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Boundary Layer Flow And Heat Transfer On A Moving Plate In A Carbon Nanotubes

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***Abstract*—The problem of boundary layer flow and heat transfer characteristics on a moving plate in a carbon nanotubes was studied in this paper. The plate is presumed to move to the free stream in the same or reverse direction. By using a similarity transformation, the governing partial differential equations are transformed into a system of nonlinear ordinary differential equations that is then solved numerically using a shooting method in Maple software. It is solved for three different base fluids which is water, kerosene and engine oil and using both SWCNT and MWCNT. While before this, only solve in water for different nanoparticles. The numerical results are presented in tables and graphs. The findings show that when the plate and the free stream move in the opposite direction, the dual solution exists.**

Keywords— Boundary layer flow; carbon nanotubes; dual solutions; heat transfer; moving plate

I. INTRODUCTION

The boundary layer relates to the fluid layer in the immediate vicinity of the boundary surface where the viscosity impacts are important.. In 1908, Blasius solved the boundary layer problem for a free stream past a fixed flat plate, while the boundary layer on continuous moving surface was first examined by Sakiadis in 1961. There are two cases when consider about moving plate. First case when the plate and the fluid move in the same direction, while the second case when the plate and fluid move in the opposite direction. Tsou et al. [1] studied the boundary layer flow on a continuous moving plate where they consider both laminar and turbulent flow conditions. Klemp and Acrivos [2] were the first who discussed the momentum boundary layer problem for a moving semi-infinite flat plate. After that, many authors interested to study on boundary layer flow and heat transfer on a moving plate [3-7].

In previous papers, they did not consider nanofluid. Nanofluid was first introduced by Choi [8], where it is a fluid containing nanometer-sized particles, called nanoparticles. There are two model of nanofluids which are Buongiorno [9] where he studied the boundary layer of nanofluids, and has seven slip mechanisms capable of producing a relative velocity between nanoparticles and base fluid, but only thermophoresis and Brownian motion have been efficient in modeling nanofluid. The other model was Tiwari and Das [10], where they consider solid volume fraction. There are several papers studied on boundary layer flow and heat transfer on moving plate in nanofluids where they consider one of the model mention [11-19].

Differ from previous paper who consider nanofluid, this paper consider carbon nanotubes (CNTs). CNTs is an allotropes of carbon, tube-shaped material, made of carbon and consist of single-wall and multi-wall CNTs. Some of the authors who studied boundary layer flow and heat transfer in a carbon nanotubes, which are Khan et al. [20], where they investigated flow and heat transfer of carbon nanotubes along a flat plate subjected to Navier slip and uniform heat flux boundary conditions. After that, many authors developed an interest to study about carbon nanotubes on different surfaces [21-27]. But neither of them study on moving plate. So, the aim of this paper is to investigate boundary layer flow and heat transfer in a carbon nanotubes on a moving plate.

II. PROBLEM FORMULATION

2D flow over a moving plate with heat transfer in a water/oil-based nanofluid containing single-wall and multi-wall CNTs was considered. The flow is assumed to be laminar, steady, and incompressible. The base fluids and the CNTs are assumed to be n thermal equilibrium. The standard boundary layer equations for this problem can be written as follows:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial v}{\partial y}=\frac{\mu_{nf}}{\rho_{nf}}\frac{\partial^2 u}{\partial y^2} \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha_{nf}\frac{\partial^2 T}{\partial y^2} \tag{3}$$

subject to the boundary conditions:

$$u=U_w,\ v=0,\ T=T_w \ \text{ at } y=0$$
$$u\to U_\infty,\ T\to T_\infty \ \text{ as } y\to\infty \tag{4}$$

where U_w and U_∞ are constants and correspond to the plate velocity and the free stream velocity, respectively. Further, u and v are the velocity components along x- and y-directions, respectively, T is the temperature of the nanofluid, μ_{nf}, α_{nf} and ρ_{nf}

are the viscosity, thermal diffusivity and density of the nanofluid, respectively, which are given by Oztop and Abu-Nada [28]:

$$\alpha_{nf} = \frac{k_{nf}}{(\rho C_p)_{nf}}, \mu_{nf} = \frac{\mu_f}{(1-\varphi)^{2.5}}, \rho_{nf} = (1-\varphi)\rho_f + \varphi\rho_{CNT}, (\rho C_p)_{nf} = (1-\varphi)(\rho C_p)_f + \varphi(\rho C_p)_{CNT},$$

