Decision making in power trading in Brazilian electricity market using multiciteria analysis

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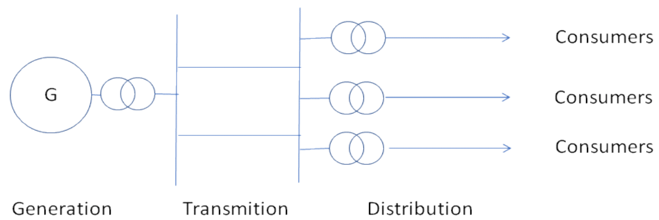
***Abstract*— This articles, which is an experimental research, focuses on the development of a metodology for decision making in power trading in Brazil´s electricity market, using multicriteria analysis. A common problem faced by a consumer is how to define what the ideal percentage of forward contracts, avoiding spot price spark financial impacts, but, at the same time, making a decision that secures cost competitiveness. This research proposes the construction of objective functions for cost and risk of a certain amount of electricity to be purchased in the market and the definition of alternative solutions. The solution of the deterministic problem (for concrete alternatvies) will be based on Bellman-Zadeh approach for decision making in fuzzy environment. Its application produces harmonious multicriteria solutions. Solution sets for different anternatives (optimal local solutions) are processed through construction and analysis of payoff matrix. This technique denotes the effect of each alternative solution, depending on the combination of initial data, state of nature and market conditions. This path allows for yielding robust solutions for decision making in power trading, increasing decision maker confidence and cost efficiency.**

Keywords—Power trading, free power market, multicriteria decision making, robust solutions

I. INTRODUCTION

Until 1995 Brasil´s electricity market was bundled. Generation, transmition, distribution and trading activities were performed by state owned companies (Federal, State and Municial). By that time there was just one option to purchase power: Regulated Market (ACR) [1]

Exhibit. 1 – Electricity market value chain



After 1995, in the wave of market liberalization occured globally, electricity market legal reform paved the way for an unbundled market with the aims to attract investments for the electricity sector. That made possible for a large consumer (above 0,5 MW) connected to the grid (above 13,8kV) to select its electricity provider. Such measure triggered off competition in generation activity, although maintaining natural monopoly for transmition and distribution activities. From an electricity trading perspective, that was the dawn of a new environment: Free Power Market (ACL) [2].

In this environment free consumers (conditions apply) – focus of this article – can select its electricity provider and negotiate terms and conditions (such as price, term, flexibilities) of the power supply agreement. Consumers can purchase electricity from generator, traders or even from other consumers with excess electricity [3].

Having the choice to choose supplier and terms and conditions means making decision and managing risks. Decision making and risk management in electricity market is not trivial. Many businesses are not prepared for this. A wrong decision can lead to millions of dollars in losses. Most of the times this process requires highly qualified personnel in risk assessment and management. Such experts are not always part of some consumers organisation chart, who are focused on their core business – which is rarely energy.

The objective of this work is to propose a simple and easy to apply method for decision making in power trading in Brasil´s free power free market as an alternative to value at risk methodology. Alternative options (optimum local solutions), using payoff matrices and deterministic multicriteria analysis (cost and risk) based on Bellman-Zadeh approach, are used for decision making in fuzzy environment.

The hope is that this work can serve as a base for consumers to develop their own capability and increase their confidence in defining long-term power purchase strategy with the aims to reduce costs and risks associated with the decision making process and their power portfolio management.

