Extended Poisson Theory with Fourier Sinusoidal Series

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***Abstract*—Recently developed Extended Poisson Theory with thickness-wise polynomial functions for analysis of plates is modified with possible sinusoidal functions. Use of polynomial functions in the analysis of bending of plates is completely eliminated through proper sine and cosine functions. In the analysis of extension problems, it is shown that they can be used only for corrective solutions to the two term polynomial distributions of in-plane displacements.**

Keywords—Elasticity; Plates; Bending; Extension

I INTRODUCTION

A beginning towards the development of a plate theory by the present author was made in the year 1988 through publication of a technical note [1] in which Reissner's expected sixth order equation [2] was derived through an iterative procedure. He did not paid much attention about other aspects mentioned in the Reissner's article due to several investigators including scientists of great authority involved in the development of plate theories. The interest in these aspects was revived after a gap of more than 12 years since retirement in the year 1995 due to inducement from his Grand-children.

In spite of several review articles on plate theories reported in the literature, the present work is essentially due to Jemielita's 217 page survey [3] in which an attempt was made to answer the general question, *'To study or to create'*. . In this context, recent development of Extended Poisson Theory (EPT) is presumably a significant contribution in the analysis of laminated composite plates with anisotropic plies within small deformation theory of elasticity.

Small deformation theory of Elasticity consists of three equations of equilibrium in terms of six symmetric stress components along with three surface conditions This three dimensional problem is converted in to three equations of equilibrium governing three displacements (*u, v, w*) using six stress-strain constitutive relations and six strain-displacement relations. Alternately, these field equations are derived through calculus of variations using stationary property of relevant total potential. In solving this 3-D problem, a sequence of 2-D problems is generated by making suitable assumptions about thickness-wise distributions of displacements and/or stresses (or strains). In these energy methods, equations governing 2D variables correspond to plate element equilibrium equations (PEEES).

PEEES are eliminated through an adapted iterative procedure in the recently developed Extended Poisson Theory (EPT) of plates [4]. In EPT, however, it appears that one cannot avoid initial solutions of displacements with two and one term representation of in-plane displacements (*u, v*) with transverse (vertical) displacement *w* from z-integration of constant and linear ε_z(x, y) with respect to z-coordinate in extension and bending problems, respectfully. Displacement *w* is used as face variable in these problems and domain variable in the associated torsion problems.

Extended Poisson theory (EPT) appears to be most suitable theory to overcome lacuna in the classical theories of primary plate problems. Disadvantage in the application of EPT is in the development of software for generation of polynomial f_k(z) functions necessary for analysis of plates with thickness ratio varying up to unit value. New theories of plates are proposed here to overcome this problem of software development replacing f_k(z) functions with Fourier series in terms of proper sinusoidal functions.

Analysis of plates with different geometries and material properties under different kinematic and loading conditions does not provide much scope for development of new theories other than those with the analysis of primary problems of a square plate. As mentioned by Ghugal and Shimpi in their review article [5], the development of refined structural theories for laminated plates (made up from advanced fiber reinforced composite materials) has their origins in the refined theories of isotropic plates. Hence, analysis is confined here to primary problems of isotropic square plates.

II. PRELIMINARIES

A square plate bounded within $0 \le X, Y \le a$, $-h \le Z \le h$ with reference to Cartesian coordinate system (X, Y, Z) is considered. Material of the plate is homogeneous and isotropic with elastic constants E (Young's modulus), ν (Poisson's ratio) and G (Shear modulus) that are related to one other by $E = 2(1+\nu)G$. For convenience, coordinates X, Y, Z and displacements (U, V, W) in non-dimensional form $x = X/a$, $y = Y/a$, $z = Z/h$ and half-thickness ratio $\alpha = (h/a)$ are used.

With the above notation, equilibrium equations in terms of stress components are:

$$\alpha\,(\sigma_{x,x} + \tau_{xy,y}) + \tau_{xz,z} = 0 \qquad (1a)$$
$$\alpha\,(\sigma_{y,y} + \tau_{xy,x}) + \tau_{yz,z} = 0 \qquad (1b)$$
$$\alpha\,(\tau_{xz,x} + \tau_{yz,y}) + \sigma_{z,z} = 0 \qquad (2)$$

in which suffix after ',' denotes partial derivative operator.