$$\frac{k_{nf}}{k_f} = \frac{1-\varphi+2\varphi\frac{k_{CNT}}{k_{CNT}-k_f}\ln\frac{k_{CNT}+k_f}{2k_f}}{1-\varphi+2\varphi\frac{k_f}{k_{CNT}-k_f}\ln\frac{k_{CNT}+k_f}{2k_f}} \quad (5)$$

where φ is the nanoparticle volume fraction, $(\rho C_p)_{nf}$ and $(\rho C_p)_{CNT}$ are the heat capacity of the nanofluid and carbon nanotubes, k_{nf} is the thermal conductivity of nanofluid, k_f and k_{CNT} are the thermal conductivities of the fluid and carbon nanotubes, respectively, and ρ_f and ρ_{CNT} are the densities of the fluid and carbon nanotubes, respectively. The use of the term for k_{nf}/k_f were taken from Xue [29] where the Maxwell theory model takes into account the impact of CNT space distribution on thermal conductivity.

The governing equations (1)-(3) subject to the boundary conditions (4) can be expressed in a simpler form by introducing the following transformation:

$$\eta = \left(\frac{U}{v_f x}\right)^{1/2} y, \psi = (v_f x U)^{\frac{1}{2}} f(\eta), \theta(\eta) = \frac{T-T_\infty}{T_w - T_\infty} \quad (6)$$

where U is the composite velocity defined as $U = U_w + U_\infty$. ψ is the stream function defined as $u = \frac{\partial\psi}{\partial y}$ and $v = -\frac{\partial\psi}{\partial x}$, which identically satisfies (1). Employing the similarity variables (7), (2)-(3) reduce to the following ordinary differential equations as:

$$\frac{1}{Pr}\left[\frac{k_{nf}/k_f}{[1-\varphi+\varphi(\rho C_p)_{CNT}/(\rho C_p)_f]}\right] + \frac{1}{2} f\theta' = 0 \quad (8)$$

subject to the boundary conditions (4) which becomes

$$f(0) = 0, f'(0) = \lambda, \theta(0) = 1,$$
$$f'(\eta) \to 1-\lambda, \theta(\eta) \to 0 \text{ as } \eta \to \infty \quad (9)$$

The physical quantities of interest, which are the skin friction coefficient C_f and the local Nusselt number Nu_x, are defined as

$$C_f = \frac{\tau_w}{\rho_f U^2}, \quad Nu_x = \frac{x q_w}{k_f(T_w - T_\infty)} \quad (10)$$

where the surface shear stress τ_w and the surface heat flux q_w are given as

$$\tau_w = \mu_{nf}\left(\frac{\partial u}{\partial y}\right)_{y=0}, \quad q_w = -k_{nf}\left(\frac{\partial T}{\partial y}\right)_{y=0} \quad (11)$$

with μ_{nf} and k_{nf} being the dynamic viscosity and thermal conductivity of the nanofluids, respectively. By using the similarity variables (6) into (10), we obtain

$$C_f Re_x^{1/2} = \frac{1}{(1-\varphi)^{2.5}} f''(0), \quad (12)$$

$$Nu_x / Re_x^{\frac{1}{2}} = -\frac{k_{nf}}{k_f}\theta'(0), \quad (13)$$

where $Re_x = Ux/v_f$ is the local Reynolds number.

III. DISCUSSION

The nonlinear ordinary differential equations (7)-(8) subject to the boundary conditions (9) has been solved numerically using the shooting method. The effects of the solid volume fraction of carbon nanotubes φ and the Prandtl number Pr are analyzed for three different base fluids, namely water, kerosene and engine oil. We consider both single-wall and multi-wall CNTs. The Prandtl numbers taken to be $Pr = 6.2$, $Pr = 21$ and $Pr = 155$ for base fluids water, kerosene and engine oil, repectively. The thermophysical properties of the base fluid and the carbon nanotubes are listed in Table 1. abc

Table 1 : The thermophysical properties of the base fluid and CNTs [28, 30-33]

