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Analysis and Design of a Robust Control System of Armature-Controlled DC Motor and a Type-Driving Mechanism

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Abstract—The contribution of this paper is in suggesting a useful technique for analysis and design of a robust control system in circumstances of variable gain and time-constants. If applying the method of the Advanced D-partitioning the effects of parameter variations on system's stability can be well analyzed. The method employs the possibility to define regions of stability in the space of the system's parameters. The approach to the robust controller design is compensation with two degrees of freedom, enforcing desired system performance. As a case study, a system consisting of an armature-controlled dc motor and a type-driving mechanism is discussed. The suggested technique for analysis and design is essential and beneficial for the further development of control theory in this area.

Keywords—Control System analysis, Parameter variations, Regions of stability, Advanced D-partitioning, Robust control, System performance;

I. INTRODUCTION

Control systems must yield performance that is robust or insensitive to parameter variations. In the process of design of a robust control system, it is important to determine the regions of stability, related to the variation of the system parameters. Considering initial research of Neimark [1], the method was further extended by the author in previous published work [2], [3], [4]. The upgraded by the author method, dealing with the effects of parameters variations on the system's stability, is classified as Advanced D-partitioning. It is an effective tool for system stability analysis in case of variation of any of the system's parameters. The analysis tool, further upgraded and demonstrated in this research, can be used when one, two or more parameters are varied independently or simultaneously.

This research is also suggesting a method for design of a broad-spectrum robust controller, achieved by applying forward-series compensation. It can suppress the influence of any parameters variations of the control system. Innovation is demonstrated in the unique property of the designed robust controller that can operate effectively for variations of any one of the system's parameters within prescribed limits. It rejects the impact of the simultaneous parameters uncertainties. The design of the robust controller is based on the ITAE criterion [5], [6]. It has a minimum value at damping ratio $\zeta = 0.707$. If a system is of higher order, a pair of dominant poles represents the system dynamics. Then, ζ still specify the location of these poles and is considered as the relative damping ratio of the system. Its value will be taken as a performance objective targeted by the proposed optimization design.

II. SISTEM WITH VARIABLE TIME-CONSTANT

In order to illustrate the application of the method of the Advanced D-partitioning analysis, a control system with a variable time-constant is suggested. The system, as shown in Figure 1, consists of an armature-controlled dc motor and a type-driving mechanism [7], [8]. Due to variation of the load and the mechanism time-constant, the system experience problems in maintaining stability.

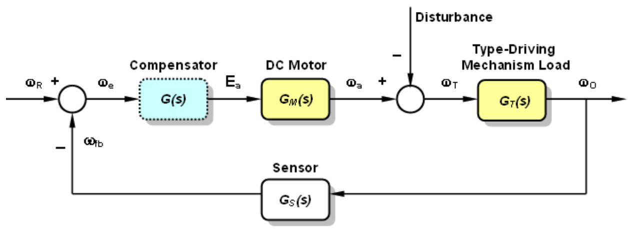

Figure 1: Armature-Controlled DC Motor and a Type-Driving Mechanism

Initially, the transfer function of the controller is considered *G(s)* = 1, while the DC motor transfer function can be presented as:

$$G_M(s) = \frac{\omega_a}{E_a} = \frac{K_T}{JL_a s^2 + (R_a J + L_a B)s + (R_a B + K_e K_T)} = \frac{10}{(1+0.5s)(1+0.8s)} \quad (1)$$

Where J = 0.01 kg.m^2 is the load inertia
B = 0 is the load damping
R_a = 0.5Ω is the armature resistance

L_a = 0.4H;	is the armature inductance
K_e = 0.1V/rad/sec	is the motor constant
K_T = 10 Nm/A	is the motor gain constant