In displacement based models, stress components are expressed in terms of displacements, via, six stress-strain constitutive relations and six strain-displacement relations. These relations within the classical small deformation theory of elasticity are:

Strain-stress and semi-inverted stress-strain relations:

$$E\,\varepsilon_x = \sigma_x - \nu\,(\sigma_y + \sigma_z) \qquad (3a)$$
$$E\,\varepsilon_y = \sigma_y - \nu\,(\sigma_x + \sigma_z) \qquad (3b)$$
$$E\,\varepsilon_z = \sigma_z - \nu\,(\sigma_x + \sigma_y) \qquad (3c)$$
$$G\,[\gamma_{xy}, \gamma_{xz}, \gamma_{yz}] = [\tau_{xy}, \tau_{xz}, \tau_{yz}] \qquad (4)$$
$$\sigma_x = E'(\varepsilon_x + \nu\,\varepsilon_y) + \mu\,\sigma_z \qquad (5a)$$
$$\sigma_y = E'(\varepsilon_y + \nu\,\varepsilon_x) + \mu\,\sigma_z \qquad (5b)$$
$$\varepsilon_z = -\mu\,e + (1 - 2\,\nu\,\mu)\,\sigma_z/E \qquad (6)$$

Here, $e = (\varepsilon_x + \varepsilon_y)$, $E' = E/(1-\nu^2)$ and $\mu = \nu/(1-\nu)$.

Strain-displacement relations:

$$[\varepsilon_x, \varepsilon_y, \varepsilon_z] = [\alpha\,u_{,x}, \alpha\,v_{,y}, w_{,z}] \qquad (7a)$$
$$\gamma_{xy} = \alpha\,(u_{,y} + v_{,x}) \qquad (7b)$$
$$[\gamma_{xz}, \gamma_{yz}] = [u_{,z} + \alpha\,w_{,x}, v_{,z} + \alpha\,w_{,y}] \qquad (8)$$

In-plane equilibrium equations in terms of displacements are

$$E'\,[\alpha^2\Delta u - \tfrac{1}{2}(1+\nu)\,\alpha^2\,(v_{,xy} - u_{,yy})] + \mu\,\alpha\,\sigma_{z,x} + \tau_{xz,z} = 0 \qquad (9a)$$
$$E'\,[\alpha^2\Delta v + \tfrac{1}{2}(1+\nu)\,\alpha^2\,(v_{,xx} - u_{,xy})] + \mu\,\alpha\,\sigma_{z,y} + \tau_{yz,z} = 0 \qquad (9b)$$

Prescribed upper and bottom face conditions along with edge conditions can be modified such that even functions $f_{2n}(z)$ and odd functions $f_{2n+1}(z)$ in the z-distribution of (u, v) are for analysis of extension and bending problems, respectively.

Correspondingly, vertical displacement $w(x, y, z)$ is odd in extension and even in bending problems due to transverse shear strain-displacement relations.

III INITIAL SETS OF SOLUTIONS IN PRIMARY PLATE PROBLEMS FROM EPT

In EPT of primary plate problems, in-plane displacements [u, v] require two term representation in extension problems and one term representation in bending (or associated torsion) problems. Prescribed conditions at each of x = constant edge (with analogous conditions along y = constant edge) in the primary problems are

$$u = u_n(y) \quad \text{or} \quad \sigma_{xn}(y) = T_{xn}(y) \qquad (10a)$$
$$v = v_n(y) \quad \text{or} \quad \tau_{xyn}(y) = T_{xyn}(y) \qquad (10b)$$

in which '$_n$' is '$_0$' in extension and '$_1$' in bending problems. Similarly, prescribed transverse stresses along z = ± 1 faces of the plate are $[T_{xz}(x, y), T_{yz}(x, y), T_z(x, y)]_n$. Due to odd and even z-distribution, however, $[T_{xz1}, T_{kyz1}, T_{z0}]$ correspond to extension problems and vice versa in bending problems.