Physical properties	Base fluids			Carbon nanotubes	
	Water	Kerosene	Engine oil	SW CNT	MW CNT
$\rho(kg/m^3)$	997	783	884	2600	1600
$c_p(J/kgK)$	4179	2090	1910	425	796
$k(W/mK)$	0.613	0.145	0.144	6600	3000

Figs. 1 and 2 illustrate the variation of $f''(0)$ and $-\theta'(0)$ with some values of the velocity ratio parameter λ, for three different values of carbon nanotube volume fraction φ, where $\varphi = 0$, 0.1 and 0.2 for water base fluid with $Pr = 6.2$. While Figs. 3 and 4 illustrate the variation of $f''(0)$ and $-\theta'(0)$ with some values of the velocity ratio parameter λ, for three different base fluids, which are water, kerosene and engine oil when $\varphi = 0.1$.

There are regions of unique solutions for $\lambda > 0$, dual solutions for $\lambda_c < \lambda \le 0$ when the plate and the free stream move in the opposite direction, and no solutions for $\lambda < \lambda_c < 0$. Therefore, the solutions exist up to the critical value $\lambda = \lambda_c < 0$, beyond which no solutions exist. Based on our computation, $\lambda_c = -0.5482$. this value of λ_c is in agreement with those reported by Bachok et al.[15].

Figs. 5 and 6 show the variations of skin friction coefficient and the local Nusselt number, given by (12) and (13) with the carbon nanotube volume fraction φ, for three different base fluids: water, kerosene and engine oil with $\lambda = 0.2$. It shows that these quantities increase almost linearly with φ. In addition, it is noted that the lowest heat transfer rate is water. This is because water has the highest thermal conductivity compared to kerosene and engine oil, as shown in Table 1.

The velocity and temperature profiles for some values of parameters are presented in Figs. 7-12. The term for both first and second solutions refer to the curves shown in Figs. 1-4, where the first solution has larger value of $f''(0)$ and $-\theta'(0)$ compared to the second solution. These profiles satisfy the boundary conditions (9) asymptotically, which supporting the existence of dual solutions shown in Figs. 1-4.

IV. CONCLUSION

In this paper, we have studied theoretically and analysed the effects of carbon nanotubes volume fraction on the fluid flow and heat transfer characteristics. The problem was solved by using a shooting method by Maple software. The results indicated that:

1. Dual solutions were found to exist when the plate and the free stream move in the opposite direction.
2. The value of $\lambda_c = -0.5482$ for carbon nanotube and nanofluid (Tiwari and Das) are the same.
3. The skin friction coefficients and the local Nusselt number increases as the carbon nanotube volume fraction increases,
4. Engine oil-SWCNT have higher heat transfer rates than water- and kerosene-SWCNT.
5. SWCNT more efficient than MWCNT in both skin friction coefficient and local Nusselt number.

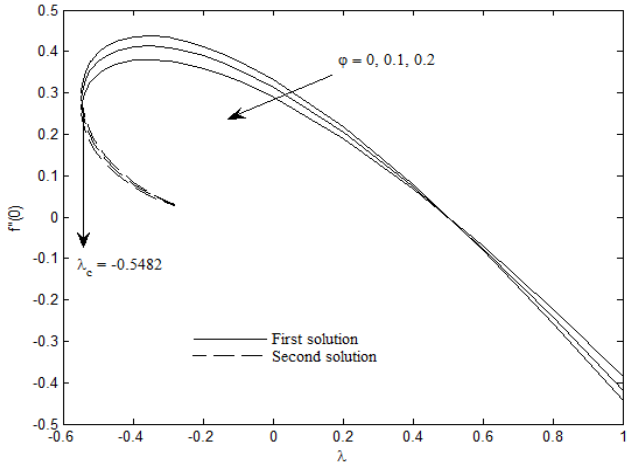

Fig. 1 : *Variation of $f''(0)$ with λ for some values of φ for water-SWCNT.*

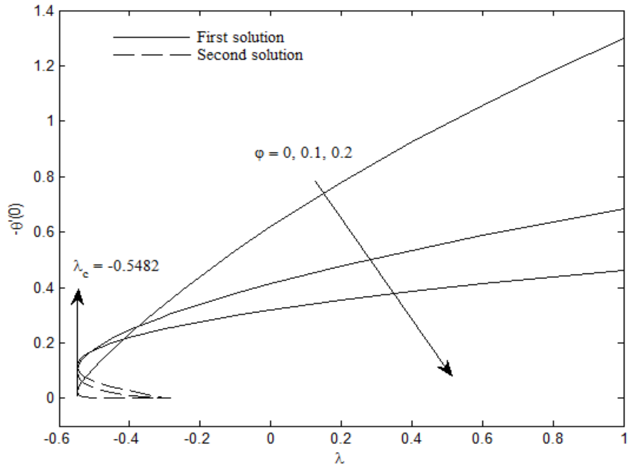

Fig. 2: *Variation of -$\theta'(0)$ with λ for some values of φ for water-SWCNT.*

