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Magnetohydrodynamics Newtonian Fluid Flow over a Stretching Surface

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***Abstract*—This paper describes the numerical study of magnetohydrodynamics (MHD) Newtonian fluid flow, which is subjected to the exponentially stretching sheet and affected by Soret-Dufour parameter. The governing basic equations (flow, momentum, energy and concentration equations) are converted to nonlinear ordinary differential equations (ODEs) by using non-similarity method. Subsequently, the ODE are solved numerically by bvp4c program in Matlab software. The graphs of velocity, temperature and concentration profiles are presented due to different controlling parameters, namely as magnetic field parameter and suction parameter.**

Keywords—magnetohydrodynamics; stretching surface; suction; Soret number; Dufour number

I. INTRODUCTION (*Heading 1*)

There are various benefits of boundary layer flow and transmission of heat instigated by a stretching sheet, to the engineering process. The applications are in cooling of metallic plate in a bath, glass fibre and paper production, extrusion of polymer sheet from a die, and continuous casting [1]. Magyari and Keller [2] were the pioneers who studied the flow and thermal boundary layers confined by an exponential stretching sheet. Normally, mass transfer caused by temperature differences in fluid flow is known as Soret effect, whereas Dufour effect refers to the heat transfer due to the concentration differences in the flow of fluid. Therefore, these parameters must not be neglected if the transmission of heat and mass occur in fluid flow. So, the reports regarding to the exponential variations of stretching velocity and temperature distributions have been published, with the presence of Soret-Dufour parameters [3-7].

This paper concentrates on the impacts of Soret and Dufour parameters over an exponentially flat stretching sheet. The governing equations are transformed to ordinary differential equations using dimensionless similarity transformation parameter and are solved numerically by Matlab. Computed numerical results are performed through graphs of velocity, temperature and concentration profiles. This study is organized as follows: Problem Formulation, Results and Discussion, finally Conclusion.

II. METHODOLOGY

Consider the two-dimensional incompressible, viscous and electrically conducting magnetohydrodynamics Newtonian fluid over an exponentially stretching surface, as depicted in Fig. 1. The mathematical formulation which represent the modeled problem are stated as below:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \quad (1)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\upsilon\frac{\partial^2 u}{\partial y^2}+g\beta_T\left(T-T_\infty\right)+g\beta_c\left(C-C_\infty\right)+\frac{\sigma\beta_0^2}{\rho}u \quad (2)$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}+\frac{DK_T}{C_sC_P}\frac{\partial^2 C}{\partial y^2} \quad (3)$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D\frac{\partial^2 C}{\partial y^2}+\frac{DK_T}{T_m}\frac{\partial^2 T}{\partial y^2} \quad (4)$$

where u and v are the components of velocity in the x and y directions, $\upsilon=\mu/\rho$ is the kinematic viscosity, μ is the viscosity, ρ is the fluid density, g is the acceleration due to gravity, β_T is the coefficient of thermal expansion, β_C is the coefficient of solutal expansions, β_0 is the constant strength of magnetic field, T is the temperature of the fluid, C is the concentration of the fluid, α is the thermal diffusivity, D is the solutal diffusivity of the medium, K_T is the thermal diffusion ratio, C_s is the concentration susceptibility, C_p is the specific heat at constant pressure and T_m is the mean fluid temperature. The subscripts of w and ∞ at parameters T and C indicate the conditions at the wall and at the outer edge of the boundary layer.

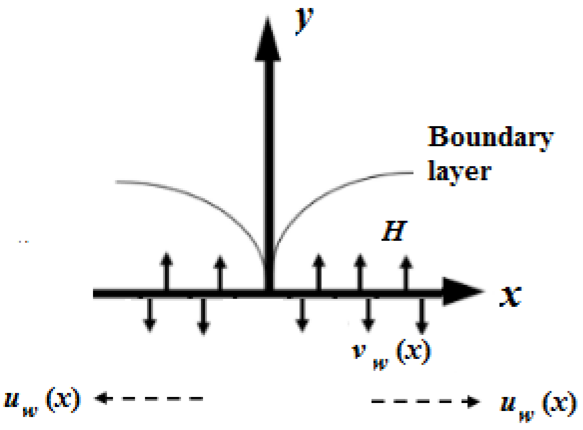

Fig. 1. *Physical model and coordinate system*

The appropriate boundary conditions are

$$\left.\begin{array}{ll} u = u_w(x) = \lambda U_0 \exp(x/L), & \\ v = v_w(x), & \\ T_w(x) = T_\infty + T_0 \exp(x/2L), & \\ C_w(x) = C_\infty + C_0 \exp(x/2L), & \text{at } \; y = 0, \\ u \to 0, \quad T \to T_\infty, \quad C \to C_\infty & \text{as } \; y \to \infty, \end{array}\right\} \quad (5)$$

where $\lambda > 0$ is the stretching parameter and $v_w(x) < 0$ is the wall mass suction velocity. the subscript of 0 at parameters U, T and C denote the initial conditions.

