Electrothermal Instability in an Anisotropic Porous Medium Saturated by Nanofluid with Magnetic Field

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***Abstract*— The combined effects of magnetic field and electric field on the onset of thermal convection in an anisotropic saturated by nanofluid has been studied. The nanofluid layer bounded by rigid-rigid, rigid-free and free-free boundaries with zero flux of nanoparticles volumetric fraction. Linear stability theory is employed and the characteristic equation for Rayleigh number is obtained by using the single term Galerkin approximation. The results are presented graphically to observe the effect of electric Rayleigh number, magnetic field, mechanical anisotropy parameter as well as thermal anisotropy parameter.**

Keywords*— *thermal convection, anisotropy, nanofluid, magnetic field, electric field

I. INTRODUCTION

Electrothermal instability is a natural convection under the influence of alternate current (AC) electric field in a dielectric fluid. There are few practical applications of electrohyrodynamics discussed in [1] and [2]. The stability of dielectric fluid in the presence of electric field has been studied and it is found that electric force plays an important role in driving the motion of poor electrically conducting fluid [3,4]. The study of electrothermal convection attracts researchers due to its wide and growing applications in electronic devices and electrical equipment. Reference [5] and [6] considered rotational effect on the electrohydrodynamic convection in dielectric fluid with the effect of velocity and temperature boundary conditions. Then, [7] studied the onset of electroconvection in a dielectric nanofluid saturated porous layer while [8] considered anisotropy effect and thermal modulation on the electro thermal instability.

The existence of nanoparticle in a base fluid like water and oil making the fluid called as nanofluid [9]. Nanofluid is a promising fluid that has been used widely in many applications such as drug delivery, electronic application and energy supply. The fact is due to the nanoparticle that presence in fluid help to increase the thermal conductivity of fluid, thus it enhances the transfer of heat. The nanofluid model developed by [10] is used in [11-16] to study the thermal convection in a nanofluid layer. Reference [17] considered mechanical and thermal anisotropy parameter in the problem of the onset of convection in porous medium saturated by a nanofluid. These literatures have been employing the Buongiorno model which involves the conservation equations of a non-homogeneous equilibrium model of nanofluid that incorporates the Brownian motion and thermophoresis. [18] revisited the study on the onset of convection in nanofluid saturated porous layer by imposing zero-flux for nanoparticle fraction on the boundary which is more realistic. Zero-flux nanoparticle means the value of nanoparticle fraction on the boundary adjust accordingly. This motivates [19] to revise [17] as well as considering the more realistic boundary condition.

The study of magnetic field effect on Rayleigh Benard convection in fluid pioneered by [20]. Since then, Chandrasekhar number is introduced to represent non-dimensional parameter of magnetic field. The investigation on the effect of magnetic field on the thermal instability of nanofluid under different situation can be referred to literatures [21-24]. The researches on nanofluid convection under the influence of magnetic field seem to be significant due to its wide range applications in physics and engineering. There are several wide applications are such as magnetic storage media, magnetohydrodynamics generators and magnetic field sensors.

To the best of our knowledge, none of the literature above consider magnetic field effect on the electrothermal convection in a dielectric nanofluid saturated anisotropic porous medium. The present problem attempts to extend the study of paper [7] and [8] by considering the effect of magnetic field along with electric field effect on the thermal instability in an anisotropic porous medium saturated by a dielectric nanofluid. This investigation incorporates three different type of boundary conditions which are free-free, rigid-free and rigid-rigid. We performed linear stability analysis and obtained Rayleigh number as eigenvalue by using the Galerkin method.

II. MATHEMATICAL FORMULATION

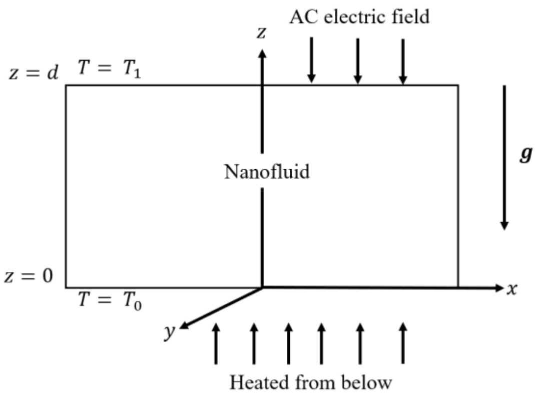


Fig.1. *Configuration of the problem.*

Use a Cartesian coordinate system (x, y, z) in which $z-$ axis points vertically upward, we consider a horizontal layer of nanofluid saturated anisotropic porous medium confined between two plates $z \in (0, d)$ is heated from below and is subjected to a vertical AC electric field as illustrated in Fig.1. The temperature at lower and upper plates are denoted by T_0 and T_1 respectively, which T_0 is greater than T_1 and the normal component of nanoparticle flux has to vanish at an impermeable boundary. T_1 and ϕ_0 are taken to be the reference scale for temperature and nanoparticle fraction respectively. A uniform vertical magnetic field $\boldsymbol{H} = (0, 0, H_0)$ acts on the system. Following [7,19,24] the governing equations of the problem under Boussinesq approximation are:

