# Stagnation Point Flow and Heat Transfer of Casson Nanofluid past a Permeable Stretching Sheet

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***Abstract*—The main objective of the present work is to examine the stagnation point flow and heat transfer of a Casson nanofluid due to a permeable stretching surface. The sheet is permeable to allow the impact of wall mass suction/injection. Nanoparticles dispersion in a working fluid can inflate the heat transfer characteristics based on the previous studies on nanofluid. Hence, in the present work, the combination of non-Newtonian Casson fluid with Buongiorno's model of nanofluid is modelled. The nonlinear partial differential equations (PDEs) are transformed into a set of ordinary differential equations (ODEs) using a similarity transformation. Numerical results are then computed via the aid of bvp4c function in MATLAB software. The effect of the pertinent parameters (suction, Casson, Brownian motion and thermophoresis) are examined through the velocity, temperature and concentration profiles. The addition of suction and Casson fluid parameters can enhance the flow and subsequently, maintain the boundary layer flow. The enhancement of Brownian motion and thermophoresis parameters augment the temperature profile.**

***Keywords—Casson nanofluid; stagnation point flow; stretching sheet; suction***

## I. INTRODUCTION

There are many engineering and technological applications that are related to the boundary layer flow due to the stretching sheet for example the aerodynamic extrusion of plastic sheets, glass fiber production along a liquid film and the cooling/drying process of paper and textiles. Hiemenz [1] being the earliest to study the stagnation point flow for two-dimensional problem while Chiam [2] initiated the stagnation point flow due to the stretching sheet. Since then, many researchers investigated the quality of final product by enhancing the heat transfer performance. Viscous fluid did not seem to be a good choice of working fluids. Nanofluid was invented to improve the thermophysical properties of the working fluid and therefore, increase the heat transfer performance. Buongiorno [3] formulated the nanofluid model which incorporated the coupled effects of Brownian motion and thermophoresis. Buongiorno [3] investigated seven types of slip which can contribute relative velocity between nanoparticles and the base fluid; however only thermophoresis and Brownian motion are suitable to model the nanofluid. Khan and Pop [4] scrutinized the boundary layer flow of nanofluid past a stretching sheet using the Buongiorno's model.

There are many non-Newtonian fluid models that are formulated due to the advancement in the manufacturing and processing industries. Generally, the two types of non-Newtonian fluids are dilatant (shear thickening) fluid and pseudoplastic (shear thinning) fluid. Casson fluid is one of the dilatant fluid which has infinite viscosity when the shear rate is zero. Nadeem et al. [5], Hayat et al. [6], Bhattacharya et al. [7], Mukhopadyay [8] and Mukhopadyay et al. [9] analyzed the Casson fluid flow and heat transfer over various surfaces. Later, few researcher extended the Casson fluid flow problem with the inclusion of nanoparticles. The combined effects of velocity slip and convective condition on magnetohydrodynamics (MHD) stagnation point flow over a stretching sheet was studied by Ibrahim and Makinde [10]. They used Casson nanofluid model in their work. The results showed that the skin friction coefficient increased whereas both local Nusselt and Sherwood numbers decreased with the augmentation of Casson parameter. Oyelakin et al. [11] reported the Dufour and Soret effects on the unsteady Casson nanofluid flow, heat and mass transfer over a stretching sheet. Thermal radiation, heat generation, velocity slip and convective condition were also contemplated in the study. Both modeled the governing problem using Buongiorno's model of nanofluid.

In the present work, stagnation point flow with heat and mass transfer of Casson nanofluid due to a permeable stretching sheet is examined. The permeable sheet is considered to allow the wall mass suction/injection parameter. Buongiorno's model of nanofluid is utilized to represent the nanoparticles in the problem. The main interest of the study is to investigate the impact of the suction parameter, Casson fluid parameter, Brownian motion and thermophoresis parameters towards the velocity, temperature and concentration profiles. The authors are confident that there are no other studies have been reported due to the present work and the results are new.