In this problem, the stream function is stated as:

$$\left.\begin{array}{l} \psi(x,y) = (2\upsilon L U_0)^{1/2} \exp(x/2L) \;\; f(\eta), \\ u = \dfrac{\partial \psi}{\partial y}, \qquad v = -\dfrac{\partial \psi}{\partial x}, \end{array}\right\} \quad (6)$$

This will introduce new similarity variables:

$$\left.\begin{array}{l} \theta(\eta) = \dfrac{T - T_\infty}{T_w - T_\infty}, \qquad \phi(\eta) = \dfrac{C - C_\infty}{C_w - C_\infty}, \\ \eta = y\left(\dfrac{U_0}{2\upsilon L}\right)^{1/2} \exp(x/2L), \\ u = U_0 \exp(x/L) \; f'(\eta), \\ v = -\left(\dfrac{\upsilon U_0}{2L}\right)^{1/2} \exp(x/2L)\left[f(\eta) + \eta f'(\eta)\right], \end{array}\right\} \quad (7)$$

where prime denotes differentiation with respect to η.

When (7) is substituted into (2) – (5), the following form are occurred:

$$f''' + ff'' - 2(f')^2 - 2H\left[\exp(-X)\right] f' + 2Ri\left[\exp\left(\frac{-3X}{2}\right)\right](\theta + N\phi) = 0 \quad (8)$$

$$\frac{1}{\Pr}\theta'' + f\theta' - f'\theta + Db\,\phi'' = 0. \quad (9)$$

$$\frac{1}{Sc}\phi'' + f\phi' - f'\phi + Sr\,\theta'' = 0. \quad (10)$$

$$\left.\begin{array}{lll} f'(\eta) = \lambda, & f(\eta) = S, & \\ \theta(\eta) = 1, & \phi(\eta) = 1 & \text{at } \; \eta = 0, \\ f'(\eta) \to 0, & \theta(\eta) \to 0, & \\ \phi(\eta) \to 0 & & \text{as } \; \eta \to \infty, \end{array}\right\} \quad (11)$$

where the parameters involved in this problem are formulated in Table 1. From this table, negative Ri for the case of opposing flow, otherwise it indicates the case of aiding flow. Besides, the profiles of velocity, temperature and concentration are denoted by $f'(\eta)$, $\theta(\eta)$ and $\varnothing(\eta)$.

III. Results and Discussion

Numerical computations have been performed for the velocity, temperature and concentration profiles for various values of suction and magnetic field parameters. First solution is presented by solid line, whereas dashed line denoted as second solution. In order to validate the numerical method used in this paper, we have compared the present results with those obtained by Magyari and Keller [2] and Srinivasacharya and RamReddy [3] for wall-temperature gradient for the following values: $Ri = Sr = Sc = H = S = N = D_f = X = 0$ and $Pr = 1$. This physical parameter is calculated by using the formulation:

$$Nu_x / \sqrt{\mathrm{Re}_x} = -\sqrt{X/2} \;\; \theta'(0). \quad (12)$$

We obtained the same value with the previous reports, which is 0.95478. As a conclusion, the present value of wall-temperature gradient is in excellent agreement with those reported by the previous investigators. Therefore, the good comparison proves that our bvp4c MATLAB is applicable to our numerical computation.

In the current research, the governed parameters are fixed as $\lambda = 1$, $N = 0.5$, $Ri = 1$, $Sc = 1$, $Pr = 1$, $Db = 0.03$, $Sr = 2$ and X=0.1 for numerical computations. Figs. 2-4 show the impact of magnetic field H on the velocity, temperature and concentration profiles. For the first solution, temperature and concentration values increase with the increment of magnetic field parameter but act oppositely for velocity profile. In all cases, first solution tends to reach zero level for larger η. On the other hand, in velocity and concentration profiles, second solution goes down until it reaches minimum point and after that it shows opposite trend until it reaches zero value. Values of concentration in second solution decreases with the increments of H. The values of velocity of second solution decreases at small η and increases at large η, by the effect of H. However, the pattern of temperature profile for the second solution shows different pattern with the velocity profile, by the effect of the same parameter.

