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Manifestation In Fins Of Fishs (Carp, Catfish, Kutum,) The Geometric Image Of Hierarchical Cayley Tree

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Annotation- Based on the manifestation of the Fibonacci series in the structures of living organisms, as well as the analysis of the self-organized natural structures of the fins of fish, it was shown that in the formation of the branches in the paired and unpaired fins at the micro level, the rules for constructing the Cayley's tree appear. The morphology of the fins of fish shows signs of the simplest regular "trees" of Cayley. Using the example of some fish (catfish, kutum, carp), it was shown that these formations have the form of flat trees, describing a chain of bifurcations, sequentially forming the structure of the fins. Such morphology of all fins gives them high plasticity and mobility when performing certain motor functions in the aquatic environment.

INTRODUCTION

A binary tree is an ordered tree, each vertex of which has at most two subtrees. A binary tree is a recursive structure, since each of its subtrees is itself a binary tree and, therefore, each of its nodes in turn is the root of a tree [1–6]. A tree node that has no descendants is called a leaf.

Analysis of the morphology of the surface of fins with signs of binary trees should be compared with Fibonacci trees, and from a methodological point of view, start the consideration with the constructions of Cantor.

The organization of these systems using binary trees can often significantly reduce the time to search for the desired item. The maximum number of steps in a tree search is equal to the height of a given tree, i.e. the number of levels in the hierarchical tree structure.

A binary search tree is a tree for which an additional condition is satisfied when both subtrees is binary search trees.

The Fibonacci tree [1, 3] is somewhat more like a real bush than the trees considered earlier, perhaps because many natural processes satisfy the Fibonacci law (see Fig. 1).

Speaking of fractals in a condensed medium, one should bear in mind the use of the concept rather than the description of the observed geometric image. This circumstance represents the central idea of the exposition. Initially, the fractal was introduced as a geometric object in ordinary physical space [7,8]. Therefore, from a methodological point of view, it is advisable to begin consideration with the visual constructs of Cantor. Their choice is due to the fact that in the first case the fractal dimension D is greater than the topological d, and in the second D <d. By gradual complication of the procedure for constructing a fractal, it is possible to find a general expression for the fractal dimension of an arbitrary mono-fractal characterized by a single value D. The construction scheme itself is clearly represented by the Cayley's hierarchical tree.

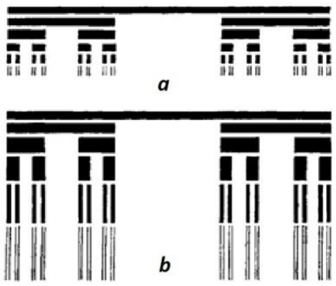


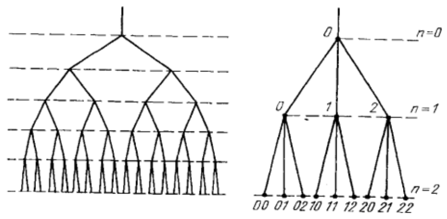
Fig.1. *Construction of the triad Cantor set: a - Geometric method, b - Coagulation method [10].*

Cayley's hierarchical tree. The procedure for constructing the Cantor set (see Fig. 1) can be represented by the Cayley hierarchical tree shown in Fig. 2, a. Let us show that the topological equivalence ([1*]) of the figures shown in fig. 1 and 2, a, allows each element of a fractal set to associate a point of an ultrametric space, the geometric image of which is represented by the Cayley's tree. For this purpose, the parameterization of the hierarchical tree should be entered, i.e. to build a way of analytic description [9,10].

