MHD Mixed Convection Stagnation Point Flow along a Vertical Stretching Permeable Surface with Heat Source/Sink

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In this paper, we consider a two-dimensional stagnation point flow and heat transfer due to suction or injection towards a stretching surface. The governing nonlinear boundary layer equations are transformed into the system of nonlinear ordinary differential equation (ODE) using similarity transformation. The equations are then solved numerically by using the bvp4c function in MATLAB software. The effects of the governing parameters, namely velocity ratio parameter, Hartmann number, mixed convection parameter, Prandtl number, heat generation/absorption coefficient and suction/injection parameter are discussed and presented graphically.

Keywords— MHD; Mixed convection; vertical stretching; stagnation point flow; bvp4c

I. INTRODUCTION

The stagnation flow represents the fluid motion near the stagnation point. According to Wang [1], the stagnation region encounters the highest pressure, the highest heat transfer, and the highest rates of mass deposition. The study of stagnation point flow towards a solid surface in moving fluid traced back to Hiezmenz [2]. He was the pioneer to analyze the two-dimensional stagnation point flow on stationary plate by using a similarity transformation to reduce the Navier-Stokes equations to non-linear ordinary differential equations. The existence and uniqueness of the solution in [2] was shown by Tam [3] and Craven and Peletier [4]. Ramachandran et al. [5] analyzed the laminar mixed convection in two-dimensional stagnation flows around heated surfaces for both cases of an arbitrary wall temperature and arbitrary surface heat flux variations, and they found the existence of dual solutions for a certain range of the buoyancy parameter.

Ishak et al. [6], [7] studied the mixed convection boundary layer flow near the two-dimensional stagnation-point flow of an incompressible viscous fluid over a stretching vertical sheet and vertical porous plates, respectively, by considering both cases of assisting and opposing flows. Noor et al. [8] considered the flow of mixed convective magnetohydrodynamic (MHD) fluid flow through vertical porous channel with radiation effect and used a shooting method to obtain the numerical solutions. They found that the lower fluid flow concentration on the inclined surface is caused by the high value of thermophoretic parameter. The problem of the axisymmetric stagnation flows of micropolar fluids towards a shrinking sheet has been solved numerically by Shafique [9], while Mburu et al. [10] investigated the MHD flow between two parallel infinite plates with inclined magnetic field under applied pressure gradient by solving it analytically with appropriate boundary conditions.

Recently, Liu et al. [11] investigated the three-dimensional MHD mixed convection under volumetric heat source in vertical square duct with wall effects and they found that the wall conductance ratio dominates the jet flow at low Gr number and high wall conductance ratio, while Akinshilo [12] considered the flow of mixed convective magnetohydrodynamic (MHD) fluid flow through vertical porous channel with radiation effect he managed to prove the usefulness of mixed convection MHD flow in practical applications. In additional, Ramana Reddy et al. [13] did a comparative study on the combined influences of frictional and irregular heat on MHD Casson and Maxwell fluid flows due to a stretching surface and they observed that the mass transfer rate in Casson fluid flow is better than that of Maxwell fluid. Furthermore, the effects of heat generation/absorption on MHD mixed convective stagnation point flow along a vertical stretching sheet in the presence of external magnetic field is investigated by Sharma et al. [14].

In this paper, we consider the numerical solutions and suction or injection by extending the study done in [14]. To our best knowledge, the case of permeable surface has not been conveyed yet, hence the presented numerical results are original.