TABLE 1. LIST OF PARAMETERS AND PROFILES

Parameter/ Profiles Name	Formula/Denotation
Mixed convection parameter	$Ri = \dfrac{Gr}{\mathrm{Re}^2}$
Magnetic field parameter	$H = \dfrac{2\sigma L B_0{}^2}{\rho U_0}$
Thermal Grashof number	$Gr = \dfrac{g\beta_T\left(T_0 - T_\infty\right)L^3}{\nu^2}$
Reynolds number	$\mathrm{Re} = \dfrac{u_0 L}{\nu}$
Buoyancy ratio	$N = \dfrac{\beta_C\left(C_0 - C_\infty\right)}{\beta_T\left(T_0 - T_\infty\right)}$
Dimensionless coordinate along the plate parameter, where *L* is the length of the sheet	$X = \dfrac{x}{L}$
Prandtl number	$\mathrm{Pr} = \dfrac{\upsilon}{\alpha}$
Schmidt number	$Sc = \dfrac{\upsilon}{D}$
Soret number	$Sr = \dfrac{DK_T\left(T_0 - T_\infty\right)}{T_m\,\upsilon\left(C_0 - C_\infty\right)}$
Dufour number	$Db = \dfrac{DK_T\left(C_0 - C_\infty\right)}{C_s C_p \upsilon\left(T_0 - T_\infty\right)}$
Suction parameter	$S = \dfrac{v_w\left(x\right)}{\exp\left(x/2L\right)} \times \sqrt{2L/\upsilon U_0} > 0$

Figs. 5-7 show the impact of suction parameter S on the velocity, temperature and concentration profiles. For the both solutions, the effects of suction parameter on velocity, temperature and concentration profiles are in decrement pattern. For second solution, variation of the velocity and concentration are descending until it achieves minimum point. After that it shows a reverse trend and gradually meet with zero.

IV. CONCLUSION

Observing the results above, important findings are stated as follows:

1. From the comparison with the previous works, it is clear that MATLAB problem solver is acceptable to do numerical computations.

2. For the first solution, temperature and concentration values increase with the increment of magnetic field parameter but velocity decreases. This variation of the profiles due to the magnetic field parameter are clearly seen at the small thickness of boundary layer η. In addition, first solution graphs for all profiles are reduced due to the effect of suction parameter.

3. In second solution, effect on the velocity, temperature and concentration values for different suction parameter are descending until it achieves minimum point. However, the effect of magnetic field parameter is to decrease the variation of velocity and concentration profiles. Instantaneous velocity for the second solution is increased at large η and with the additional rate of magnetic field.