In auxiliary problems in EPT, transverse shear stresses in bending and extension problems are expressed as

$$[\tau_{xz}, \tau_{yz}]_{0b} = -\alpha\,[\psi_{0,x}, \psi_{0,y}] \qquad (11a)$$
$$[\tau_{xz}, \tau_{yz}]_{1e} = -\alpha\,[\psi_{1,x}, \psi_{1,y}] \qquad (11b)$$

Note that $\alpha^2\,\Delta\,\psi_0 = q_1(x, y)/2$ in bending problems and $\Delta\,\psi_1 = 0$ in extension problems

In the classical plate theories, w is linear in 'z' in extension problems whereas ε_z is linear in bending problems. Here, we consider expansion of z in Fourier sine series in the form, with $\lambda_{2n-1} = 2/[(2n-1)\pi]$,

$$z = \sum A_{2n-1} \sin(z/\lambda_{2n-1}) \quad \text{(sum on n)} \qquad (12)$$

in which

$$A_{2n-1} = \int_0^1 \sin(z/\lambda_{2n-1})\, z\, dz = \lambda_{2n-1}^2 \qquad (13)$$

Successive integrations of $f_1(z) = z$ gives

$$f_{2k-1}(z) = \sum \lambda_{2n-1}^{2k} \sin(z/\lambda_{2n-1}) \qquad (14)$$
$$f_{2k}(z) = -\sum \lambda_{2n-1}^{2k+1} \cos(z/\lambda_{2n-1}) \qquad (15)$$

In bending problems, displacements [u, v] and ε_z, thereby, w are assumed in the form

$$[u, v, \varepsilon_z]_b = \sum [u, v, \varepsilon_z]_{2n-1} \lambda_{2n-1}^{2} \sin(z/\lambda_{2n-1}) \qquad (16)$$
$$w_b = w_0 - \sum w_{2n}\, \lambda_{2n-1}^{3} \cos(z/\lambda_{2n-1}) \qquad (17)$$

In extension problems,

$$w_e = \sum w_{2n-1} \lambda_{2n-1}^{2} \sin(z/\lambda_{2n-1}) \qquad (18)$$
$$[\tau_{xz}, \tau_{yz}]_e = \sum [\tau_{xz}, \tau_{yz}]_{2n-1} \lambda_{2n-1}^{2} \sin(z/\lambda_{2n-1}) \qquad (19)$$

Correspondingly, σ_z and in-plane displacements [u, v] are

$$\sigma_{ze} = \sigma_{z0} + \sum \sigma_{z2n} \lambda_{2n-1}^3 \cos(z/\lambda_{2n-1}) \quad (20)$$
$$[u, v]_e = [u, v]_0 - \sum [u, v]_{2n} \lambda_{2n-1}^3 \cos(z/\lambda_{2n-1}) \quad (21)$$

Note that σ_{z0} does not participate in the equilibrium equations but contributes in the semi-inverted stress-strain relations neglected in the classical theories.

In the case of bending problems, transverse shear stresses $[\tau_{xz}, \tau_{yz}]$ and σ_z are

$$[\tau_{xz}, \tau_{yz}]_b = [\tau_{xz}, \tau_{yz}]_0 \lambda_{2n-1} \cos(z/\lambda_{2n-1}) + \sum [\tau_{xz}, \tau_{yz}]_{2n} \lambda_{2n-1}^3 \cos(z/\lambda_{2n-1}) \quad (22)$$
$$\sigma_{zb} = [\sigma_{z1} \lambda_1^2 \sin(z/\lambda_{2n-1})] - \sum \sigma_{z2n-1} \lambda_{2n-1}^4 \sin(z/\lambda_{2n-1}) \quad (23)$$

One should note that the first term σ_{z1} in σ_{zb} is due to in-plane stresses from constitutive relations and second term is due to equilibrium equation in z-direction.