## II. Mathematical Formulation

Consider two-dimensional stagnation point flow of Casson nanofluid towards a permeable (porous) stretching sheet. The reference velocity for the stretching sheet is in a linear form $u_w(x) = ax$ while $u_e(x) = bx$ is the free stream velocity with $b > 0$. For the coordinate system, $x-$ axis is along with the sheet whereas $y-$ axis is orthogonal to the sheet as illustrated in Fig. 1. The constant wall temperature and concentration are denoted by $T_w$ and $C_w$ while $T_\infty$ and $C_\infty$ are the constant ambient temperature and concentration, accordingly. Buongiorno's model is used to represent the nanofluid by incorporating the Brownian motion and thermophoresis effect. The continuity, momentum, energy and concentration equations of a Casson nanofluid for the present problem is given by (see Ibrahim and Makinde [10] and Oyelakin et al. [11]):

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

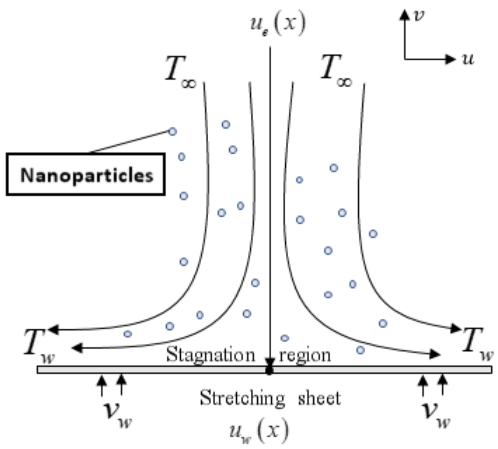


Fig. 1. *The physical model and coordinate system.*

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = u_e\frac{du_e}{dx} + \nu\left(1+\frac{1}{\gamma}\right)\frac{\partial^2 u}{\partial y^2}, \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} + \tau\left[D_B\frac{\partial C}{\partial y}\frac{\partial T}{\partial y} + \frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial y}\right)^2\right], \tag{3}$$

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D_B\frac{\partial^2 C}{\partial y^2} + \frac{D_T}{T_\infty}\frac{\partial^2 T}{\partial y^2}. \tag{4}$$

along with the initial and boundary conditions:

$$\text{at } y = 0$$
$$u = u_w(x), \quad v = v_w, \quad T = T_w, \quad C = C_w, \tag{5}$$

$$\text{as } y \to \infty$$
$$u \to u_e(x), \quad T \to T_\infty, \quad C \to C_\infty, \tag{6}$$

where $u$ and $v$ are the fluid velocities in the $x-$ and $y-$directions, respectively. Further, $T$ and $C$ are the fluid temperature and concentration, accordingly, $\nu = \mu_B/\rho$ is the kinematic viscosity of the Casson fluid, $\gamma = \mu_B\sqrt{2\pi_c}/p_y$ is the Casson fluid parameter, $\mu_B$ is the plastic dynamic viscosity of the non-Newtonian fluid, $\rho$ is the fluid density, $\pi_c$ is the critical value of the non-Newtonian fluid model, $p_y$ is the yield stress of the fluid, $\alpha = k/\rho C_p$ is the thermal diffusivity, $k$ is the fluid thermal conductivity, $\tau = (\rho C_p)_p/(\rho C_p)_f$ is the ratio of the heat capacity of the nanoparticles to the base fluid. Further, $v_w$ is the wall mass transfer velocity while $D_B$ and $D_T$ are the Brownian and thermophoresis diffusion coefficients, correspondingly.