$$\nabla \cdot \boldsymbol{v} = 0, \tag{1}$$

$$\nabla p + \mu \boldsymbol{K}^{-1}\boldsymbol{v} = \left[\phi\rho_p + (1-\phi)\rho_0\{1-\beta(T-T_1)\}\right]\boldsymbol{g} + \boldsymbol{f_e} + \frac{\mu_m}{4\pi}(\nabla \times \boldsymbol{h}) \times \boldsymbol{H}, \tag{2}$$

$$(\rho c)_m \frac{\partial T}{\partial t} + (\rho c)_f (\boldsymbol{v}\cdot\nabla) T = k_m \nabla^2 T + \varepsilon(\rho c)_p \left[D_B \nabla\phi \cdot \nabla T + \frac{D_T}{T_1}(\nabla T)^2\right], \tag{3}$$

$$\frac{\partial \phi}{\partial t} + \frac{1}{\varepsilon}\boldsymbol{v}\cdot\nabla\phi = D_B \nabla^2 + \frac{D_T}{T_1}\nabla^2 T, \tag{4}$$

magnetic induction equations

$$\frac{d\boldsymbol{h}}{dt} = (\boldsymbol{H}\cdot\nabla)\frac{v}{\varepsilon} + \tau\nabla^2\boldsymbol{h}, \tag{5}$$

$$\nabla \cdot \boldsymbol{h} = 0, \tag{6}$$

where $\nabla = \frac{\partial}{\partial x}\hat{\imath} + \frac{\partial}{\partial y}\hat{\jmath} + \frac{\partial}{\partial z}\hat{k}$, $\boldsymbol{v} = (u, v, w)$ is the Darcy velocity, p is the pressure, ρ_p is the nanoparticle density, ρ_0 is the nanofluid density at the lower boundary, μ is the viscosity, μ_m is the magnetic permeability, β is the thermal expansion coefficient, ε is the porosity of the porous medium, $\boldsymbol{g}$ is the gravitational force, ϕ is the nanoparticle volume fraction, T is the temperature, $(\rho c)_m$ is the heat capacity of fluid in porous medium, $(\rho c)_f$ is the heat capacity of nanofluid, $(\rho c)_p$ is the heat capacity of nanoparticles, D_T is the thermophoretic diffusion coefficient and D_B is the Brownian diffusion coefficient, $\boldsymbol{h}$ is the component of magnetic field and τ is the resistivity of the fluid. The anisotropic permeability tensor and thermal conductivity tensor are denoted by $\boldsymbol{K}^{-1} = K_x^{-1}(\hat{\imath}\hat{\imath} + \hat{\jmath}\hat{\jmath}) + K_z^{-1}(\hat{k}\hat{k})$ and $k_m = k_{mx}^{-1}(\hat{\imath}\hat{\imath} + \hat{\jmath}\hat{\jmath}) + k_{mz}^{-1}(\hat{k}\hat{k})$, respectively, where K_x and K_z are the permeability, k_{mx} and k_{mz} are the thermal conductivity in the x and z direction, respectively.

$\boldsymbol{f_e}$ is the force of electrical origin which can be stated as:

$$\boldsymbol{f_e} = \rho_e \mathbf{E} - \frac{1}{2}(\mathbf{E}\cdot\mathbf{E})\nabla\epsilon + \frac{1}{2}\nabla\left(\rho\frac{\partial\epsilon}{\partial p}\mathbf{E}\cdot\mathbf{E}\right) \tag{7}$$

where $\mathbf{E}$ is the electric field, ρ_e is the charge density and ϵ is the dielectric constant. It is noted that the last term in (7) can be grouped into pressure term in (2) and it does not affect on the incompressible fluid. The Coulomb force which is the first term on the right-hand side is of negligible order compared with dielectrophoretic force term for most dielectric fluids in a 60-Hz AC electric field thus it is neglected. The dielectrophoretic force is denoted in second term which the only retain and it depends on $(\mathbf{E}\cdot\mathbf{E})$ rather $\mathbf{E}$. Considering the variation of $\mathbf{E}$ is very rapid, the root mean square value of $\mathbf{E}$ is applied as the effective value [5-8].