II. PROBLEM DEFINITION

In a simple way the rules state that a consumer shall purchase 100% (one hundred percent) of its consumption. CCEE [4]. Power purchase agreements can be signed way ahead of liquidation month (forward contract) or even during the liquidation month. Sellers and buyers at any tie set up their contract into a chamber (Câmara de Comercialização de Energia Elétrica – CCEE) system, but they are going to be reconciled only when the liquidation month comes. In the liquidation month, unders will be paid and overs will be received at spot prices. Spot price, in turn, is set weekly based on an optimization models run by Operador Nacional do Sistema – ONS, national power grid operator. This optimization model aims at securing power supply and reducing short-run marginal cost in each of four regions of the national grid, in each week, for the next 5 years, taking into account mainly the available generation capacity (thermal, hydro and renewables), transmition lines capacity and load, as well as reservoir current levels and future raining conditions. Each generation has its own shor-run marginal cost (custo variável unitário – CVU). Spot price will be defined based on a supply x demand balance. Generators are piled up in a low to high price merit order. For each week, the spot price is going to be set as the CVU of the last generation necessary to generate to meet demand. All generators with CVU under spot price are going to be dispatched, and receive spot price for their uncontracted electricity. All consumers pay or receive their unders and overs (uncontracted electricity) at spot price.

Power generation in Brazil relies havily on hydro generation. According to national research bureau (Empresa de Pesquisa Energética - EPE), 65% of the electricity produced in 2017 came from hydro generation. EPE [5]. This source, despite huge advantages from environment (low emission) and competitiveness (low cost) points of view) is volatile as its depends on reservoir levels and storage capacity and raining conditions.

As hydro generation is not always available and depending on reservoir levels this source of power is not enough to meet power requirement (load), most of the times the spot price is going to be set by thermal generations, whoses costs are higher than hydro ones. Exhibit 2 shows spot price volatility over the last 7 (seven) years.

Exhibit 2 – Spot price

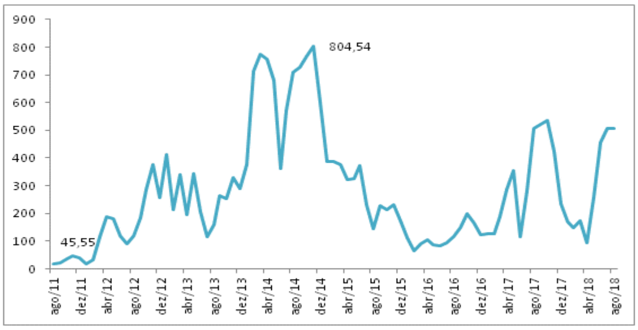


Exhibit 2 shows spot price variation of 1.670% (one thousand, six hundred and seventy percent) in only 3 years, from november 2011 (R$ 45,55/MWh) to november 2014 (R$ 804,54/MWh).

Consumers are unlikely to be able to pass this cost increase through to their customers. Therefore, it is important, for the sake of industry competitiveness, to develop a long-term power purchase strategy in order to avoid price volatility (as per shown in exhibit 2) and cost increase.

Analysis and decision making processes in power trading in Brazil´s electricity market [10 - 13] are complex and require high skill from analysts and decision makers. Hence, consumers prefer to outsource that work from traders, who also trade electricity. Trader, in turn, when doing business with consumores, on the top of electricty price and its inherent volatility, also price in ther expertise.

Table I shows statistics of january 2020 forecast spot prices.

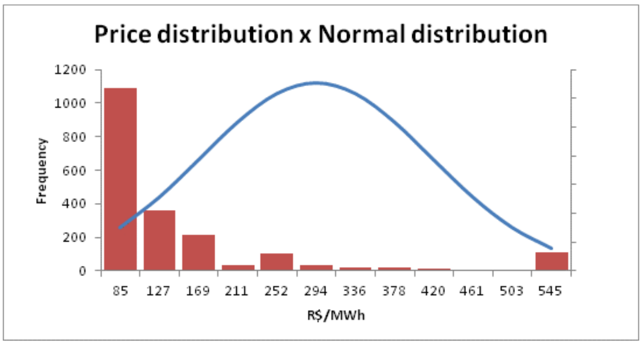
TABLE I. FORECAST SPOT PRICE FOR JAN/2020 (R$/MWH)