II. PROBLEM FORMULATION

A. *The Governing Equations*

We consider a two-dimensional steady laminar flow of a viscous incompressible fluid along a vertical stretching sheet placed in $x-$direction and $y-$axis is normal to the sheet. u and v are the velocity components in x and y directions, respectively. $u = u_e(x) = ax$ is the free stream velocity and the velocity by which the sheet is stretching is $u = u_w(x) = cx$ where both a and c are positive constants. An external magnetic field H_0 is applied normal to the sheet in the presence of heat sink/source. By these assumptions, the governing equations of continuity, momentum and energy can be expressed as

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{dp}{dx} + \nu\frac{\partial^2 u}{\partial y^2} - \frac{\sigma\mu_e^2 H_0^2}{\rho}u + g\beta(T - T_\infty), \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{k}{\rho C_p}\frac{\partial^2 T}{\partial y^2} + \frac{Q}{C_p}(T - T_\infty), \tag{3}$$

where μ_e is the magnitude permeability, σ is the electrical conductivity, ρ is the fluid density, T is the fluid temperature, T_∞ is the free stream temperature, and the temperature of the sheet is denoted as $T_w = T_\infty + bx, b$ is a constant for heated surface $b > 0$ so that $T_w > T_\infty$ and for cooled surface $b < 0$ and $T_w < T_\infty$, β is the volumetric coefficient of thermal expansion, $\nu = \frac{\mu}{\rho}$ is the kinematic viscosity, g is the acceleration due to gravity, k is the thermal conductivity and C_p is the specific heat at constant pressure.

The Eqs. (1)-(3) are subjected to the following boundary conditions,

$$\begin{aligned} &v = v_w,\ u = u_w = cx,\ T = T_w(x) = T_\infty + bx \ \text{ at } \ y = 0, \\ &u \to u_e(x) \to ax,\ T \to T_\infty \ \text{ as } \ y \to \infty. \end{aligned} \tag{4}$$

Due to hydrostatic and magnetic pressure gradient, the forces will be equilibrium as given below

$$-\frac{1}{\rho}\frac{dp}{dx} = u_e\frac{\partial u_e}{\partial x} + \frac{\sigma\mu_e^2 H_0^2}{\rho}u. \tag{5}$$

Hence, the equation of momentum (2) becomes

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = u_e\frac{\partial u_e}{\partial x} + \frac{\sigma\mu_e^2 H_0^2}{\rho}(u - u_e) + \nu\frac{\partial^2 u}{\partial y^2} + g\beta(T - T_\infty). \tag{6}$$

B. *Similarity Transformation*

The Eqs. (1), (3) and (6) subject to boundary conditions (4) can be expressed in a simpler form by applying the following transformation [14]

$$\eta = \sqrt{\frac{a}{\nu}}y,\ \psi = \sqrt{a\nu}\,x f(\eta),\ \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \tag{7}$$

where η is the similarity variable and ψ is the stream function which defined as

$$u = \frac{\partial \psi}{\partial y},\ v = -\frac{\partial \psi}{\partial x}, \tag{8}$$

Using (7), Eq. (1) is satisfied, while Eqs. (6) and (3) are reduced to the following ordinary differential equations

$$f''' + ff'' - (f')^2 + 1 + Ha^2(1 - f') + \lambda\theta = 0, \tag{9}$$

$$\theta'' + \Pr(f\theta' - f'\theta + \delta\theta) = 0, \tag{10}$$

along with the corresponding boundary conditions

$$\begin{aligned} &f(0) = s,\ f'(0) = \frac{c}{a} = \gamma,\ \theta(0) = 1, \\ &f'(\eta) \to 1,\ \theta(\eta) \to 0 \ \text{ as } \ \eta \to \infty, \end{aligned} \tag{11}$$

where $Ha = \mu_e^2 H_0^2 \sqrt{\frac{\sigma}{\rho a}}$ is the Hartmann number, $\lambda = \frac{g\beta(T_w - T_\infty)}{a^2 x}$ is the mixed convection parameter, $Pr = \frac{v}{\alpha} = \frac{\mu C_p}{k}$ is Prandtl number, $\delta = \frac{Q}{\rho a C_p}$ is the heat generation/absorption coefficient, $s = \frac{-v_w}{\sqrt{a v}}$ is the suction/injection parameter and $\gamma = \frac{c}{a}$ is the velocity ratio parameter.