The Maxwell equations are

$$\nabla \times \mathbf{E} = 0, \tag{8}$$

$$\nabla \cdot (\epsilon\mathbf{E}) = 0. \tag{9}$$

Employing (8), the electric field can be expressed as

$$\mathbf{E} = -\nabla\psi \tag{10}$$

where ψ is the root mean square value of the electric potential. The electrical conductivity is considered to vary linearly function with temperature in the form

$$\epsilon = \epsilon_0[1 - \delta(T - T_1)] = 0, \tag{11}$$

where $\delta(> 0)$ is the thermal coefficient of electrical conductivity.

We assume the temperature is fixed and there is no vertical nanoparticle flux on the boundaries which is physically more realistic. The boundary conditions for:

free-free boundary

$$w = 0, \ \frac{\partial^2 w}{\partial z^2} = 0, \ T = T_0, \ D_B\frac{\partial\phi}{\partial z} + \frac{D_T}{T_1}\frac{\partial T}{\partial z} = 0 \tag{12a}$$

at $z = 0$,

$$w = 0, \ \frac{\partial^2 w}{\partial z^2} = 0, \ T = T_1, \ D_B\frac{\partial\phi}{\partial z} + \frac{D_T}{T_1}\frac{\partial T}{\partial z} = 0 \tag{12b}$$

at $z = 1$,

rigid-free boundary

$$w = 0,\ \frac{\partial w}{\partial z} = 0,\ T = T_0,\ D_B\frac{\partial \phi}{\partial z} + \frac{D_T}{T_1}\frac{\partial T}{\partial z} = 0 \quad (13a)$$

at $z = 0$,

$$w = 0,\ \frac{\partial^2 w}{\partial z^2} = 0,\ T = T_1,\ D_B\frac{\partial \phi}{\partial z} + \frac{D_T}{T_1}\frac{\partial T}{\partial z} = 0 \quad (13b)$$

at $z = 1$,

rigid-rigid boundary

$$w = 0,\ \frac{\partial w}{\partial z} = 0,\ T = T_0,\ D_B\frac{\partial \phi}{\partial z} + \frac{D_T}{T_1}\frac{\partial T}{\partial z} = 0 \quad (14a)$$

at $z = 0$,

$$w = 0,\ \frac{\partial w}{\partial z} = 0,\ T = T_1,\ D_B\frac{\partial \phi}{\partial z} + \frac{D_T}{T_1}\frac{\partial T}{\partial z} = 0 \quad (14b)$$

at $z = 1$.

The basic state of the nanofluid is taken to be quiescent layer

$$\boldsymbol{v} = 0,\ p = p_b(z),\ T = T_b(z),\ \phi = \phi_b(z),$$
$$\epsilon = \epsilon_b(z),\ \mathbf{E} = \mathbf{E}_b(z),\ \psi = \psi_b(z),$$
$$\boldsymbol{H} = \boldsymbol{H}e_z \quad (15)$$

where subscript b indicates the basic state. The solution of basic state by referring to [8] are given by

$$T_b = T_0 - \frac{\Delta T}{d}z,$$

$$\phi_b = \phi_0 + \left(\frac{D_T\Delta T}{D_B T_1}\right)z$$

$$\epsilon_b = \epsilon_0\left(1 + \frac{\delta\Delta T}{d}\right)z\hat{k}$$

$$E_b = E_0\left(\frac{1}{1 + \frac{\delta\Delta T}{d}z}\right)\hat{k}$$

$$\psi_b = -\frac{E_0 d}{\delta\Delta T}\log\left(1 + \frac{\delta\Delta T}{d}\right)\hat{k}$$

where $\Delta T = T_0 - T_1$ and $E_0 = -\frac{\psi_1\frac{\delta\Delta T}{d}}{\log(1+\delta\Delta T)}$ is the root mean square value of the electric field at the lower boundary.

The basic solution is slightly being perturbed and the perturbed state in the form

$$\boldsymbol{v} = \boldsymbol{v}',\ p = p_b + p',\ T = T_b + T',$$
$$\phi = \phi_b + \phi',\ \epsilon = \epsilon_b + \epsilon',$$
$$\mathbf{E} = \mathbf{E}_b + \mathbf{E}',\ \psi = \psi_b + \psi', \quad (16)$$

where $\boldsymbol{v}', p', T', \phi', \epsilon', \mathbf{E}'$ and ψ' are the perturbed quantities.