The governing model of the problem is simplified by utilizing these similarity transformations:

$$\eta = y\sqrt{\frac{b}{\nu}},\quad \psi = \sqrt{b\nu}\,xf(\eta),$$
$$\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty},\quad \phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty}. \tag{7}$$

Equation (1) is satisfied when

$$u = \frac{\partial \psi}{\partial y} = bxf'(\eta) \quad \text{and}$$
$$v = -\frac{\partial \psi}{\partial x} = -\sqrt{b\nu}\, f(\eta). \tag{8}$$

Therefore, using Equations (7) and (8), the partial differential equations (PDES) in Equations (2)–(4) are reduced into a set of nonlinear ordinary differential equations (ODEs) as follows:

$$\left(1 + \frac{1}{\gamma}\right) f''' + ff'' - f'^2 + 1 = 0, \tag{9}$$

$$\theta'' + \Pr\left(f\theta' + Nb\theta'\phi' + Nt\theta'^2\right) = 0, \tag{10}$$

$$\phi'' + \Pr Le f\phi' + \frac{Nt}{Nb}\theta'' = 0, \tag{11}$$

and the transformed conditions are

$$f(0) = S,\quad f'(0) = \lambda,\quad \theta(0) = 1,\quad \phi(0) = 1,$$
$$f'(\eta) \to 1,\quad \theta(\eta) \to 0,\quad \phi(\eta) \to 0 \ \text{ as } \ \eta \to \infty \tag{12}$$

The governing parameters are Brownian motion $Nb$, Lewis number $Le$, velocity ratio $\lambda$, thermophoresis $Nt$, Prandtl number $\Pr$ and suction $S$ which are defined by

$$Nb = \frac{\tau D_B (C_w - C_\infty)}{\nu},\quad Le = \frac{\alpha}{D_B},\quad \lambda = \frac{a}{b},$$
$$Nt = \frac{\tau D_T (T_w - T_\infty)}{\nu T_\infty},\quad \Pr = \frac{\nu}{\alpha},\quad S = -\frac{v_w}{\sqrt{b\nu}} \tag{13}$$

and it is worth to mention that $\lambda > 0$ and $\lambda < 0$ correspond to the stretching and shrinking parameters, accordingly while $S > 0$ and $S < 0$ correspond to the suction and injection parameters, respectively. The sheet is impermeable if $S = 0$.

The physical quantities of interest are the skin friction coefficient, the local Nusselt and Sherwood numbers which are defined as

$$C_{fx} = \frac{\tau_w}{\rho u_e^2},\quad Nu_x = \frac{xq_w}{k(T_w - T_\infty)},$$
$$Sh_x = \frac{xh_w}{D_B (C_w - C_\infty)}, \tag{14}$$

respectively. The wall shear stress $\tau_w$, the wall heat flux $q_w$ and the wall mass flux $h_w$ are given by

$$\tau_w = \left(\mu_B + \frac{p_y}{\sqrt{2\pi_c}}\right)\left(\frac{\partial u}{\partial y}\right)_{y=0},\quad q_w = -k\left(\frac{\partial T}{\partial y}\right)_{y=0},$$
$$h_w = -D_B\left(\frac{\partial C}{\partial y}\right)_{y=0}. \tag{15}$$

Using Equations (7), (8), (13) and (14), the reduced skin friction coefficient, local Nusselt and Sherwood numbers are:

$$C_{fx}\,\mathrm{Re}_x^{1/2} = \left(1 + \frac{1}{\gamma}\right) f''(0),\quad Nu_x\,\mathrm{Re}_x^{-1/2} = -\theta'(0),$$
$$Sh_x\,\mathrm{Re}_x^{-1/2} = -\phi'(0), \tag{16}$$

where $\mathrm{Re}_x = xu_e/\nu$ is the local Reynolds number.