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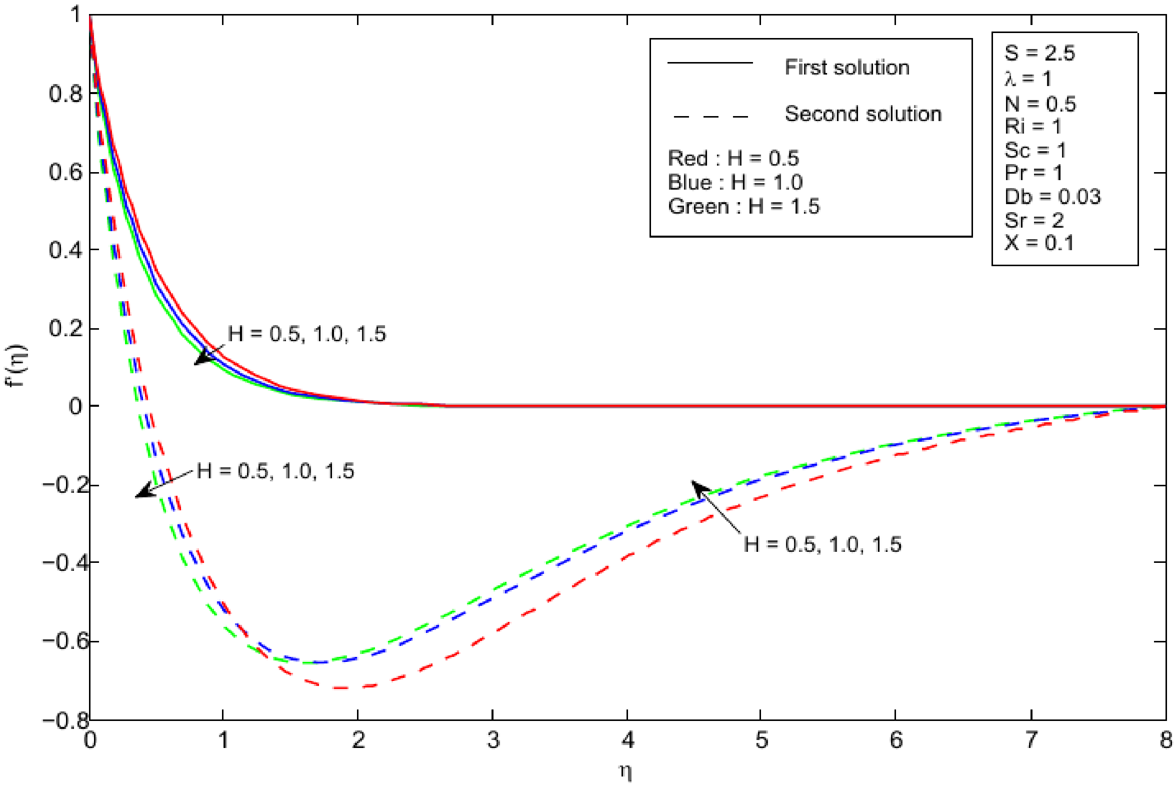