	Spot	Market
Maximum	544,93	244,63
Average	122,25	
Minimum	43,32	

Tablel I shows maximum, average and minimum price for january 2020. Consider a certain consumer with a need of 1.000 MWh for january 2020. At market price this consumer pay R$ 244.630,00 for its needs, if it decides to buy forward contract in the market. This consumer can also take the risk and not buy anything. In this case, if average price occurs, the cost will be R$ 122.250,00. Therefore, if the consumer hadn´t purchased in advance it would pay half the cost of the market. The additional cost is commonly called “regret cost”. But, on the other hand, if it doesn´t rain much and price sparks, spot price will reach R$ 544,93, and the cost will be R$ 544.930,00; which means more than R$ 300 thousand above market price. This is commonly called “risk avoided”. Then, instead of leaving all its need exposed to spot price, consumers can develop its purchase strategy to better balance regret costs and risks. Consumers can assess the possibility of purchasing from 0% to 100% of its needs that reduces regret costs (in case of spot price shrunks) and reduces risk (in case spot price sparks).

Exhibit 3 shows a frequency histogram of prices for january 2020 (in bars) to which a consumer portfolio is exposed to and a normal distribution (in lines).

Exhibit 3 – Frequency histogram



As we can see in Exhibit 3, prices for a certain month (january 2020) don´t fall into a normal distribution, but a exponential distribution.

The Value at Risk metodology (VaR) methodology calculates the risk of a portfolio. Commonly used in risk assessment it presupposes a normal distribution whereas probable prices are not distributed that way. Therefore, the use of this metodology can induce the decision maker to adopt a more conservative approach as the risk is calculated using a long tail distribution. Probility of occurence of a long tail event is rare, but the use of VaR methodology can induce the decision maker to make wrong and more conservative decisions if decision maker is risk averse.

Thus, purchasing electricity in forward contracts implies an opportunity to reduce cost (if the price in the liquidation month turns out to be higher than negotiated price) and a threat (regret cost; if price turns out to be lower than negotiated price). If a consumer decides not to take risk and buys 100% of its needs in forward contracts for january 2020, for instance, it will have saved money if when january 2020 comes and the spot price is a lot higher than forward price (negotiated price). But, on the other hand, it will regret if when january 2020 comes spot price is a lot lower than negotiate ones.

Next sections of this work i) describe different approaches and methodologies to assess risk, ii) present VaR methodology and its results in electricity market in Brazil, and iii) present the proposed methodolgy (Bellman-Zadeth approach) an its results .

III. RELATED WORKS

AVEN [9] presented an extensive work with vast definition, concept and terminology very useful for risk understanding, assessment and management..

RIBEIRO [10] presented a work using Value at Risk (VaR) and its variance Conditional Value at Risk (CVaR) largely employed in power trading by many companies in Brazil. Spot price is stocastic. The methodology suggests a purchasing value considering the occurence of 5% worst case cenario (P95) of price distribution. Thus, this method is criticised for overestimating the consequences of occurence of such rare event (likely to happen only 5% of the time). Yet, the method is still criticised for not obeying additive property (P97% ≠ P95% + P2%). CVaR metric customize the risk taking portion of VaR drawbakck. But, both, are still biased in terms of long-tail distribution. In practical terms, VaR and CVaR methodolgies calculated the value at risk for a certain exposed portfolio and monitors this risk. If the risk is above what the company is able or willing to carry, decision makers should buy forward contracts and avoid risks.

MUNHOZ [11] set up an optimum long-term purchase strategy, defining long and short term exposure, using Markowitz portfolio mean-variance approach. HAUGH [12]. Although results are simple to understand, problem formulation seems to be very complex.

BOSA e TORTELI [13] criticised Markowitz mean-variance methodolgy for iimature market, as Brazil´s electricity market is the case, and suggested the use of CVaR (conditional value at risk) to better assess financial impacts using historical prices for comparison.

RIBEIRO [14] proposed CVaR method using an interval confidence of 99%, instead of a commonly used 95%, with the aims to better capture the consequences of a extremely rare event.