C. *Physical Quantity of Interest*

The important physical quantities of interest in this problem are the skin friction coefficient, C_f and the local Nusselt number, Nu_x which are defined as

$$C_f = \frac{\tau_w}{\rho\mu_e^2},\ Nu_x = \frac{xq_w}{k(T_w - T_\infty)}, \tag{12}$$

where the wall shear stress τ_w and the heat flux q_w are given by

$$\tau_w = \mu\left(\frac{\partial u}{\partial y}\right)_{y=0},\ q_w = -k\left(\frac{\partial T}{\partial y}\right)_{y=0}. \tag{13}$$

Using (7) and (13), Eq. (12) becomes

$$\sqrt{\mathrm{Re}_x}C_f = f''(0),\ \frac{Nu_x}{\sqrt{\mathrm{Re}_x}} = -\theta'(0), \tag{14}$$

where $Re_x = \mu_e(x)\frac{x}{v}$ is the Reynolds number.

III. Results and Discussion

The system of nonlinear ordinary differential equations (9) and (10) subject to the boundary conditions (11) have been solved numerically using the MATLAB boundary value problem solver bp4vc [15]. The relative error tolerance was fixed to 10^{-5} throughout the numerical computation. The boundary layer thickness is set to $\eta = 12$ to ensure the profiles generated satisfy the boundary conditions (11) asymptotically. In order to verify the accuracy of the present method, a comparison with a previous literature is made. The comparison, as shown in Table

I, is observed to be in a good agreement, thus we are confident that our method is accurate.

Table 1. Comparison of numerical values of skin friction coefficient $f''(0)$ and local nusselt number $-\theta'(0)$ for different values of Pr when $\lambda = 1, s = Ha = \delta = \gamma = 0$.

	[14]		Present	
Pr	$f''(0)$	$-\theta'(0)$	$f''(0)$	$-\theta'(0)$
1	1.67550	0.8708	1.67544	0.8708
10	1.49282	1.9448	1.49284	1.9446
20	1.44840	2.4579	1.44848	2.4576
40	1.41010	3.1017	1.41006	3.1011
60	1.39030	3.5524	1.39027	3.5514
80	1.37740	3.9108	1.37739	3.9095
100	1.36800	4.2133	1.36803	4.2116

Figs. 1 and 2 illustrate the velocity profiles $f'(\eta)$ for various values of velocity ratio parameter γ for the cases of injection $s = -2$ and suction $s = 2$, respectively. Both cases of assisting $(\lambda > 0)$ and opposing $(\lambda < 0)$ flows are considered when these figures are plotted. It is observed that the boundary layer thickness is thinner for assisting flow compared to the opposing flow. It can also be seen that the boundary layer thickness is smaller for the case of suction, as illustrated in Fig. 2. These phenomena is caused by the effect of suction itself, where it sucks the slow moving fluid molecules to the plate, thus making the fluid moves closer and closer to the surface, hence thinning the boundary layer thickness.

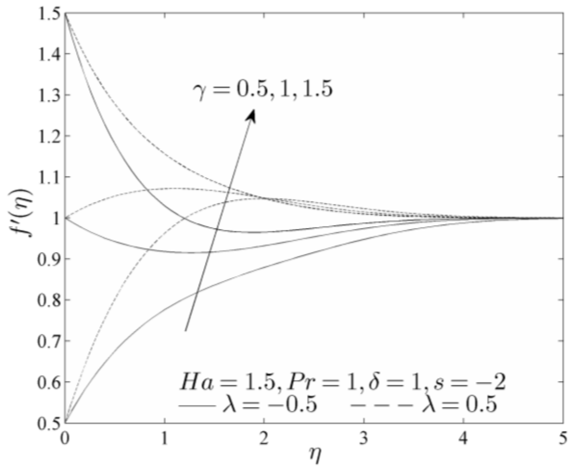

Fig. 1. Velocity profiles $f'(\eta)$ for various values of γ when $s = -2$.