IV ANALYSIS OF FLEXURE (BENDING) PROBLEM

From face load condition,

$$[\sigma_{z1} \lambda_1^2 \sin(z/\lambda_1)]_{z=1} = \lambda_1^2 q_0(x, y) \quad (24)$$

Static equilibrium equations governing $[u, v]_{2n-1}$ are

$$\lambda_{2n-1}^3 E'\alpha^2 \Delta u_{2n-1} + \mu \lambda_{2n-1}^2 \alpha \sigma_{z2n-1} = \lambda_{2n-1}^3 \tau_{xz2n} \quad (25)$$
$$\lambda_{2n-1}^3 E'\alpha^2 \Delta v_{2n-1} + \mu \lambda_{2n-1}^2 \alpha \sigma_{z2n-1,y} = \lambda_{2n-1}^3 \tau_{yz2n} \quad (26)$$

We have from z direction-equilibrium equation

$$\lambda_{2n-1}^2 \alpha^2 \Delta [E' e_{2n-1} + \mu \sigma_{z2n-1}] = \lambda_{2n-1}^4 \sigma_{z2n-1} \quad (27)$$

With $[u, v]_{2n-1} = -\alpha [\psi_{2n,x} - \varphi_{2n,y},\ \psi_{2n,y} + \varphi_{2n,x}]$, the above equation becomes

$$\lambda_{2n-1}^2 \alpha^2 \Delta [E' \lambda_{2n-1}^2 \alpha^2 \Delta \psi_{2n} - \mu \sigma_{z2n-1}] = \lambda_{2n-1}^4 \sigma_{z2n-1} \quad (28)$$

For n=1,

$$E' \lambda_1^4 \alpha^4 \Delta \Delta \psi_2 - \mu \alpha^2 \Delta q_0 = \lambda_1^2 q_0(x, y) \quad (29)$$

Above equation (29) has to be solved along with $\Delta \varphi_2 = 0$ subjected to edge conditions

$$\psi_2 = 0 \text{ if } \psi_0 = 0 \quad \text{or} \quad \tau_{xz2} = 0 \quad (30)$$
$$u_1 = 0 \quad \text{or} \quad \sigma_{x1} = T_{x1}(y) \quad (31a)$$
$$v_1 = 0 \quad \text{or} \quad \tau_{xy1} = T_{xy1}(y) \quad (31b)$$

Due to transverse stresses from auxiliary problem, in-plane displacements $[u, v]_1$ are determined from satisfying both static and z-integrated equilibrium equations.

In higher order corrections, σ_{z2n+1} $(n \geq 1)$ has to be a free variable. For this purpose, we modify the term $\sigma_{z2n+1} \lambda_{2n+1}^4 \sin(z/\lambda_{2n+1})$ in the form $\sigma_{z2n+1} \lambda_{2n+1}^4 [\sin(z/\lambda_{2n-1}) + \sin(z/\lambda_{2n+1})]$.

Corresponding in-plane displacements and transverse shear stresses are

$$[u, v] = [(\lambda_{2n+1}^4/\lambda_{2n-1}^2) \sin(z/\lambda_{2n-1}) + \lambda_{2n+1}^2 \sin(z/\lambda_{2n+1})] [u, v]_{2n+1} \quad (32)$$
$$[\tau_{xz}, \tau_{yz}] = [(\lambda_{2n+1}^4/\lambda_{2n-1}) \cos(z/\lambda_{2n-1}) + \lambda_{2n+1}^3 \cos(z/\lambda_{2n+1})] [\tau_{xz2n+1}, \tau_{xz2n+1}] \quad (33)$$

It is to be noted that the role of $\sigma_{z2n+1} \lambda_{2n+1}^4 \sin(z/\lambda_{2n-1})$ is in rectifying error in the semi-inverted stress-strain laws in static equilibrium equations and does not participate in the integrated equilibrium equations.

With $[u, v]_{2n+1} = -\alpha [\psi_{2n+2,x} - \varphi_{2n+2,y},\ \psi_{2n+2,y} + \varphi_{2n+2,x}]$, equation governing ψ_{2n+2} becomes

$$\alpha^2 \Delta [E' \alpha^2 \Delta \psi_{2n+2} - \mu \sigma_{z2n-1}] = \sigma_{z2n+2} \quad (34)$$

Above equation (34) has to be solved along with $\Delta \varphi_{2n+2} = 0$ subjected to edge conditions

$$\Psi_{2n+2} = 0 \text{ if } \psi_0 = 0 \quad \text{or} \quad \tau_{xz2n+2} = 0 \quad (35)$$
$$u_{2n+1} = 0 \quad \text{or} \quad \sigma_{x2n+1} = 0 \quad (36a)$$
$$v_{2n+1} = 0 \quad \text{or} \quad \tau_{xy2n+1} = 0 \quad (36b)$$