TEIVE et al [15] proposed the use of a multiobjective genetic algorith using VaR and CVaR methodologies to determine optimum portfolio risk and return based on Pareto´s efficiency frontier.

KETTUNEN [16] developed a work for power purchase optimization in norwegian power market (Nordpool) under uncertain conditions, for different consumers risk appetites.

EKEL [17] described Bellman-Zadeh approach [8] for multiobjective and multiattribute decision making in fuzzy environment [6] [7] for engineering systems application.

WOJT [18] presented a work about advantages of application of Low Partial Moments (LPM) metrics compared to mean-variance ones.

MERICÓ [19] described the applicability of Ordered Weigthed Average (OWA) e PEREIRA [20] used it for the construction of an aggregated metric using VaR, CVaR e Low Partial Moments (LPM) in power trading in Brazil.

IV. VALUE AT RISK (VAR) METHODOLOGY

Value at Risk (VaR) was initially used in financial market, but is largely used nowadays in many market, including power [10, 15, 20]. O objective of the methodology is to determine the “value at risk” of a portfolio, which is, what the loss of a portfolio would be if a worst case scenario materializes. O VaR is usely calculated for a confidence interval of 95%, 97,5% ou 99%. Thus, when using a confidence interval of 99%, it is expected that in 100 observations, in at least 1 (one) the loss exceeds the loss calculated. Confidence interval of 95% is more commonly used.

V. RESULTS USING VAR

A purchase of 1000 MWh is used througout this work. Thus, any amount not purchased in forward contract is will be paid at spot price on liquidation day. Spot price can vary between minimum and maximum. Table II shows spot price statistics for the period of interest.

TABLE II. PRICES AND COSTS

	R$/MWh	R$
Minimum	43,32	43.320,19
Average	122,25	122.246,72
Std.deviation	122,38	122.379,04
Market	244,63	244.625,77
5% worst case	513,51	513.510,00
Maximum	544,93	544.932,59

For each scenario of percentage of purchase of forward contracts, the total cost, comprised of cost of forward contract and spot contracts is calculated.

Total cost is obtained multiplying the amount of electricity being purchased in forward contracts by market price, as per Table II, added to the amount of electricity being purchase in spot contracts multiplyied by average price, as per Table II.

TABLE III. SCENÁRIOS AND COSTS

Scenario	Forward		Spot	Total
	%	R$	R$	R$
C0	0%	-	122.246,72	122.246,72
C1	10%	24.462,58	110.022,05	134.484,63
C2	20%	48.925,15	97.797,38	146.722,53
C3	30%	73.387,73	85.572,71	158.960,44
C4	40%	97.850,31	73.348,03	171.198,34
C5	50%	122.312,88	61.123,36	183.436,24
C6	60%	146.775,46	48.898,69	195.674,15
C7	70%	171.238,04	36.674,02	207.912,05
C8	80%	195.700,61	24.449,34	220.149,96
C9	90%	220.163,19	12.224,67	232.387,86
C10	100%	244.625,77	-	244.625,77

For each scenario maximum and minimum cost is also calculated. Maximum and minimum costs are calculated considering that in each scenario a certain amount is purchased at market price and the remainder is paid at maximum (5% worst case) or minimum price, respectively, as per Table II. Then, avoided cost and regret cost are calculated. Avoided cost is calculated as the difference between maximum cost for each alternative (scenario) and R$ 24.625,77 (which is the market cost for whole power requirement. And regret cost is calculated as the difference between each scenario minimum cost and the lowest minimum cost among all alternatives (scenarios C0 to C10), which is R$ 43.320,19.

Table IV shows the results of maximum, minimum, avoided and regret cost for all scenarios.