We apply the small disturbance on the initial state by substituting (16) into (1)-(14). Vanishing the pressure from the momentum equation by operating curl twice and retaining the z-component. Thus, (2) becomes

$$\mu\left(\nabla_H^2 + \frac{1}{\xi}\frac{\partial^2}{\partial z^2}\right)w' = \nabla_H^2\phi'[\rho_p - \rho_0]\boldsymbol{g} - \nabla_H^2(T' - T_1)\rho_0\beta\boldsymbol{g} + \frac{\epsilon_0 E_0^2\delta^2\Delta T}{d}\nabla_H^2 T' - \frac{\epsilon_0 E_0\delta\Delta T}{d}\frac{\partial}{\partial z}\nabla_H^2\psi' - \boldsymbol{H}\frac{\partial}{\partial z}\nabla_H^2\boldsymbol{h}' \quad (17)$$

Nondimensionalizing the governing equations (3-14) and (17) by scaling

$$(x^*, y^*, z^*) = (d)(x', y', z')$$

$$(u^*, v^*, w^*) = \left(\frac{d}{\kappa_{Tz}}\right)(u', v', w')$$

$$t^* = \left(\frac{\kappa_{Tz}}{\sigma d^2}\right)t'$$

$$p^* = \left(\frac{K_z}{\mu\kappa_{Tz}}\right)p'$$

$$T^* = \frac{T' - T_1}{\Delta T}$$

$$\phi^* = \frac{\phi' - \phi_0}{\phi_0}$$

$$\psi^* = \left(\frac{1}{\delta\Delta T E_0 d}\right)\psi'$$

$$h^* = \left(\frac{\tau}{H_0\kappa_{Tz}}\right)h' \quad (18)$$

where $\kappa_{Tz} = \frac{k_{mz}}{(\rho c)_f}$ and $\sigma = \frac{(\rho c)_m}{(\rho c)_f}$. We obtain the nondimensional form after eliminating the asterisk $(*)$ for simplicity in the form:

$$\left(\frac{1}{\xi}\frac{\partial^2}{\partial z^2} + \nabla_H^2\right)w - Q\nabla^2\frac{\partial h_z}{\partial z} - Rt\nabla_H^2 T + Rn\nabla_H^2\phi$$
$$+Re\nabla_H^2\left(T - \frac{\partial\psi}{\partial z}\right) = 0, \quad (19)$$

$$w + \left[\left(\eta\nabla_H^2 + \frac{\partial^2}{\partial z^2}\right) - \left(\frac{\partial}{\partial t} + \frac{N_A N_B}{Le}\frac{\partial}{\partial z}\right)\right]T$$
$$-\frac{N_B}{Le}\frac{\partial}{\partial z}\phi = 0, \quad (20)$$

$$\frac{N_A}{\varepsilon}w - \frac{N_A}{Le}\nabla^2 T + \left(\frac{1}{\sigma}\frac{\partial}{\partial t} - \frac{1}{Le}\nabla^2\right)\phi = 0, \quad (21)$$

$$\frac{\varepsilon}{\sigma}\frac{Pr_2}{Pr_1}\frac{\partial\chi}{\partial t} - \frac{\partial\varsigma}{\partial z} - \varepsilon\nabla^2\chi = 0, \quad (22)$$

$$\frac{\varepsilon}{\sigma}\frac{Pr_2}{Pr_1}\frac{\partial h_z}{\partial t} - \frac{\partial w}{\partial z} - \varepsilon\nabla^2 h_z = 0, \quad (23)$$

$$\frac{\partial T}{\partial z} - \nabla^2\psi = 0, \quad (24)$$

where $\nabla_H^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$ is the two-dimensional horizontal Laplacian operator. (19), (23) and (24) have been obtained by eliminating ∇p and introducing $\chi = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$ and $\varsigma = \frac{\partial h_y}{\partial x} - \frac{\partial h_x}{\partial y}$.

The dimensionless parameters in (19)-(24) are

$Le = \frac{\kappa_{Tz}}{D_B}$ is the Lewis number,

$Rt = \frac{dK_z\Delta T\rho_0\beta\boldsymbol{g}}{\mu\kappa_z}$ is the thermal Rayleigh number,

$Re = \frac{K_z\epsilon_0 E_0^2\delta^2\Delta T}{\mu\kappa_z}$ is the electric Rayleigh number,

$Rn = \frac{dK_z\phi_0[\rho_p - \rho_0]\boldsymbol{g}}{\mu\kappa_z}$ is the concentration nanoparticle Rayleigh number,

$N_A = \frac{D_T \Delta T}{D_B T_1 \phi_0}$ is the modified diffusivity ratio,

$N_B = \varepsilon \frac{(\rho c)_p \phi_0}{(\rho c)_f}$ is the modified particle density,

$\xi = \frac{K_x}{K_z}$ is the mechanical anisotropy parameter,

$\eta = \frac{k_{mx}}{k_{mz}}$ is the thermal anisotropy parameter,

$Q = \frac{\mu_m H_0^2 d^2}{4\pi\mu\tau}$ is the Chandrasekhar number,

$Pr_1 = \frac{\mu}{\rho_0 \kappa_{Tz}}$ is the Prandtl number,

$Pr_2 = \frac{\mu}{\rho_0 \tau}$ is the magnetic Prandtl number.