V ANALYSIS OF EXTENSION PROBLEM

In a primary extension problem, the plate is subjected to symmetric normal stress $\sigma_{z0} = q_0(x, y)/2$, asymmetric shear stresses $[\tau_{xz1}, \tau_{yz1}] = \pm [T_{xz1}(x, y), T_{yz1}(x, y)]$ along top and bottom faces of the plate. Here, $\sigma_{z0} = q_0/2$ satisfying face condition does not participate in equilibrium equation of transverse stresses and the corresponding specified face shears $[T_{xz1}, T_{yz1}]$ are gradients of a given harmonic function ψ_1 so that $[T_{xz}, T_{yz}] = \alpha [\psi_{1,x}, \psi_{1,y}]$. Transverse shear stresses and normal stress satisfying face conditions are $[\tau_{xz}, \tau_{yz}] = \alpha z [\psi_{1,x}, \psi_{1,y}]$ and $\sigma_{z0} = q_0(x, y)/2$. With the inclusion of the above gradients of the known ψ_1 in the normal stresses, in-plane equilibrium equations are, with $(v_{0,x} - u_{0,y}) = 0$,

$$(E'/3) \alpha^2 \Delta [u, v] + \mu \alpha [\sigma_{z0,x}, \sigma_{z0,y}] = 0 \quad (37)$$

.
in which $[u, v] = [u, v]_0 + [\psi_{1,x}/G, \psi_{1,y}/G]$. We have from constitutive relation

$$\varepsilon_{z0} = -\mu e_0 + (1 - 2 \nu \mu) q_0/2E \quad (38)$$

We have $w = z\, \varepsilon_{z0}$ so that $w(x, y, z)$ in sinusoidal series is

$$w = \sum w_{2n-1} \lambda_{2n-1}^2 \sin(z/\lambda_{2n-1}) \quad (39)$$

with corresponding corrective transverse shear stresses

$$[\tau_{xz}, \tau_{yz}] = \sum [\tau_{xz}, \tau_{yz}]_{2n-1}\lambda_{2n-1}^2 \sin(z/\lambda_{2n-1}) \quad (40)$$

Here, displacements $[u, v]_e$ and σ_{ze} take the form

$$[u, v] = [u, v]_0 - \sum [u, v]_{2n} \lambda_{2n-1}^3 \cos(z/\lambda_{2n-1}) \quad (41)$$
$$\sigma_z = \sigma_{z0} + \sum \sigma_{z2n} \lambda_{2n-1}^3 \cos(z/\lambda_{2n-1}) \quad (42)$$

It is to be noted that series in eq. (41) is due to $\int[\tau_{xz}, \tau_{yz}]dz$ whereas series from $[\tau_{xz}, \tau_{yz}]_{,z}$ is not a complete set due to absence of constant term. Hence, one needs $[\tau_{xz}, \tau_{yz}]_{2n-1}\lambda_{2n-1}^4 \sin(z/\lambda_{2n-1})$ to satisfy static equilibrium equations with $[u, v]$ in eq.(41).

In view of the above observation, sinusoidal representation of solutions in primary extension problems is possible only as corrective solutions to analysis with two term polynomial representation of in-plane displacements, viz.,

$$[u, v] = [u, v]_0 - \tfrac{1}{2}(1 - z^2)[u, v]_2 \quad (43)$$

Corresponding analysis with the above displacements is presented earlier [6]. It involves solutions for $[\tau_{xz1}, \tau_{yz1}, \sigma_{z2}]$ and $[\tau_{xz3}, \tau_{yz3}, \sigma_{z4}]$ which are coefficients of polynomial functions $f_3(z) = ½(z - z^3/3)$ and $f_4(z) = (5 - 6z^2 + z^4)/24$. Here, errors in the analysis are due to $f_3(z)$ associated with $[\tau_{xz3}, \tau_{yz3}]$ and $f_4(z)$ associated with σ_{z4}. To rectify these errors, one proceeds with sinusoidal series for transverse stresses in the form $\sum [\tau_{xz}, \tau_{yz}]_{2n-1}\lambda_{2n-1}^4 \sin(z/\lambda_{2n-1})$ and $\sum \sigma_{z2n-1}\lambda_{2n-1}^5 \cos(z/\lambda_{2n-1})$. Here,