## III. Results And Discussion

In the present work, bvp4c programme in MATLAB software is used as the medium to solve the ordinary (similarity) differential equations (see Equations (9)–(11)) together with the boundary condition (see Equation (12)). In the present work, $\eta_\infty$ is set up to 15 while the values of $\lambda = Nb = Nt = 0.5$, $S = 0.2$, $\gamma = 0.1$, $\Pr = 10$ and $Le = 2$ are fixed except the varied parameters as demonstrated in the figures. The effect of the pertinent parameters namely suction $S$, Casson fluid $\gamma$, Brownian motion $Nb$ and thermophoresis $Nt$ on the velocity, temperature and concentration profile are presented in Figs. 2–11. All the profiles fulfilled the boundary conditions asymptotically, hence we are confident that the results are correct. In addition, for the present model validation, we also manage to compare the values of $f''(0)$ between present and previously reported results by Wang [12] and Bachok et al. [13] as elucidated in Table I. It shows that the comparisons are in an excellent agreement.

TABLE I. COMPARISON VALUES OF $f''(0)$ WHEN $\Pr = 1$, $\gamma = 1000000$, $S = 0$ AND VARIOUS $\lambda$ FOR VISCOUS FLUID

| $\lambda$ | Present | Wang [12] | Bachok et al. [13] |
|---|---|---|---|
| 2 | -1.887306 | -1.88731 | -1.887307 |
| 0.5 | 0.713294 | 0.71330 | 0.713295 |
| 0 | 1.232587 | 1.232588 | 1.232588 |
| -0.5 | 1.495669 | 1.495670 | 1.495670 |

Figures 2–4 portray the effect of wall mass suction/injection parameter on the velocity, temperature and concentration profiles. As $S$ enhances, the fluid velocity slightly inclines while both temperature and concentration profiles decline. The application of suction is important in delaying the boundary layer separation by enhancing or inducing the flow near the wall (see Fig. 2). In addition, the heated fluid moves towards the wall surface as the suction parameter augments and subsequently, develops both thermal and concentration boundary layer thickness. This process leads to the reduction of fluid temperature and nanoparticles volume fraction profiles as depicted in Figs. 3 and 4.

The velocity profile increase with the enlargement of $\gamma$ as can be seen from Fig. 5. It means that the fluid velocity is higher for a viscous nanofluid as compared to Casson nanofluid because as $\gamma \to \infty$, the present Casson nanofluid model will reduce to the viscous nanofluid model. An upsurge of $\gamma$ seems to degenerate the temperature and nanoparticles volume fraction profiles as portrayed in Figs. 6 and 7.