The set of (19)-(24) will be subjected to normal mode analysis to analyze the behavior of the nanofluid system. Upon normal mode analysis, the perturbation quantities are taken to be

$$[w, T, \phi, h_z] = [W(z), \Theta(z), \Phi(z), K(z)] \times e^{i(a_x x + a_y y) + nt} \quad (25)$$

where a_x and a_y are wave number in (x, y) plane and n is the growth rate.

Setting $n = i\omega$, since the real part of n is zero for neutral stability (ω is real and is a dimensionless frequency). As we consider the case of stationary convection, n is equal to zero since $\omega = 0$. Then, substituting (25) into (19)-(24) yields the set of the stability equations:

$$\left(\frac{1}{\xi}D^2 - a^2\right) - Q(D^2 - a^2)DK + Rta^2\Theta - Rna^2\Phi + Rea^2(\Theta - D\Psi) = 0, \quad (26)$$

$$W + \left(D^2 - \eta a^2 - \frac{N_A N_B}{Le}D\right)\Theta + \frac{N_B}{Le}D\Phi = 0, \quad (27)$$

$$-\frac{N_A}{\varepsilon}W + \frac{N_A}{Le}(D^2 - a^2)\Theta + \frac{1}{Le}(D^2 - a^2)\Phi = 0, \quad (28)$$

$$D\Theta - (D^2 - a^2)\Psi = 0, \quad (29)$$

$$DW + \varepsilon(D^2 - a^2)K = 0, \quad (30)$$

where $D = \frac{d}{dz}$ and $a = \left(a_x^2 + a_y^2\right)^{1/2}$ is the wavenumber.

Upon eliminating (30), we let $(D^2 - a^2)K = -\frac{DW}{\varepsilon}$, then substitute it into (26) and we obtain

$$\left(\frac{1}{\xi}D^2 - a^2\right) + \frac{Q}{\varepsilon}D^2W + Rta^2\Theta - Rna^2\Phi + Rea^2(\Theta - D\Psi) = 0. \quad (31)$$

Thus, for further analysis we will be using (31) in exchange with (26)

Now (31) and (27)-(29) are solved subjected to the appropriate boundary conditions

free-free boundary

$$W = 0, \quad D^2W = 0, \quad \Theta = 0,$$
$$N_A D\Theta + D\Phi = 0, D\Psi = 0 \text{ at } z = 0,1 \quad (32)$$

Lower rigid- upper free boundary

$$W = 0, \quad DW = 0, \quad \Theta = 0,$$
$$N_A D\Theta + D\Phi = 0, D\Psi = 0 \text{ at } z = 0$$
$$W = 0, \quad D^2W = 0, \quad \Theta = 0,$$
$$N_A D\Theta + D\Phi = 0, D\Psi = 0 \text{ at } z = 1 \quad (33)$$

rigid-rigid boundary

$$W = 0, \quad DW = 0, \quad \Theta = 0,$$
$$N_A D\Theta + D\Phi = 0, D\Psi = 0 \text{ at } z = 0,1 \quad (34)$$

We solve the set of (31) and (27)-(29) numerically by using the Galerkin method in order to get an approximate solution to the system of equations. The basis functions W, Θ, Φ and Ψ were chosen accordingly:

$$W = \sum_{i=1}^{N} A_i W_i, \quad \Theta = \sum_{i=1}^{N} B_i \Theta_i,$$
$$\Phi = \sum_{i=1}^{N} C_i \Phi_i, \quad \Psi = \sum_{i=1}^{N} D_i \Psi_i \quad (35)$$

where A_i, B_i, C_i and D_i are constants and $i = 1,2,3 \ldots$. The basis function represented the power series satisfies boundary conditions (32)-(34) and W_i, Θ_i, Φ_i, and Ψ_i are assumed in the following form:

free-free boundary

$$W = \Theta = \sin(\pi z), \Phi = -N_A \sin(\pi z),$$
$$\Psi = \cos(\pi z) \quad (36)$$

lower rigid- upper free boundary

$$W = 3z^2 - 5z^3 + 2z^4, \; \Theta = z - z^2,$$
$$\Phi = -N_A(z - z^2), \Psi = z^3 - z^2 - z \quad (37)$$

rigid-rigid boundary

$$W = z^2 - 2z^3 + z^4, \Theta = z - z^2,$$
$$\Phi = -N_A(z - z^2), \Psi = z - z^2. \quad (38)$$

Substituting (35) into set of (31) and (27)-(29) and making the expressions on the left-hand sides of those equations (the residuals) orthogonal to the trial functions, we obtain a system of 4N linear algebraic equations in the 4N unknowns. Eliminating the determinant of the coefficient leads to the characteristic equation giving the thermal Rayleigh Rt as the eigenvalue for the system.

