), (3.16), (3.18), (3.19) Knowable that when $\Re_0 < 1$,

The age equilibrium of the system is globally asymptotically stable

V The existence of age equilibrium

Section 2 proved that when $\Re(\varphi) > 1$, Age equilibrium is not stable, in fact, in this case there is a non-trivial equilibrium, that is, the following theorem

C Theorem 3: When $\Re(\varphi) > 1$,system (1.8) There is at least one age equilibrium, seeking system (1.8) .The equilibrium (1.8) point is to find a time-independent positive solution that satisfies the following system of (1.8) equations $\left(m^*(a), s^*(a), e^*(a), i^*(a), r^*(a)\right)$

Partial differentiation into ordinary differential equations

$$\frac{dm^*(a)}{da} = -\delta(a)m^*(a) \qquad (4.1a)$$

$$\frac{ds^*(a,t)}{da} = \delta(a)m^*(a) - \left[\varphi(a)+k(a)V^*\right]s^*(a) \qquad (4.1b)$$

$$\frac{de^*(a)}{da} = -\alpha(a)e^*(a) + k(a)\left[s^*(a)+\sigma r^*(a)\right]\cdot V^* \quad (4.1c)$$

$$\frac{di^*(a)}{da} = -\theta(a)i^*(a) + \alpha(a)e^*(a) \qquad (4.1d)$$

$$\frac{dr^*(a)}{da} = \theta(a)i^*(a) + \varphi(a)s^*(a) - \sigma k(a)r^*(a)\cdot V^* \quad (4.1e)$$

$$m^*(0)=1, s^*(0)=e^*(0)=i^*(0)=r^*(0) \qquad (4.1f)$$

$$m^*(a)+s^*(a)+e^*(a)+i^*(a)+r^*(a)=1 \qquad (4.1g)$$

Here $V^* = \int_0^{+\infty} h(a)P_\infty(a)i^*(a)da \qquad (4.2)$

Theorem 3 proves as follows: Solve the (4.1) euations are obtained

$$m^*(a)=\exp\left\{-\int_0^a \delta(\tau)d\tau\right\}, \qquad (4.3a)$$

$$s^*(a)=\int_0^a \delta(\tau)\exp\left\{-\int_0^\tau \delta(v)dv\right\}\exp\left\{-\int_\tau^a\left[\varphi(v)+k(v)V^*\right]dv\right\}d\tau \qquad (4.3b)$$

$$e^*(a)=\int_0^a k(\rho)\hbar(\rho,V^*)\cdot V^*\cdot\exp\left\{-\int_\rho^a \alpha(s)ds\right\}d\rho \qquad (4.3c)$$

$$i^*(a)=\int_0^a \alpha(\xi)e^*(\xi)\exp\left\{-\int_\xi^a \theta(s)ds\right\}d\xi \qquad (4.3d)$$

$$r^*(a)=\int_0^a \varphi(\xi)s^*(\xi)\exp\left\{-\sigma V^*\int_\xi^a k(s)ds\right\}d\xi$$

$$+\int_0^a \theta(\eta)i^*(\eta)\exp\left\{-\sigma V^*\int_\eta^a k(s)ds\right\}d\eta \qquad (4.3e)$$

Here $\hbar(\rho,V^*)=s^*(\rho)+\sigma r^*(\rho) \qquad (4.3f)$

Use $(4.3g)$ into $(4.3e)$ will be substituted and exchanged order

$$r^*(a)=\int_0^a \varphi(\xi)s^*(\xi)\exp\left\{-\sigma V^*\int_\xi^a k(s)ds\right\}d\xi$$

$$+V^*\int_0^a k(\rho)\hbar(\rho,V^*)\int_\rho^a \theta(\eta)i^*(\eta)\exp\left\{-\sigma V^*\int_\eta^a k(s)ds\right\}$$

$$\cdot\int_\rho^\eta \alpha(\xi)\cdot\exp\left\{-\int_\rho^\xi \alpha(s)ds\right\}\cdot\exp\left\{-\int_\xi^\eta \theta(s)ds\right\}d\xi d\eta d\rho \qquad (4.3h)$$