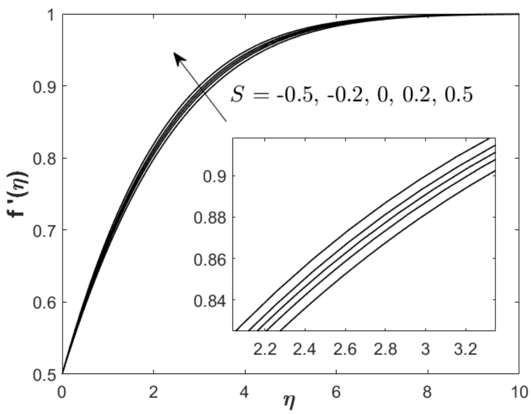


Fig. 2. *Effect of $S$ on the velocity profile.*

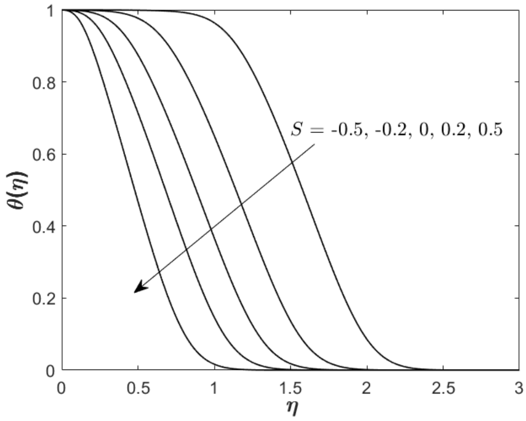


Fig. 3. *Effect of $S$ on the temperature profile.*

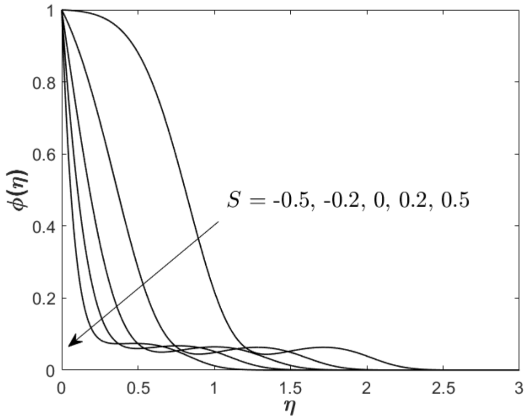


Fig. 4. *Effect of $S$ on the concentration profile.*

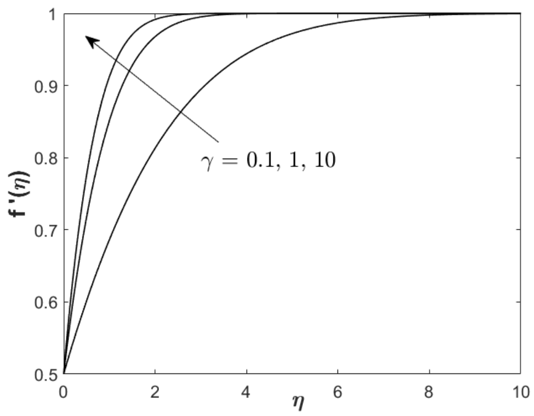


Fig. 5. *Effect of $\gamma$ on the velocity profile.*

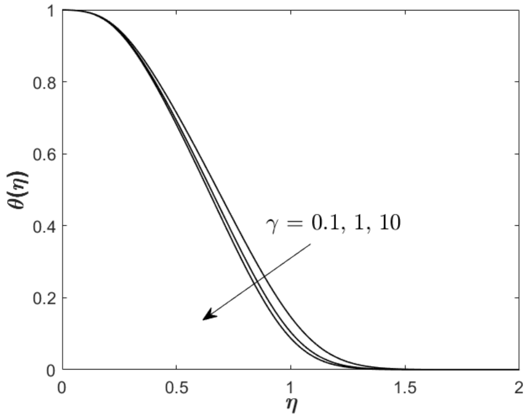


Fig. 6. *Effect of* $\gamma$ *on the temperature profile.*

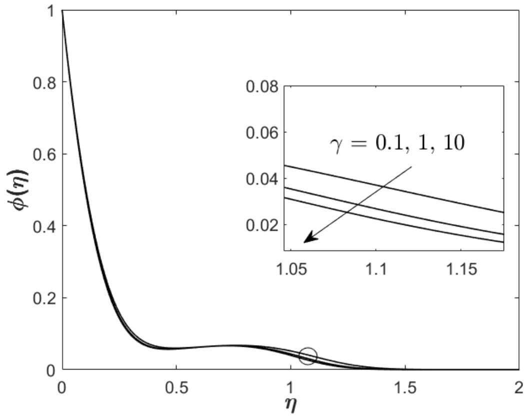


Fig. 7. *Effect of* $\gamma$ *on the concentration profile.*

Figures 8 - 11 elucidate the impact of Brownian motion and thermophoresis parameters on the temperature and concentration profiles. There are no significant results of $Nb$ and $Nt$ on the velocity profile, hence the graphs are omitted. Both $Nb$ and $Nt$ boost the temperature profiles. However, $Nb$ reduces the concentration profile while $Nt$ gives reverse impact. The inclusion of nanoparticles generally will induce thermophoresis parameter which can inflate the thermal conductivity of the base fluid (Casson fluid). Consequently, both temperature and concentration profiles enhance as displayed in Figs. 10 and 11.