Now, we perform integration by part with respect to z from zero to one. By employing boundary conditions (32)-(34), the system of linear homogeneous algebraic equations is obtained

$$A_{ij}W_i + B_{ij}\Theta_i + C_{ij}\Phi_i + D_{ij}\Psi_i = 0, \quad (39)$$

$$E_{ij}W_i + F_{ij}\Theta_i + G_{ij}\Phi_i + I_{ij}\Psi_i = 0, \quad (40)$$

$$J_{ij}W_i + L_{ij}\Theta_i + M_{ij}\Phi_i + N_{ij}\Psi_i = 0, \quad (41)$$

$$P_{ij}W_i + R_{ij}\Theta_i + S_{ij}\Phi_i + T_{ij}\Psi_i = 0 \quad (42)$$

The above set of homogeneous algebraic equations can have a non-trivial solution if and only if

$$\begin{vmatrix} A_{ij}\ B_{ij}\ C_{ij}\ D_{ij} \\ E_{ij}\ F_{ij}\ G_{ij}\ I_{ij} \\ J_{ij}\ L_{ij}\ M_{ij}\ N_{ij} \\ P_{ij}\ R_{ij}\ S_{ij}\ T_{ij} \end{vmatrix} = 0. \quad (43)$$

The eigenvalue has to be obtained from the characteristic (43). For the first approximation N=1, the expression of the thermal Rayleigh number for free-free boundary condition are

$$Rt = \frac{1}{a^2}\left(\frac{\pi^2 Q + a^2\varepsilon}{\varepsilon} + \frac{\pi^2}{\xi}\right)(\eta a^2 + \pi^2) - \frac{a^2 Re}{(a^2+\pi^2)} - N_A Rn\left(\frac{Le(\eta a^2+\pi^2)}{\varepsilon(a^2+\pi^2)} + 1\right) \quad (44)$$

III. Results and Discussions

In this paper, we investigate the combined effect of magnetic field and AC electric field on the onset of convection in an anisotropic porous medium saturated by a dielectric nanofluid heated from below. A linear stability analysis has been carried out and the expression of thermal Rayleigh number, Rt is obtained by performing the Galerkin method. The term of Rt is defined by the other parameters, and the range of the parameter value is taken as proposed by [10] to observe the stability of the nanofluid layer. The influence of magnetic field, AC electric field, mechanical and thermal anisotropy parameter on the thermal Rayleigh number Rt and critical Rayleigh number Rt_c for three types of boundary conditions such as free-free, rigid-free and rigid-rigid are depicted graphically in Fig. 2-8.

The variation of thermal Rayleigh number with wavenumber, a under the influence of magnetic field is illustrated in Fig. 2. It is observed that the value of Rt increase with an increase in Q , indicating that magnetic field has stabilizing effect on the stability of nanofluid, thus it inhibits the onset of convection. This reveals that increasing the strength of magnetic field trigger the magnetic line to be distorted by the convection due to the viscosity of the fluid is induced and hence it obstructs rate of disturbance [23]. As can be seen in Fig.2 as well, the nanofluid layer by rigid-rigid boundary has the most stable system since it dominates the upper part of the graph. This type of boundary condition delays the onset of thermal convection.

Fig.3 depicts the effect of AC electric field on the onset of electro convection in nanofluid layer. Rt seems to decrease when elevating the value of Re, showing that AC electric field effect helps to quicken the onset of convection. This occurs due to electric field act upon the temperature difference causing the constant gradient of electric constant to trigger destabilizing electrostatic energy to the system and thus the system becomes unstable.

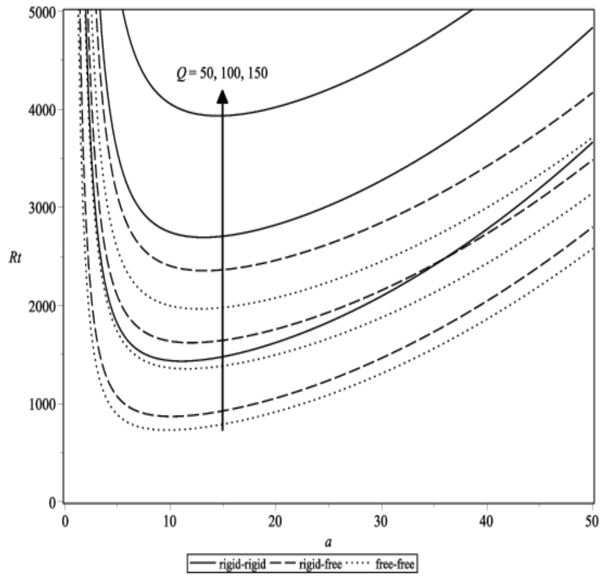
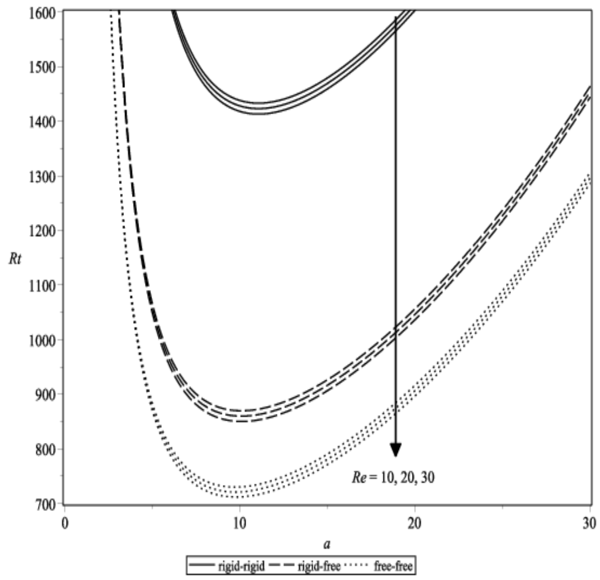