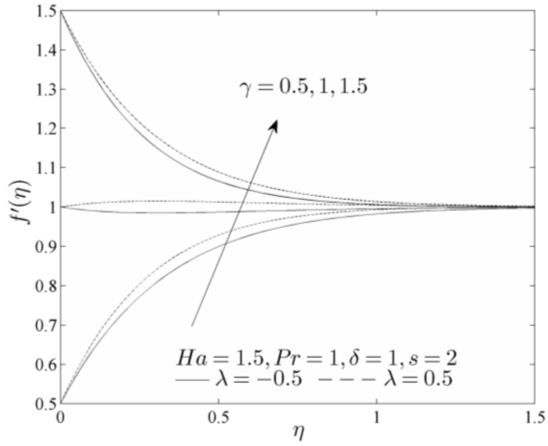

Fig. 2. Velocity profiles $f'(\eta)$ for for various values of γ when $s = 2$.

Furthermore, the temperature profiles $\theta(\eta)$ for various values of γ when $s = -2$ and $s = 2$ while other parameters are remained constant are displayed in Figs. 3 and 4. From both figures, it is observed that the boundary layer thickness decreases with the increase of γ and s. This shows that the heat transfer process between the fluid and the surface is more effective when suction is applied to the flow, because more fluid is trapped adjacent to the plate. The profiles plotted in Figs. 1-4 satisfy the far field boundary conditions (11) asymptotically, thus supporting the numerical results obtained in this study.

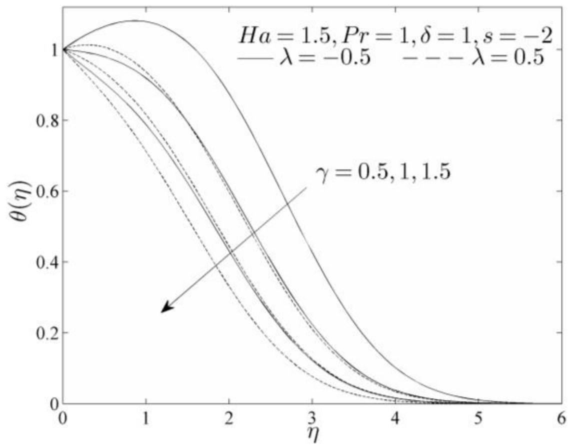

Fig. 3. Temperature profiles $\theta(\eta)$ for various values of γ when $s = -2$.

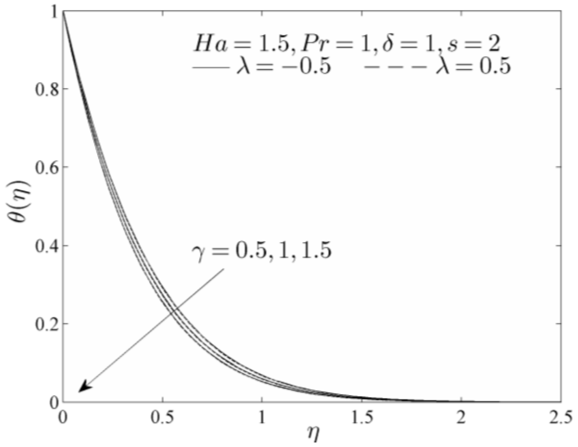

Fig. 4. Temperature profiles $\theta(\eta)$ for for various values of γ when $s = 2$.

IV. Conclusion

In this paper, the problem of magnetohydrodynamic mixed convection stagnation point flow along a vertical stretching permeable surface with heat source or sink is considered. The governing nonlinear boundary layer equations in the form of partial differential equations are transformed into the system of nonlinear ordinary differential equation using similarity transformation, which then solved numerically using the bvp4c function in MATLAB software. We found that the boundary layer thickness decrease with the increase of velocity ratio parameter and when suction is applied to the surface.

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