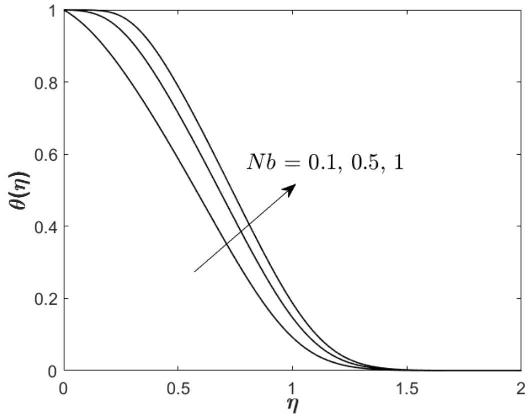


Fig. 8. *Effect of* $Nb$ *on the temperature profile.*

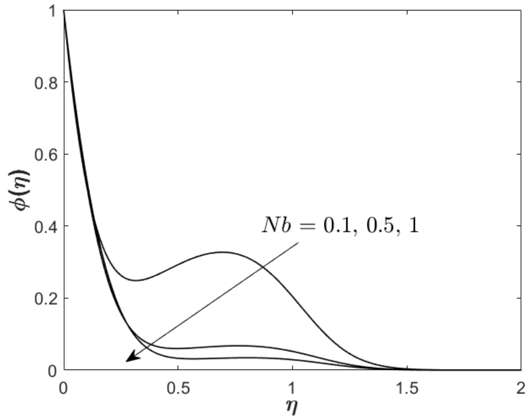


Fig. 9. *Effect of* $Nb$ *on the concentration profile.*

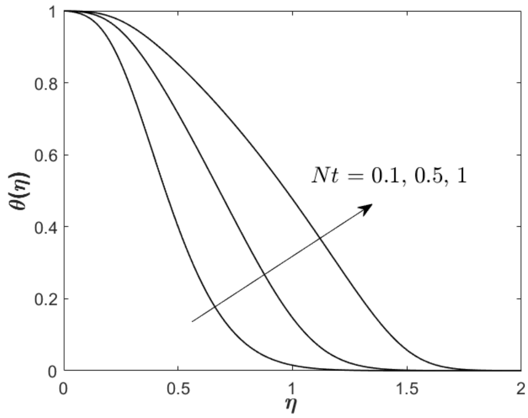


Fig. 10. *Effect of* $Nt$ *on the temperature profile.*

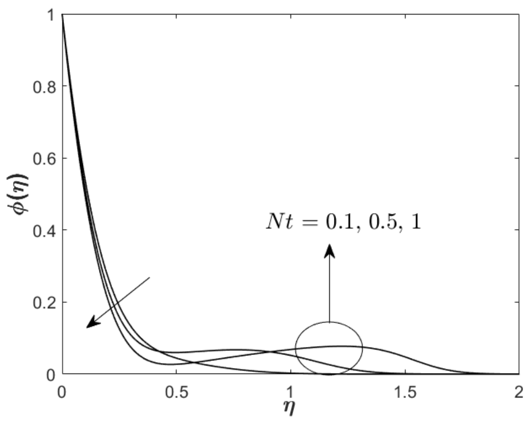


Fig. 11. *Effect of $Nt$ on the concentration profile.*

## IV. Conclusion

The effect of suction on the stagnation point flow coupled with heat and mass transfer of Casson nanofluid over a linear flat sheet is deliberated. Buongiorno's model of nanofluid is chosen to represent the appearance of nanoparticles in the Casson fluid. Using the bvp4c solver in the MATLAB software, all these results are obtained within the specific range of control parameters. The conclusions are as follows:

- The velocity profile enhances with the increment of the suction parameter and Casson fluid parameter. There are no significant impact of Brownian motion and thermophoresis parameters on the fluid velocity.
- The temperature profile decreases with the addition of suction and Casson fluid parameters whereas opposite effect is obtained when the Brownian and thermophoresis parameters are enhanced.
- The nanoparticles volume fraction profile reduces with the inflation of suction, Casson fluid and Brownian motion parameters whereas the thermophoresis parameter gives a reverse profile.

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