The following equations with $\hbar(\rho,V^*)$ parameters are satisfied

by $(4.3f)$ and $(4.3h)$

$$\hbar(a,V^*)=g(a,V^*)+\int_0^a \psi(a,\rho,V^*) \qquad (4.4)$$

$$g(a,V^*)=s^*(a)+\sigma\int_0^a \varphi(\xi)s^*(\xi)\exp\left\{-\sigma V^*\int_\xi^a k(s)ds\right\}d\xi \qquad (4.5)$$

$$\psi(a,\rho,V^*)=\sigma V^* k(\rho)\int_\rho^a \theta(\eta)\exp\left\{-\sigma V^*\int_\eta^a k(s)ds\right\}$$

$$\cdot\int_\rho^\eta \alpha(\xi)\exp\left\{-\int_\rho^\xi \alpha(s)ds\right\}\cdot\exp\left\{-\int_\xi^\eta \theta(s)ds\right\}d\xi d\eta \qquad (4.6)$$

Obviously Contains only known functions, and all known functions $g(a,V^*)$, $a\in[0,A)$ that it is continuous,it is easy to see that $g\in C([0,A)\times R,R)$, the relation V^* is continuously differentiable, and for each one $V^*\in R$,the function $\psi(\cdot,\cdot,V^*)$ is the on the continuous $Volterra$ nucleus (See [15] for details). From the result of the theorem1.2 in Section 2 of Chapter 5 of [15] we know that for each one $V^*>0$,there is a solution $\hbar(a,V^*)$ defined $[0,A_{\max})$ in the maximum interval of existence.And depends on $\hbar(a,V^*)$ it continuously V^*,so that we can get a bounded $\hbar(a,V^*)$ solution, for some one,we can get $m(a,t)+s(a,t)+e(a,t)+i(a,t)+r(a,t)=1$, all $\hbar(a,V^*)$ are bounded by the literature [15].The twelfth chapter, section one, knows that $A_{\max}=+\infty$ to put $(4.3g)$ in (4.2), both sides of the equation obtained $V^*(V^*\neq 0)$ at the same time to get

$$1=\int_0^{+\infty}h(a)P_\infty(a)\int_0^a k(\rho)\,d\hbar(\rho,V^*)\int_\rho^a\alpha(\xi)\cdot e^{-\int_\rho^\xi\alpha(s)ds}\cdot e^{-\int_\xi^a\theta(s)ds}\,d\xi\,d\rho\,da$$

$$=\int_0^{+\infty}k(\rho)\hbar(\rho,V^*)\int_\rho^{+\infty}h(a)P_\infty(a)\int_\rho^a\alpha(\xi)\cdot e^{-\int_\rho^\xi\alpha(s)ds}\cdot e^{-\int_\xi^a\theta(s)ds}\,d\xi\,da\,d\rho$$

$$=G(V^*)\qquad(4.7)$$

If there is a positive V^* solution to the (4.7) equation, there is a $\hbar(a,V^*)$ bounded solution defined above $[0,+\infty)$ in (4.4),The original system, there is an age equilibrium, easy to know when $V^*=0$, the equation (4.1) derived $i^*(a)=0$, this time the equilibrium equilibrium is the state of age, pay attention $\hbar(a,0)=Q_\varphi(a)$, and thus get $G(0)=\Re(\varphi)$ by (2.11) and (4.7). the title $\Re(\varphi)>1, G(0)>1$, and by $(4.1g)$ and (4.3) know $i^*(a)<1$, so by $(4.3g)$

$$V^*\cdot\int_0^a k(\rho)\hbar(\rho,V^*)\int_\rho^a\alpha(\xi)\cdot e^{-\int_\rho^\xi\alpha(s)ds}\cdot e^{-\int_\xi^a\theta(s)ds}\,d\xi\,d\rho<1\qquad(4.8)$$