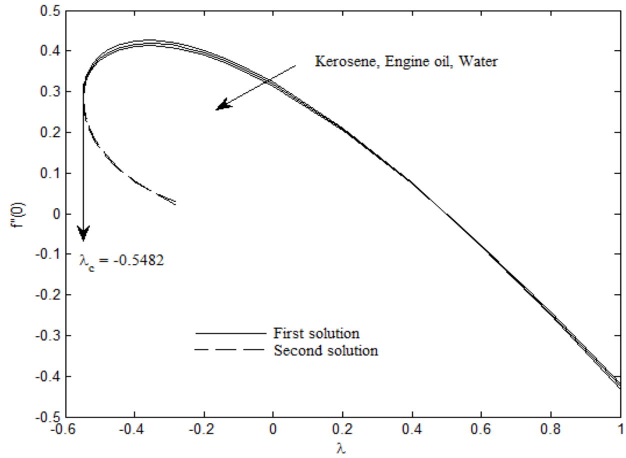

Fig. 3: *Variation of $f''(0)$ with λ for different base fluids with SWCNT and $\varphi = 0.1$.*

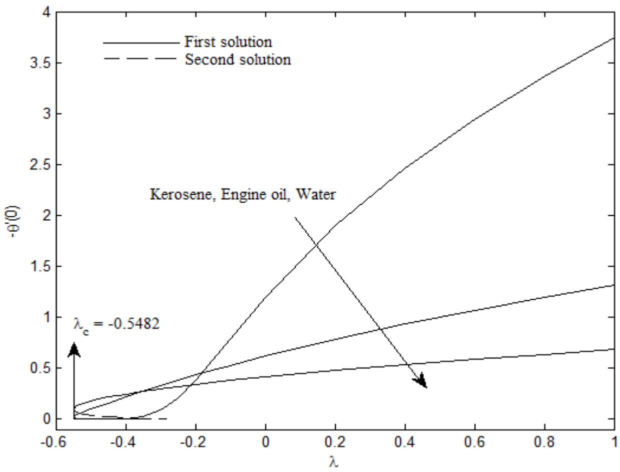

Fig. 4: *Variation of -$\theta'(0)$ with λ for different base fluids with SWCNT and $\varphi = 0.1$.*

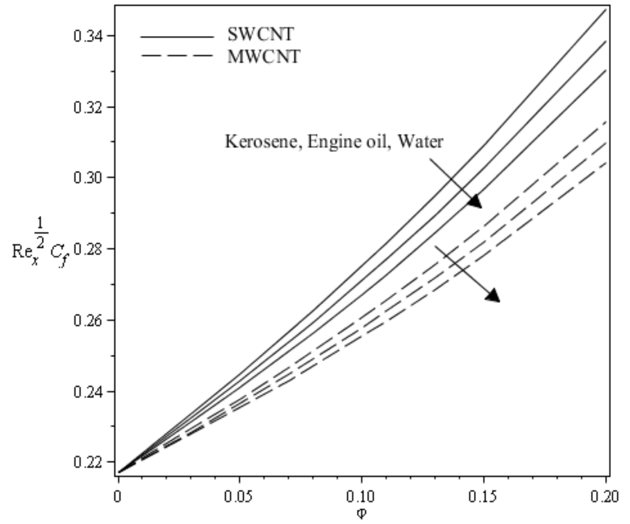

Fig. 5: *Variation of skin friction coefficient with φ for different base fluids and carbon nanotubes when $\lambda = 0.2$.*