We begin the consideration with the simplest example of a one-dimensional tree shown in Fig. 2, b. It is characterized by the number of levels n=2 and branching j = 3. It can be seen from the figure that each tree node at the lower level n = 2 can be given by n numbers a_l, where the index l ranges from 0 to n-1 and the numbers a_l themselves change from 0 to j-1. In other words, the coordinates of the nodes of n-level represent n-digit numbers in the j-number system.

$$\{a_l\}_n^j = a_0 a_1 \dots a_{n-1}, a_l = 0,1,\dots,j-1 \qquad (1)$$

They define a space with an ultrametric topology, a characteristic feature of which is that its points cannot form triangles with all different sides [6]. This property is easily seen, if it is assumed that the distance l between any belonging to nodes of the Cayley tree given level n, is determined by the number of steps to the common ancestor, located at the level n-l., E.g., distance between nodes 10 and 12 in Fig. 2, b is equal to one, and between 01 and 12 - two; points 01, 12, 20 form an equilateral triangle, and 01, 11, 12 - isosceles. From the given examples, it follows that if two nodes are numbered by sets (1) of the numbers a1 and b1, then the distance between them depends only on which of them are the first to differ from each other. So, for the tree shown in Fig. 2, b, the distance is two if $a_0 \neq b_0$, and one if $a_0 = b_0$, $a_1 \neq b_1$. With an arbitrary combination of numbers n, j, the distance between these points is equal to l= 0, 1, ..., n + 1, if the equalities $a_m = b_m$, m= 0, 1, . . . , n-l-1, but $a_{n-1} \neq b_{n-1}$



a **b**

Fig. 2. *The simplest regular Cayley trees. a - A tree describing a chain of bifurcations (branching j = 2). b - Parametrization Cayley tree with the number of hierarchical levels of n = 2 and j = 3 branching.*

The importance of the ultra-metric space concept is due to the fact that, reflecting the hierarchical structure of the system, it implements the so-called logarithmic metric for physically observable quantities. This means that in such a space the distance turns out to be a linear function of the logarithm of the observable quantity p. Since manipulation with ln p is less convenient than with linear dependence on l, instead of the usual axis of the values of p, it is convenient to introduce the corresponding ultrametric space, characterized by distance l, and all calculations should be carried out in this space [1,4].

To determine the dependence of p (1), we present the value of p in the j-ary number system (1). This is done by decomposition into a power series the first n coefficients of which are given

$$\rho(a-b) = (a_0 - b_0)j^n + (a_1 - b_1)j^{n-1} + \cdots \qquad (2)$$

by n-digit numbers (1), and the latter determines the origin of the value of p. From fig. 2, b it is easily seen that the representation (2) corresponds to the division of j^n nodes of the Cayley's tree into an $n+1$ group, each of which consists of clusters of nodes characterized by the same values of the maximum distance between them. So, the first term of the series (2.) corresponds to a group consisting of j single nodes, for which l = 0. The second term (2.) describes the contribution of clusters whose nodes are separated by a distance l = 1 (in Fig. 2, b we can distinguish three such clusters: 00, 01, 02; 10, 11, 12 and 20, 21, 22). Since each of these clusters is generated by a node lying on the previous hierarchical level n -1, it is easy to see that in the general case any such cluster consists of j nodes, and their number is l - 1. An arbitrary distance l

corresponds to a group of clusters generated by a hierarchical level *n - l*. Therefore, the number of such clusters is j^{n-1}, and each of them contains j^{1} nodes. These nodes are grouped into *j* subclusters corresponding to a smaller distance *l – 1*. Thus, we can conclude that the degree of