Fig. 2. *The impact of magnetic field parameter H on velocity profile*

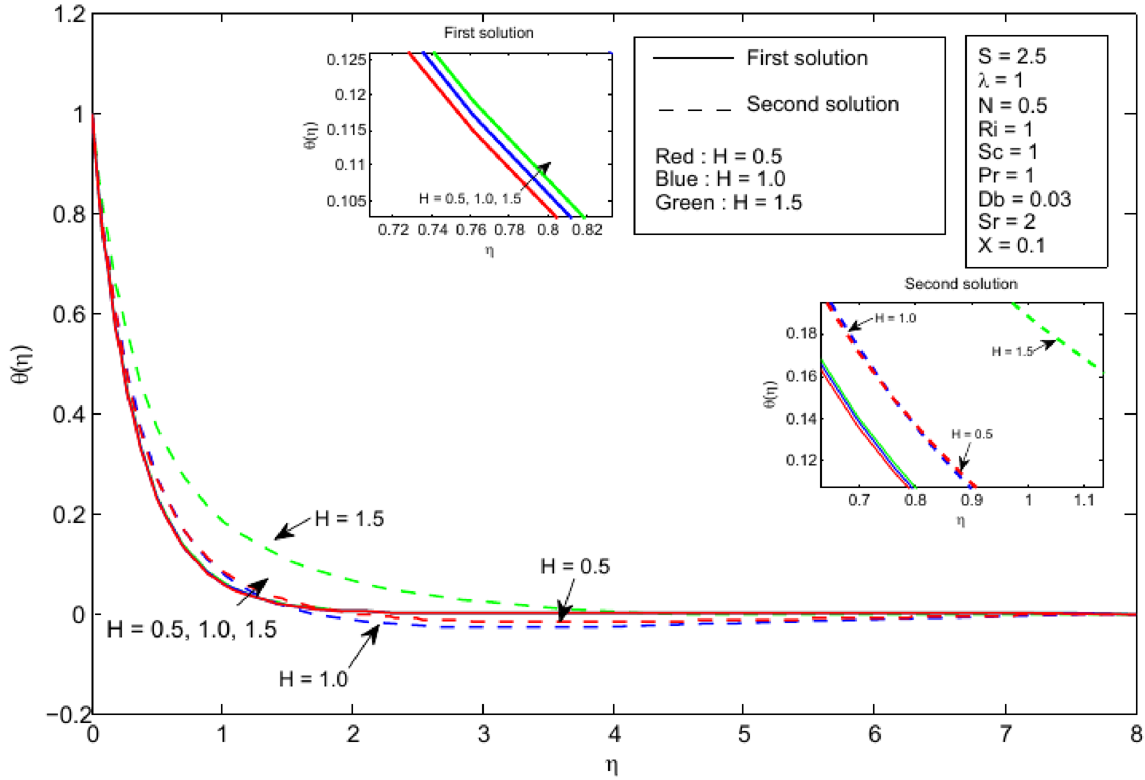

Fig. 3. *The impact of magnetic field parameter H on temperature profile*

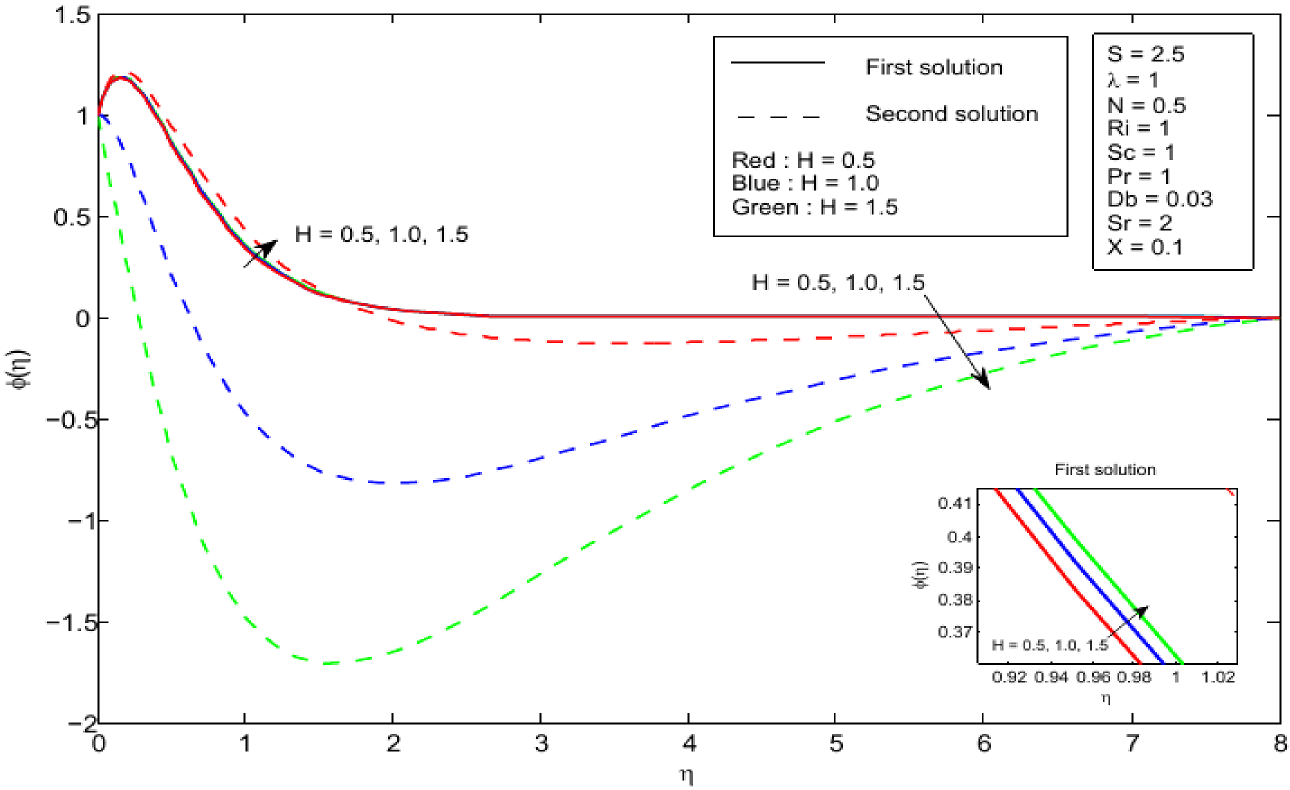