$$[u, v] = [u, v]_0 - \tfrac{1}{2}(1 - z^2)[u, v]_2 - \sum [u, v]_{2n+2} \lambda_{2n-1}^3 \cos(z/\lambda_{2n-1}) \quad (44)$$
$$[\tau_{xz}, \tau_{yz}] = [\tau_{xz}, \tau_{yz}]_1 + f_3(z)[\tau_{xz}, \tau_{yz}]_3 + \sum [\tau_{xz}, \tau_{yz}]_{2n+3} \lambda_{2n-1}^4 \sin(z/\lambda_{2n-1}) \quad (45)$$
$$\sigma_z = \sigma_{z0} + \tfrac{1}{2}(1 - z^2)\sigma_{z2} + f_4(z)\sigma_{z4} - \sum \sigma_{z2n+4} \lambda_{2n-1}^5 \cos(z/\lambda_{2n-1}) \quad (46)$$

Then, equations governing $[u, v]_{2n+2}$ satisfying both static and z-integrated equilibrium equations are, with $(v_{0,x} - u_{0,y}) = 0$,

$$(E'/3)\,\alpha^2 \Delta [u, v]_{2n+2} + \mu\, \alpha[\sigma_{z4,x}, \sigma_{z4,y}] = [\tau_{xz}, \tau_{yz}]_{2n+3} \quad (47)$$
$$\alpha\,[\tau_{xz2n+3,x} + \tau_{yz2n+3,y}] = \sigma_{z2n+4} \quad (48)$$

Equation governing ψ_{2n+3}, with $[u, v]_{2n+1} = -\alpha\,[\psi_{2n+3,x} - \varphi_{2n+3,y}, \psi_{2n+2,y} + \varphi_{2n+3,x}]$, becomes

$$\alpha^2\Delta\,[E'\alpha^2\Delta\,\psi_{2n+3} - \mu\,\sigma_{z2n+2}] = \sigma_{z2n+4} \quad (49)$$

Above equation (49) has to be solved along with $\Delta\varphi_{2n+3} = 0$ subjected to edge conditions

$$\Psi_{2n+3} = 0 \text{ if } \psi_1 = 0 \quad \text{or} \quad \tau_{xz2n+3} = 0 \quad (50)$$
$$u_{2n+2} = 0 \quad \text{or} \quad \sigma_{x2n+2} = 0 \quad (51a)$$
$$v_{2n+2} = 0 \quad \text{or} \quad \tau_{xy2n+2} = 0 \quad (51b)$$

(It is to be noted that the equations in [6, 7] corresponding to the equations in the above analysis of primary extension problems need proper modifications)

VI CONCLUDING REMARKS

Utility and effective approximation through EPT of a text book problem of bending of a square isotropic plate has been demonstrated in the earlier publications [4, 7 – 9].The plate with $\nu = 0.3$ and thickness ratio $2\alpha = 1/3$ is subjected to asymmetric load $\sigma_z = \pm(q_0/2)\sin(\pi x/a)\sin(\pi y/a)$ and zero shear stresses along $z = \pm 1$ faces. Analysis involves solution of second order auxiliary problem for transverse stresses, a fourth order system governing $[u, v]_1$ and a fourth order supplementary problem to distinguish neutral plane and face plane deformations. It is difficult to generate software for higher order approximations, in particular, for thickness ratio varying up to unit value.

Above difficulty in application of EPT through the use of polynomial functions is eliminated in the present analysis through the use of trigonometric functions. Here, second order auxiliary problem is natural part of the procedure. In-plane displacements $[u, v]$ are determined at each stage of approximation governed by a fourth order (sixth order with inclusion of a harmonic function φ) system by satisfying both static and z-integrated equilibrium equations coupled only through σ_z obtained in the preceding step to account for error in the semi-inverted constitutive relations. Moreover, there is no need for supplementary problem.

In the case of extension problem, it is to be noted that the present analysis through trigonometric functions is applicable only as corrective solutions for solutions obtained through two term polynomial representation of $[u, v] = [u, v]_0 - \tfrac{1}{2}(1 - z^2)[u, v]_2$

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