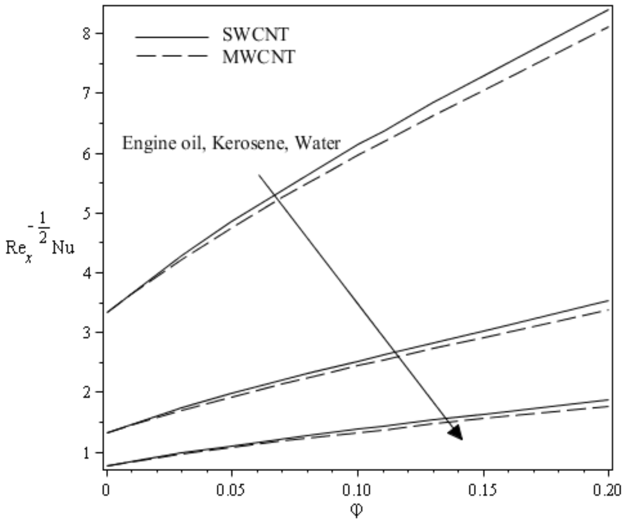

Fig. 6: *Variation of Nusselt number with φ for different base fluids and carbon nanotubes when $\lambda = 0.2$.*

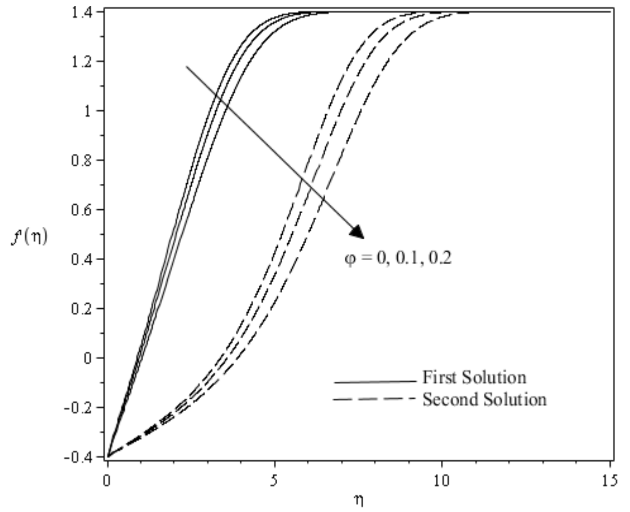

Fig. 7: *Velocity profiles for some values of φ for water-SWCNT when $\lambda = -0.4$.*

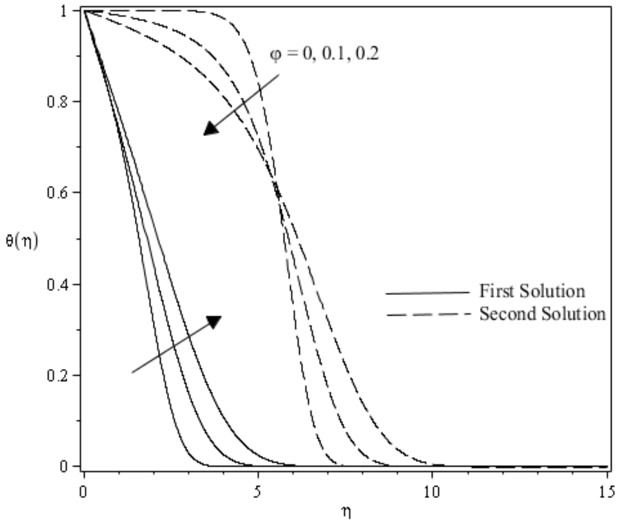

Fig. 8: *Temperature profiles for some values of φ for water-SWCNT when $\lambda = -0.4$.*

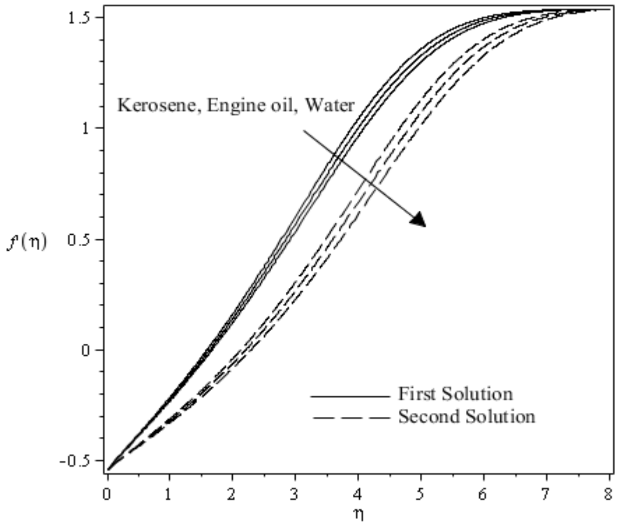

Fig. 9: *Velocity profiles for different base fluids with SWCNT, $\varphi = 0.2$ and $\lambda = 0.54$.*