Fig. 4. *The impact of magnetic field parameter H on the concentration profile*

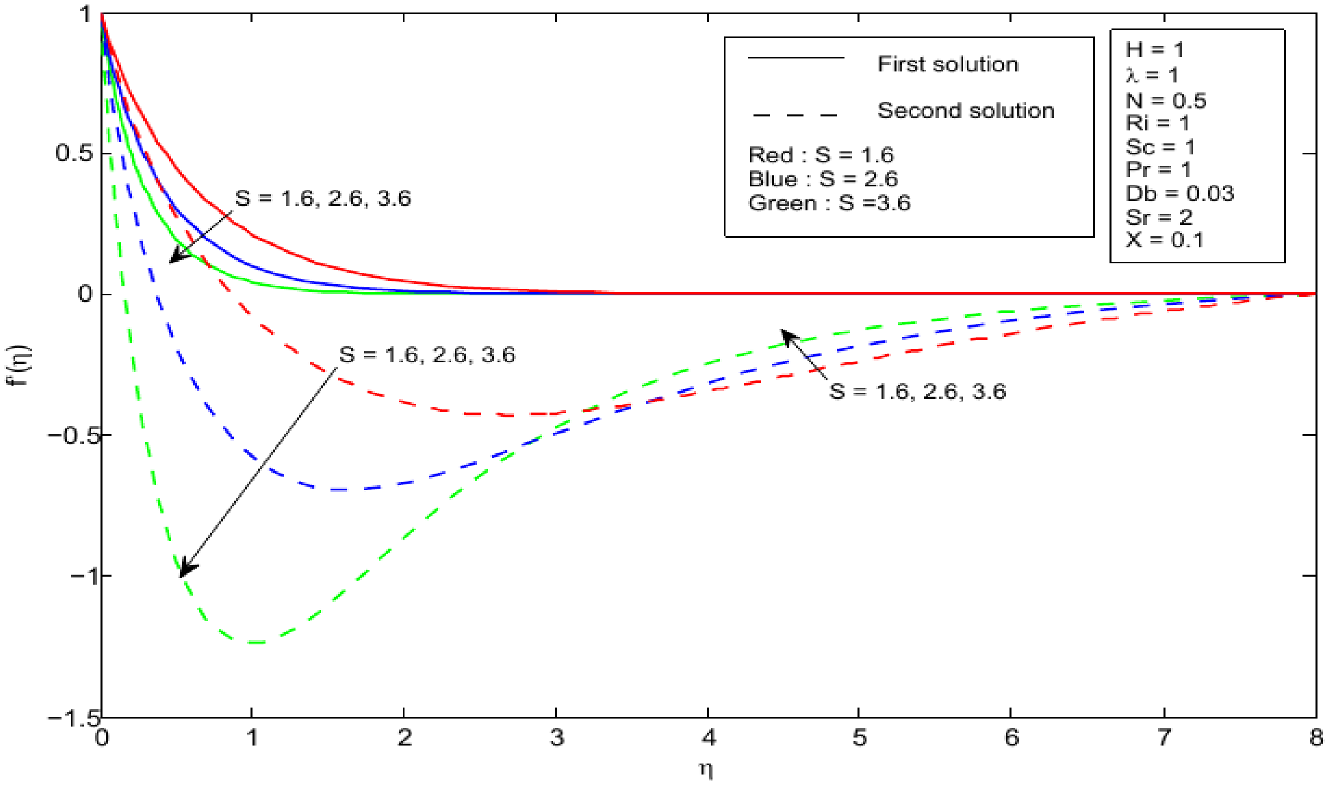

Fig. 5. *The impact of suction parameter S on velocity profile*