Thus to any $V^*>0$ get

$$V^*G(V^*)=\int_0^{+\infty}h(a)P_\infty(a)V^*\int_0^a k(\rho)\hbar(\rho,V^*)$$
$$\cdot\int_\rho^a\alpha(\xi)\exp\left\{-\int_\rho^\xi\alpha(s)ds\right\}\exp\left\{-\int_\xi^a\theta(s)ds\right\}d\xi\,d\rho\,da$$
$$<\int_0^{+\infty}h(a)P_\infty(a)da\le h^+\int_0^{+\infty}P_\infty(a)da=h^+N\qquad(4.9)$$

Here N on behalf of the number of cotton bollworm, $h^{+} = \sup_{[0,+\infty)} h(a)$

In particular, yes $V^{*} = h^{+}N$, there is $G\left(h^{+}N\right) < 1$, but since $G(0) > 1$

there is a positive solution $G(0) > 1$ on the existence of a positive solution V^{*}, this solution is not necessarily unique $G\left(V^{*}\right) = 1$ because it is not necessarily $\left(0, h^{+}N\right)$ a monotonous function, and then $\Re(\varphi) > 1$ there is at least one age mouilibrium in the system, which Is determined by the unique solution to $\left(V^{*}\right)_{+}$ which the (4.1) equation corresponds.

CONCLUSION: Because $\Re(\varphi) > 1$ it is the only sufficient condition for the existence of the equilibrium state of the age, I only prove that $\Re(\psi) < 1$ is the equilibrium state of age was locally stable at that time. Therefore, the existence of the equilibrium point in the system is a problem to be solved $\Re(\psi) < 1$. Theorem 2 shows that $\Re(\psi) < 1$

at this time, There is an equilibrium of age

References

[1] Hoppensteadt F. An age structured epidemic model. J.Frarkli In.st., 1974, 197: 325-333.
[2] Cooke K and Busenberg S. Vertically ansmitted Disease, in Nonlinear Phenomena in Mathemat-ical Sciences. V.Lakshmikantham ed. Academic Press, New York, 1982, 189-197.
[3] Busenberg S, Iannelli M and Thieme H. Global behavior of an age-structured epidemic model.SIAM J. Math. A}.al，1991，22(4):1065-1080.
[4] Anderson R M and May R M. Directly transmitted infectious diseases: control by vaccination. Sciece, 1982, 215: 1053-1060.
[5] Anderson R M and May R M. Age-related changes in the rate of disease transmission: implication for the design of vaccination programs. J. Hyg. Carra6.，1985, 94: 365-435.
[6] Anderson R M and May R M. Vaccination against rubella and measles: quantitative investigations of different policies. J. Hyg. CaTnb，1983, 90: 259-352.
[7] Iannelli M, Milner F and Pugliese A. Analytical and numerical results for the age-structured S-I-S epidemic model with mixed inter-intracohort transmission. SIAM J. Matlt. Anal，1992, 23(3):662-688
[8] Cha Y, Iannelli M and Milner E. Existence and uniqueness of endemic states for the age-structured SIR epidemic model. Math. Biosci.，1998, 150: 177-190.
[9] Greenhalgh D. Analytical threshold and stability results on age-structured epidemic models with vaccination. Theor. Popul. Biol，1988, 33: 266-290.
[10] Inaba H. Threshold and stability results for an age-structured epidemic model. J. Math. Biol.,1990, 28: 411-434.
[11] Li M Y, Muldowney J S. Global stability for the SEIR epidemic model in epidemiology. Math.Biosci., 1995, 125: 155-167.
[12] Li X, Geni G and Zhu G.Analysis of an age-structured SEIR epidemic model with vaccination. InteT. J. D 谁 Eqv,.

Appl., 2003, 7(1): 47-67.

[13]Li X. Geni G and Zhu G. The threshold and stability results for an age-structured SEIR epidemic model. Corrzput. Matlt. Ap}l.，2001, 42: 883-907.

[14]Webb G B. Theory of Nonlinear Age-Dependent Population Dynamics. New York and Basel:Marcel Dekker, 1985.

[15] Stoer J and Witzgall Chr. Covexity and Optimization in Finite Dimension I. Springer Verlag,1970.

[16] Hadeler K P. Castillo-Chavez C. A core group model for disease transmission. Math. Biosci，1995,X28: 41-56.