TABLE IV. MAXIMUM, MINIMUM COST, RISK AND REGRET

Scenario	Maximum	Minimum	Risk	Regret
	R$	R$	Avoided	
C0	513.510,00	43.320,19	268.884,23	-
C1	486.621,58	63.450,75	241.995,81	20.130,56
C2	459.733,15	83.581,30	215.107,39	40.261,12
C3	432.844,73	103.711,86	188.218,96	60.391,67
C4	405.956,31	123.842,42	161.330,54	80.522,23
C5	379.067,88	143.972,98	134.442,12	100.652,79
C6	352.179,46	164.103,53	107.553,69	120.783,35
C7	325.291,04	184.234,09	80.665,27	140.913,90
C8	298.402,61	204.364,65	53.776,85	161.044,46
C9	271.514,19	224.495,21	26.888,42	181.175,02
C10	244.625,77	244.625,77	-	201.305,58

Then, a consumer using Value at Risk (VaR) methodology, and interested in minimizing its total electricity cost, as well as minimizing higher costs (risks) and regret costs (lower costs) while deciding on how much of its portfolio to buy in forward contracts, doesn´t have a clear path to follow, as per shown in Exhibit 4. An indication of what scenario to choose can be infered as being the region where risk intercepts total and regret cost.

Exhibit 4. Total, risk and regret costs

Therefore, a robust decision using VaR methodolgy is to choose any alternative (scenario) between C4 and C6, an ambiguous solution, as they indicate a region where total cost, risk and regret costs are minimized.

VI. PROPOSED METODOLOGY

When analyzing multiobjective decision making optimization models it is necessary to first determine the set of objective functions under interest, such as $F(X) = F_1(X), \dots, F_Q(X)$, and consider that solving the problem is to optimize simultaniously each and all objective functions. Thus, problem can be presented as follows:

$$F_p(X) \to \underset{X \in L}{\text{extr}}, \quad p = 1, \dots, q \tag{1}$$

where L is a viable region in $\boldsymbol{R^n}$.

An important step in analysing the problem (2) is the determination of Pareto solution sets with $\boldsymbol{\Omega} \subseteq L$ [21]. This step is crucial for the problem solving. However, it doesn´t allow for unique solution. Hence, it is necessary to choose a particular solution, taking into account information provided but decision maker. Three approaches for the used of this information are classified as [2, 3]: a priori, a posteriori and adaptative.

When analysing multiobjective [22, 23, 24, 25, 26] problems is necessary to draft answers for especific questions. Among this questions, it is important to rise objective functions normalization, consider mportance or priority of each objective function and select the aims of opmitality.

Answers to those questions and, subsequently, the development of multiobjective methods are performed in various ways ([3-6], for instance. Nonetheless, it is important to highlight the importance of the quality of the solutions when it comes to multiobjective analysis.

Bellman-Zadeh approach in decision making in fuzzy environment using <X, F> models [27, 28, 29], the objective funcitions $F_p(X), p = 1, \dots, q$ are replaced by fuzzy sets $A_p = \{X, \mu_{A_p}(X)\}, X \in L, p = 1, \dots, q$, where $\mu_{A_p}(X)$ membership functions A_p [9, 10].

Fuzzy solution $\boldsymbol{D}$ is defined as:

$$D = \bigcap_{p=1}^{q} A_p \tag{2}$$

With membership function:

$$\mu_D(X) = \min_{1 \le p \le q} \mu_{A_p}(X), \quad X \in L \tag{3}$$

Use of (3) allows for obtaining a solution:

$$\max \mu_D(X) = \max_{X \in L} \min_{1 \le p \le q} \mu_{A_p}(X) \tag{4}$$

Thus, problem (1) is transformed into the search for:

$$X^0 = \arg \max_{X \in L} \min_{1 \le p \le q} \mu_{A_p}(X) \tag{5}$$

Obtaining solution (5) requires building $\mu_{A_p}(X), p = 1, \dots, q$, that reflect the efficiency of optimatilty reach throuh fuctions $F_p(X), X \in L, p = 1, \dots, q$. This condition is satisfied [3] with the use of the following equations:

$$\mu_{A_p}(X) = \left[\frac{\max F_p(X) - F_p(X)}{\max_{X \in L} F_p(X) - \min_{X \in L} F_p(X)} \right]^{\lambda_p}$$

(6)

When the objective interest is to minimize, or

$$\mu_{A_p}(X) = \left[\frac{F_p(X) - \min_{X \in L} F_p(X)}{\max_{X \in L} F_p(X) - \min_{X \in L} F_p(X)} \right]^{\lambda_p} \tag{7}$$

When the interest is to maximize.