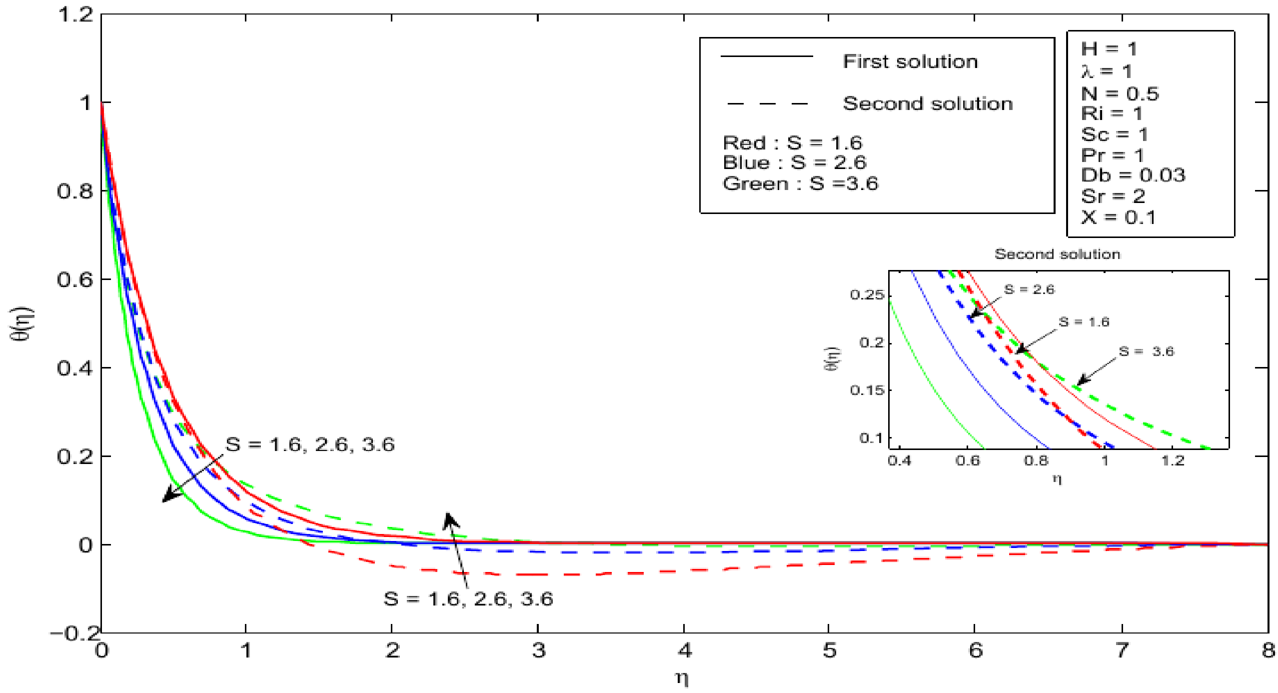

Fig. 6. *The impact of suction parameter S on temperature profile*

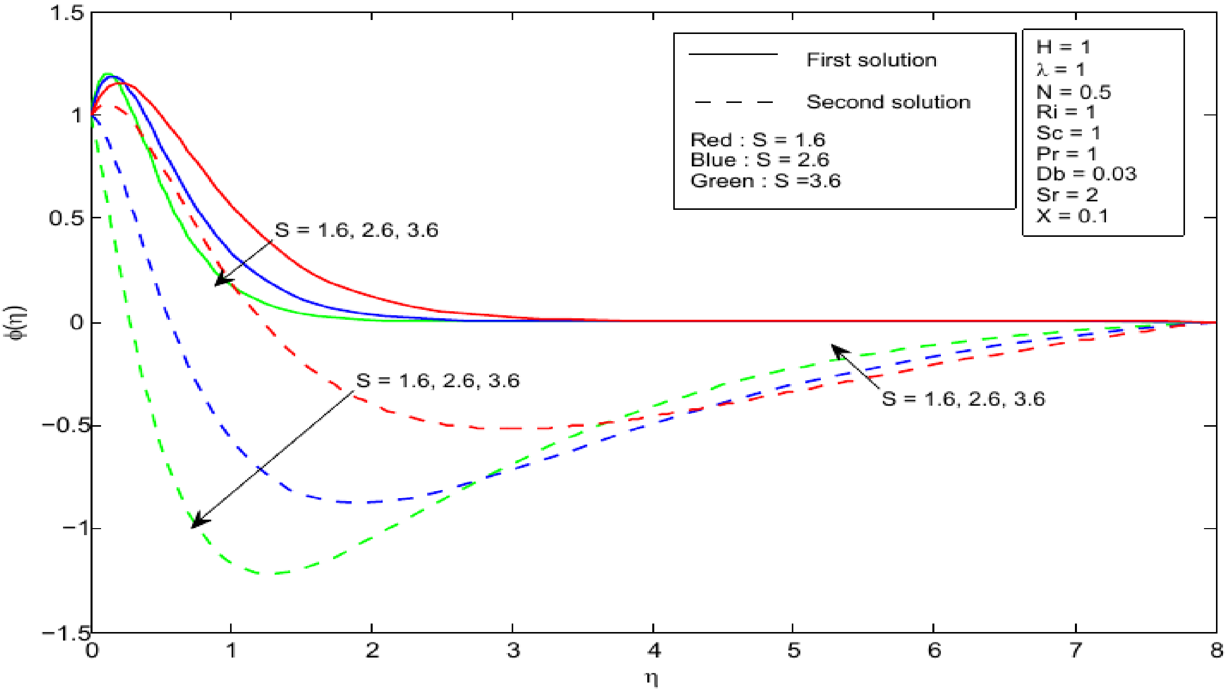

Fig. 7. *The impact of suction parameter S on concentration profile*