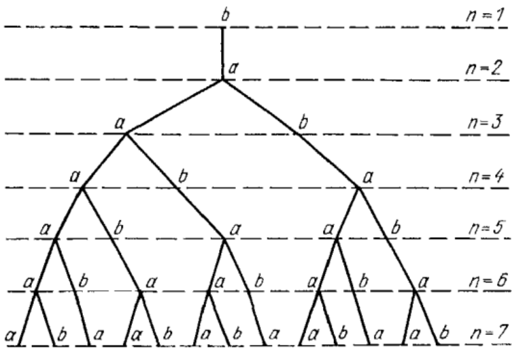


Fig. 3. *Irregular Fibonacci tree with variable branching*

the multiplier j^{n-1} of an arbitrary member of the series (2.) is given by the number of clusters of nodes, the maximum distance between which is equal to *l*, and the values of the coefficients $a_l - b_l$ are limited by the number *j* contained in them of subclusters corresponding to the distance *l* - 1. A remarkable property of the expansion (2.) is that for j »1 only one term dominates in it. Indeed, if the distance between points of the ultrametric space is l, then the first *n - l* terms containing the maximum powers of a large number j are equal to zero, since by definition $a_m = b_m$ for m = 0, 1, ..., *n — l —* 1. The last *l* members of the series contain the degrees j^{k}, *k* = l - 1, l - 2, ..., 0, whose values are negligible in comparison with j^{l}. Thus, the only term remains $(a_{n-l} — b_{n-})j^{1} < j^{l+1}$, and with logarithmic accuracy the series (2.) reduces to the form

$$ln\rho \approx (l+1)lnj \approx llnj, n, j, l >> 1 \quad (3)$$

This equality means the logarithmic metric of the ultrametric space. Above was considered a discrete ultrametric space, because only in this case can it be associated with a specific Cayley’s tree. However, just as in ordinary space, you can make the transition to the corresponding continual limit (visually, such a transition is associated with smearing a set of points that evenly fill a sheet of paper throughout its area). In the Cayley’s tree representation, this spreading is achieved by an infinite increase in the number of levels *n* and / or branching *j*. With this number of nodes falling on the level *n*,

$$N_n = j^n, \quad (4)$$

it becomes so large that the interval l_n = = *f*, separating the nearest points of the discrete space becomes infinitely small, and the ultrametric space itself becomes continuous. Accordingly, the distance *l* in the formula (4.) becomes continuous. On the Cayley’s tree, the transition to the continuum means infinite concentration of hierarchical levels. Everywhere above was meant the homogeneous Cayley’s tree, the branching of which is the same in all nodes. Obviously, the corresponding ultrametric space will have dimension d = 1. The fractional dimension D <1 is obtained only if branching disappears at each level n for some nodes. This situation is realized, for example, for the Fibonacci sequence. From the Cayley's tree answering to her, shown in fig. 3, it can be seen that non-periodic (but quite regular) alternation of nodes with branches *j* = 1, 2 is observed. In [1] it is shown that here for each nonbranching node there is a number of doubly branching nodes equal to the so-called golden section τ = (√5 +1) / 2 ≈1.618. It turns out that this leads to a decrease in the dimension of the ultrametric space corresponding to the Fibonacci tree to the value *D* = ln τ/ln 2 ≈ 0,694. The aim of the work is to identify the morphological features of the Kutum, carp and catfish fins and the procedure for their construction on the basis of the Cayley’s tree.

EXPERIMENTAL RESULTS AND DISCUSSION

As samples, self-organized structures of scaly carp, kutum and catfish were investigated. As a rule, macromolecules of fins consist of repeating units — monomers, which are combined as a result of a polymerization reaction; they are linear and branched. The graph of a branched macromolecule is a "tree". Amorphous regions include macromolecule bending regions. Consider a possible representation of a branched macromolecule in the form of a graph. At the vertices of the graph are groups of atoms, the edges correspond to the chemical bonds between the repeating units. The bold line is the trunk of the graph. Since the scales and fins are mainly composed of a mixture of collagen (natural polymers) and mineral salts, there is a possibility that the morphology of their surface structures may have certain similarities with artificial polymers. The morphology of the surface of the fins was studied with an optical microscope. Considered image (Figure 4) surface of the fin has a number of characteristics reminiscent of the crystalline state of polymers [12, 13]. It lies in the fact that, unlike low-molecular substances, polymers never completely crystallize. Along with crystallites (their size is several microns), amorphous regions are preserved in them.