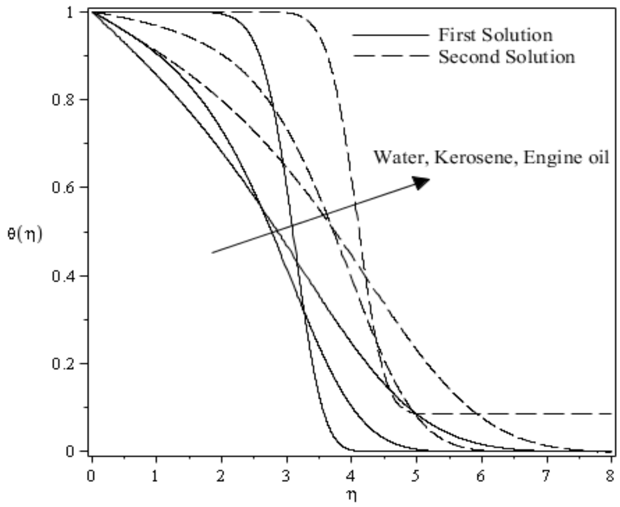

Fig. 10: *Temperature profiles for different base fluids with SWCNT, $\varphi = 0.2$ and $\lambda = 0.54$.*

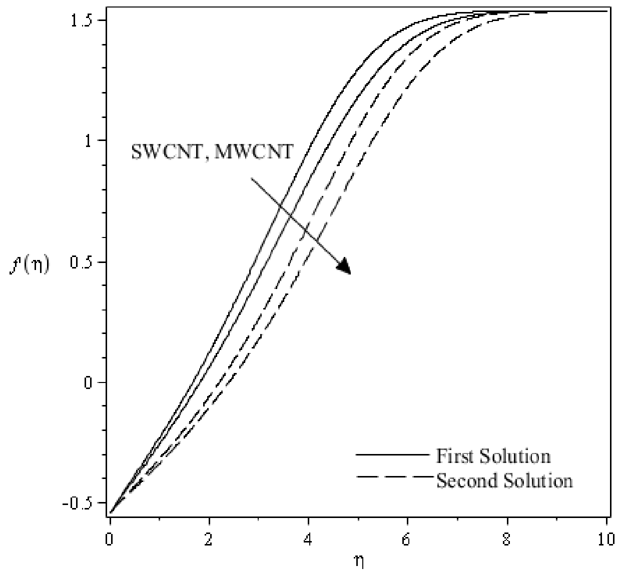

Fig. 11: *Velocity profiles for different types of carbon nanotube with water base fluid when* $\varphi = 0.2$ *and* $\lambda = 0.54$.

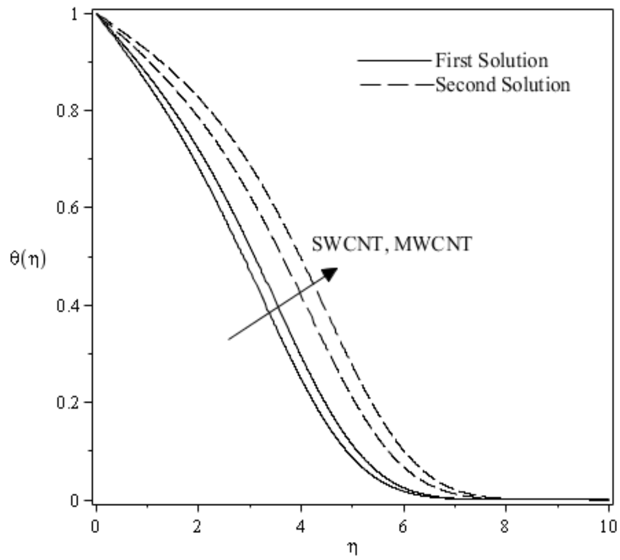

Fig. 12: *Temperature profiles for different types of carbon nanotube with water base fluid when* $\varphi = 0.2$ *and* $\lambda = 0.54$.

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