In (6) and (7), $\lambda_p, p = 1, \dots, q$ are coeficient that represent the importance of objective functions.

As an example, different coeficients are used is this work to highlight the prevailance of one objective function over another one.

VII. Results using proposed methodology

Table 5 denotes total cost, risk (maximum cost) and regret cost for different scenarios (alternatives) of forward purchases, from 0% to 100%, for january 2020.

TABLE IV. Initial Data

	C0	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10
Total cost	122	134	147	159	171	183	196	208	220	232	245
risk	269	242	215	188	161	134	108	81	54	27	-
regret cost	-	20	40	60	81	101	121	141	161	181	201

Table VI brings the results of membership functions considering the interest to minimize all objective functions.

TABLE V. Membership Functions

	C0	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10
μA_1	1,0	0,9	0,8	0,7	0,6	0,5	0,4	0,3	0,2	0,1	-
μA_2	-	0,1	0,2	0,3	0,4	0,5	0,6	0,7	0,8	0,9	1,0
μA_3	1,0	0,9	0,8	0,7	0,6	0,5	0,4	0,3	0,2	0,1	-
μD	-	0,1	0,2	0,3	0,4	0,5	0,4	0,3	0,2	0,1	-

Table VI figures show show that the best solution is C5, as this is the highest membership function μD (which is the highest membership function of the minimum membership function of A1, A2 e A3). Thus, the methodology, without ambiguity, suggests the purchase of 50% of the electricity requirements in forward contracts.

Finally, if one is interested in giving priority to the first objective function, for instance, it requires only the change of the exponent in (4) where λ_1 would then be equal to $\lambda_1 = 2$. Membership functions has been reviewed and the results are shown in Table VII.

TABLE VI. Modified Membership Functions

	C0	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10
μA_1	1,0	0,8	0,6	0,5	0,4	0,3	0,2	0,1	0,0	0,0	-
μA_2	-	0,1	0,2	0,3	0,4	0,5	0,6	0,7	0,8	0,9	1,0
μA_3	1,0	0,9	0,8	0,7	0,6	0,5	0,4	0,3	0,2	0,1	-
μD	-	0,1	0,2	0,3	0,4	0,3	0,2	0,1	0,0	0,0	-

Table VII figures show that with the introduction of the importance of one of the objective functions ($\lambda_1 = 2$) changed the final solution.

Hence, the proposed alternative, the solution for the problem, is to purchase 40% (alternative C4) in forward contracts.

VIII. Conclusion

Results show that the proposed methodology yields results equivalent to the commonly used methodology (VaR). Using january 2020 forecast spot prices the commonly used methodology suggests a robust solution with a purchase in forward contract between 40% and 60% of the electricity requirement. The proposed methodology indicates a robust solution with a purchase of 40% in forward contracts (when increasing the importance of the first objective function at the cost of risk and regret cost objective functions).

Proposed methodology aims at easing risk comprehension and at encouraging, through a simpler process, decision making in power trading by consumers, increasing market liquidity, and reducing electricity costs in Brazil´s electricity market.

Proposed methodology has the advantage of being possible to adjust the importance of each objective function (λ_p), in (3). Therefore, backtest using historical data can help to decide based on prices behaviour what objective functions should be prioritized over others.

An evolution of this work could be incorporating the idea proposed by [20], using OWA operators for decision making, using weights for different metodologies (VaR and the Bellman-Zadeth) in the search for a robust solution for the problem. Another evolution could be the construction of a consensus solution among a group of analysts and decision makers, instead of a decision being made by just one decision maker.

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