Fig.2 *Influence of magnetic field, Q on thermal Rayleigh number, Rt against wavenumber, a*



. Fig.3 *Influence of electric field number, Re on thermal Rayleigh number, Rt against wavenumber, a*

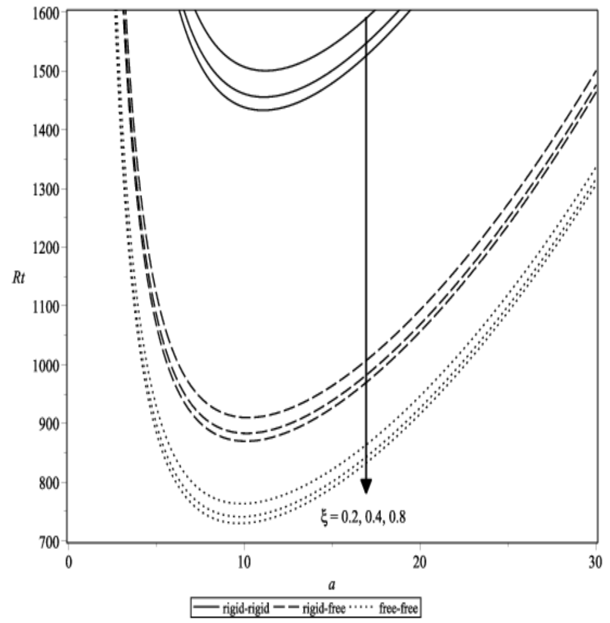


Fig.4 *Influence of mechanical anisotropy parameter,* ξ *on thermal Rayleigh number,* Rt *against wavenumber,* a*.*

The plot of Rt versus a for different value of ξ considering three different type of boundary conditions is displayed in Fig. 4. From the plot, it can be seen that an increase of ξ decrease the thermal Rayleigh number. This indicates an increase in mechanical anisotropy parameter is due to larger horizontal permeability thus it escalates the motion of fluid vertically to accelerate the onset of convective instability of dielectric nanofluid.

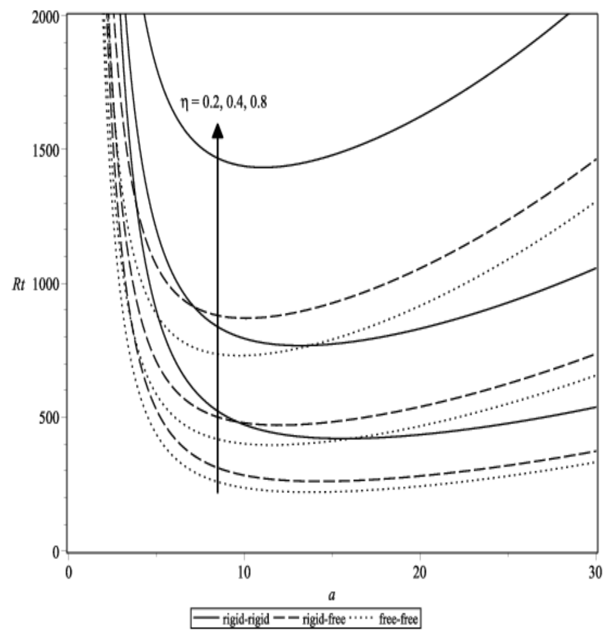


Fig.5. *Influence of thermal anisotropy parameter,* η *on thermal Rayleigh number,* Rt *against wavenumber,* a

Fig. 5 exhibits the onset of electrothermal instability in nanofluid layer due to the effect of thermal anisotropy parameter. The value of Rt is seen to decrease when reducing the value of η. This happens because when η decreases, thermal diffusivity in horizontal direction also decreases to be transferred through porous layer. Hence, it slow down the destabilizing process of the nanofluid system and inhibit the onset of convection.