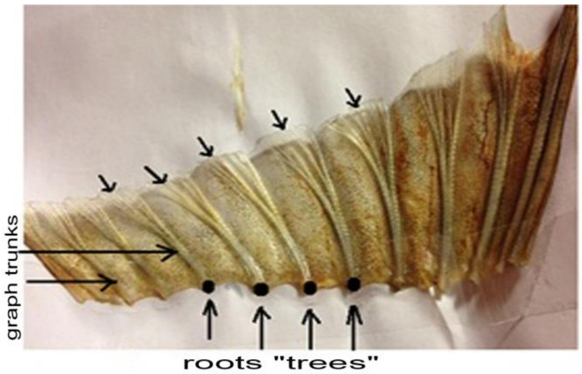


Fig.4. *Morphology of the upper fin. Legend: the vertical arrows at the bottom mark the roots of trees, which are associated with amorphous regions; from the top, vertical arrows show branched terminal functional groups. And the roots of the "trees" are linked with the skeleton of a catfish.*

Therefore, the structure of the fins of fish (see Figure 5-7) can be attributed to amorphous-crystalline. The volumetric content of the crystalline regions (region-1) in Fig. 5 can be called the degree of crystallinity.

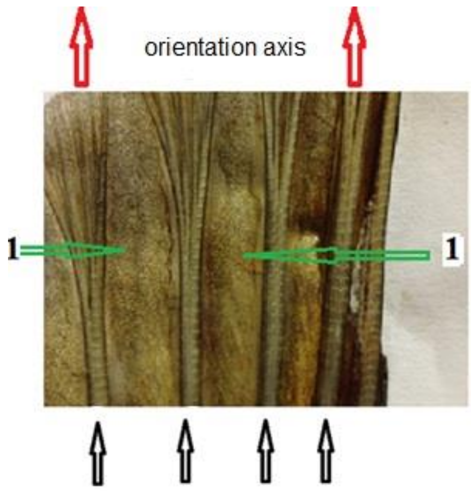


Fig.5. *An example of oriented biopolymers in the living world (for example: fish). Legend: The vertical arrows at the bottom indicate oriented trunks; the upper arrow shows the axis of orientation of the plane of the trunks.*

The morphology of the surface structures and the type of aggregation of biological objects formed for a long time in the aquatic environment with the genes embedded in them. At the same time, natural processes led to self-organized special classes of polymer composites.

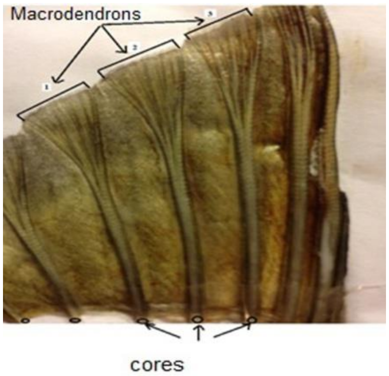


Fig.6. *The morphology of fish surface.* According to [10], this regular structure can be viewed in accordance with the chemical graph; they can also, as in [10,11], be considered a dendritic “tree” at the macro scale or Fibonacci trees with a height equal to four (see Fig. 1).

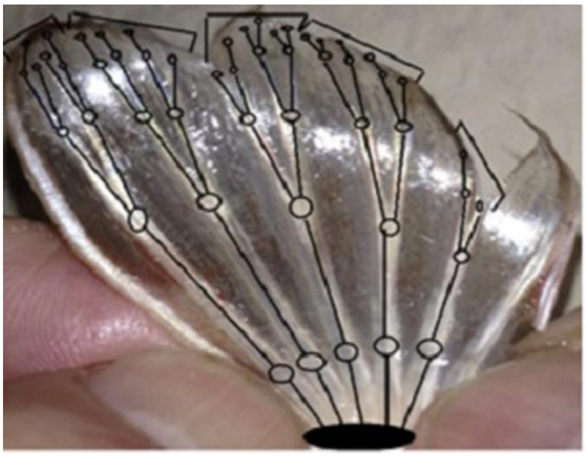


Fig.7. *The distribution of branch generations (marked with lines at the top - they are also terminal functional groups) and their generation (marked with circles) on the surface of the tail of the scaly carp.*