The combined effects of magnetic field and AC electric field on the critical Rayleigh number are shown in Fig. 6. From the graph, increasing the strength of magnetic field hinders the growth of disturbance induced by Re and thus it enhances the stability within the fluid. From the graph as well, it is noticed that the critical Rayleigh number maintain the highest value when the nanofluid system is bounded by rigid-rigid surfaces.

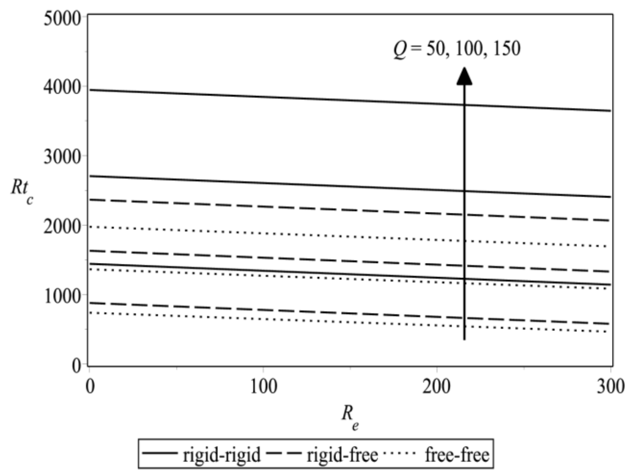


Fig.6. *Variation of the critical thermal Rayleigh number,* Rt *with electric Rayleigh number,* Re *for different value of magnetic field,* Q*.*

Fig. 7 and Fig. 8 show the impact of mechanical and thermal anisotropy parameters, respectively combining with the effect of magnetic field on the onset of convection. In Fig.7, the elevating value of ξ supress the critical Rayleigh number. In contrast, the critical Rayleigh number is supressed when reducing the value as shown in Fig. 8. From both figures, Rt_c is observed to increase and magnetic field is increasing as illustrated earlier, thus increasing the strength of magnetic field postpone the onset of the convection in dielectric fluid and stabilize the system.

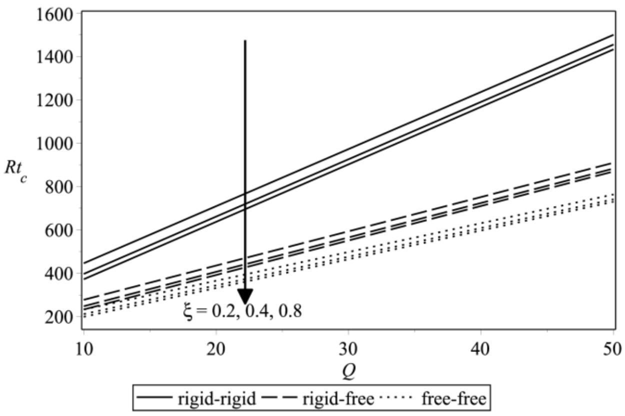


Fig.7. *Variation of the critical thermal Rayleigh number, Rt with magnetic field, Q for different value of mechanical anisotropy parameter, ξ.*

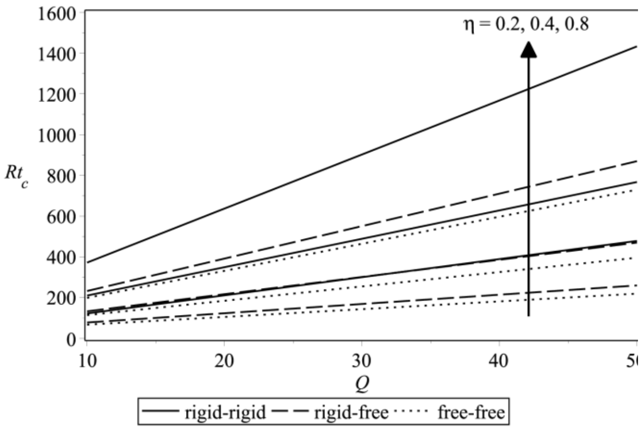


Fig.7. *Variation of the critical thermal Rayleigh number, Rt with magnetic field, Q for different value of thermal anisotropy parameter, η.*

IV. Conclusion

In this paper, the dual effect of magnetic field and electric field on the onset of thermal convection in a dielectric nanofluid saturated porous medium has been analyzed. The expression of thermal Rayleigh number is obtained using the Galerkin technique. This analysis subjected to three different boundary conditions which are free-free, rigid-free and rigid-rigid boundary. The result shows the instability of nanofluid is reinforced when elevating the value of electric field Rayleigh number, Re and mechanical anisotropy parameter, ξ. However, increasing the strength of magnetic field, Q and thermal anisotropy parameter, η slow down the destabilization process thus delay the onset of convection. Based on the boundary conditions chosen, it is found that rigid-rigid boundary has the most stable system compared to rigid-free and free-free boundary.

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