The fibers seen in the figure are biological natural crystals, these are crystals of a chemical compound of biological origin (usually proteins and nucleic acids). To establish the structure of their constituent macromolecules using x-ray structural analysis, they are grown artificially. Biological crystal (BC) is characterized by a large size of the elementary crystal cell (1-10nm). The photo of one of such macromolecules is given in fig. 8 The temperature range of the existence of BC is small; it is determined by the freezing point of the solvent, and the high-temperature part is in the range of 60–70 ° C. Many BCs have a fibrous structure — chains of macromolecules are elongated along one direction and along this direction are characterized by a certain intramolecular periodicity. Apparently, this structure has proteins that make up the materials of scales and fins of fish. Presented in Fig. 4 morphologies, the surfaces of the fins alternate with fibers, between which thin layered structures of amorphous formations are located. These structures are not true crystals. However, the presence in them of a certain periodicity makes it possible to study the structure of their constituent biomolecules (the area between the fibers marked as the trunks of the graphs in Fig. 4. For a comparative analysis of the morphology of artificial polymers with natural structures, we consider crystallization processes from a melt of synthetic crystallites. In this case, the crystallites are aggregated into supramolecular formations, in which the fibers radially diverge from the centers located on the fins (see Figure 5).

Fig.8. *Macromolecule*

According to [8, 13], in the crystallites of block samples, some regions have a folded conformation. Another part of them passes from the crystallite to the crystallite, connecting them with each other. Passing chains and regions add up and form an amorphous part of spherulites. Such similar macrostructures in Fig.5-7 are marked with passage areas (marked with arrows 1) on the fins of fish. These photographs show oriented formations of fibers that form a pronounced texture between which amorphous fin formations are located.

Here the axial texture is most common, when the axes in all scales have an oriented state. Let us analyze the state in synthetic polymers and compare them with natural structures. As fiber-forming polymers, proteins (fibroin, collagen) prove to be the most effective [8, 10].

The oriented state of polymers in fins of fishes is a state in which the axes of the straightened and parallel-laid macromolecules or supramolecular structures are located along the preferential direction — the axis of orientation (see Fig. 5).

Conclusions

Self-organized natural fin structures are a class of polymer composite materials similar to the Cayley's "trees" structures.

References

1. Olemskoi A.I., Flat A.Ya. Application of fractals in condensed-matter physics *Phys. Usp.* **36** (12) 1087–1128 (1993)
2. Cayley A. A theorem on trees, *Quart. J. Pure Appl. Math.* **23** (1889), 376-378; *Collected Mathematical Papers* Vol. 13, Cambridge University Press 1897, 26-28.
3. *Mandelbrot B.B.* The Fractal Geometry of Nature. New York, Freeman, 1982.
4. Pitman J.. Coalescent random forests. *J. Combinatorial Theory, Ser. A* 85 (1999), pp.165-193.
5. Ostilli M. Cayley trees and bethe lattices: a concise analysis for mathematicians and physicists. *Physica A: Statistical Mechanics and its Applications.* 2012. Vol. 391, Iss. 12, pp. 3417-3423
6. Peter W. Shor. A new proof of Cayley's formula for counting labeled trees. *Journal of Combinatorial Theory, Series A* 71, 154-158 (1995)
7. Imran M., Baig A.Q., Khalid W. On topological indices of fractal and Cayley tree type dendrimers. *Discrete Dynamics in Nature and Society*, vol. 2018, Article ID 2684984, 11 pages, 2018.
8. Sardar M.S., Zafar S., Zahid Z. Computing topological indices of the line graphs of Banana tree graph and Firecracker graph. *Applied Mathematics and Nonlinear Sciences,* vol. 2, no. 1, pp. 83–92, 2017.
9. Aida, T., Meijer, E. W. & Stupp, S. I. Functional supramolecular polymers. *Science* **335**, 813–817 (2012).
10. Marihin V.A., Myasnikova L.P. Non-molecular structure of polymers.L .: Chemistry, 1977. 240 p.
11. Elias H.-G. Mega Molecules. *Berlin - Heidelberg: Springer Verlag*. 1987,202 p.
12. Doi M., Edwards S. F. The theory of polymer dynamics. *Oxford: Clarendon press*. 1986. 392 p.
13. Blundell T., Johnson L., Protein Crystallography, *Academic Press*, *New York* (1976)