Considering equation (1), the gain K = 10 and the two time-constants T_1 = 0.5 sec and T_2 = 0.8 sec are known and constant values. The time-constant of the type-driving mechanism, T_3 is unstable and variable parameter due to the variation of the mechanical load. The dynamics of the type-driving mechanism is described by the following transfer function with a variable time-constant:

$$G_T(s) = \frac{1}{(1+T_3 s)} \quad (2)$$

As a result, the transfer function of the open loop system can be presented as follows:

$$G_o(s) = \frac{\omega_o}{\omega_e} = \frac{K}{(1+T_1 s)(1+T_2 s)(1+T_3 s)} = \frac{10}{(1+0.5s)(1+0.8s)(1+T_3 s)} = \frac{10}{0.4T_3 s^3 + (1.3T_3+0.4)s^2 + (T_3+1.3)s + 1} \quad (3)$$

From equation (3), the characteristic equation of the unity feedback control system is determined as:

$$G(s) = 0.4T_3 s^3 + (1.3T_3+0.4)s^2 + (T_3+1.3)s + 11 \quad (4)$$

The regions of stability reflecting the variation of the system parameter T_3 are determined by applying the method of the Advanced D-Partitioning. The time-constant T_3 is presented from equation (4):

$$T_3 = -\frac{T_1T_2 s^2 + (T_1+T_2)s + K + 1}{T_1T_2 s^3 + (T_1+T_2)s^2 + s} = -\frac{0.4s^2 + 1.3s + 11}{0.4s^3 + 1.3s^2 + s} \quad (5)$$

The D-Partitioning curve, shown in Figure 2, is plotted for the frequency variation within the range $-\infty \le \omega \le +\infty$. It is obtained by the following code:

```
>> T3=tf([-0.4 -1.3 -11],
        [0.4 1.3 1 0])
Transfer function:
-0.4 s^2 - 1.3 s - 11
---------------------
0.4 s^3 + 1.3 s^2 + s
   >> dpartition(T3)
```

The D-Partitioning determines four regions in the complex T3-Plane: D1(0), D2(0), D(1) and D(2). Only D1(0) and D2(0) are regions of stability, since they are always on the left-hand side of the D-Partitioning curve. If T_3 is varied within a range of 0 to 0.265 sec, corresponding to the segment AB, the system will be stable, since AB is within the region of stability D1(0). If T_3 is varied within a range of 0.265 sec to 1.5 sec, related to the segment BC, the system will be unstable, since BC is within the region of instability D(2). If T_3 > 1.5 sec, the system becomes stable and is operating in the region of stability D2(0).

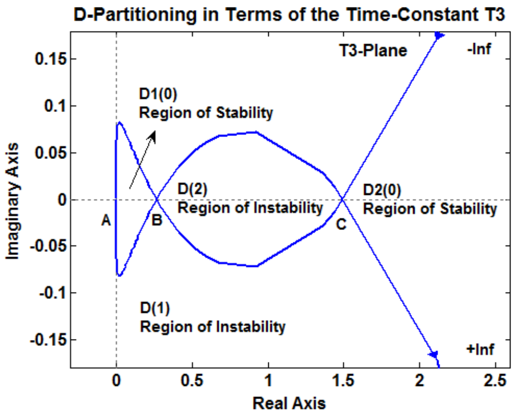

Figure 2: Advanced D-Partitioning in Terms of the Time-Constant T_3

At points B and C the system becomes marginal. Three cases are considered, each one corresponding to one of the defined by the D-Partitioning regions. If the time-constant is T_3 = 0.1 sec, the system operates within the region of stability D1(0).

```
>> T3=0.1
>> Go01=tf([10],[0.04 0.53 1.4 1])
>> margin(Go01)
GM=4.89, PM=4.48
```

Both the system gain and phase margins are positive, being G_m = 4.89 dB and P_m = 4.48 rad/sec. This confirms the results from the D-Partitioning analysis on the stability of the closed-loop system.

There are continuous oscillations with constant amplitude at the marginal points A and B. As seen from the codes below, both gain and phase margins are equal to zero, G_m = 0 dB, P_m = 0 rad/sec.

```
>> T3=0.25
>> Go02=tf([10],[0.1 0.725 1.55 1])
>> margin(Go02)
GM=0, PM=0
```

```
>> T3=1.5
>> Go03=tf([10],[0.6 2.35 2.8 1])
 >> margin(Go03)
GM=0, PM=0
```

Further, the variable time-constant is set to T_3 = 0.7 sec, close to the middle of the region of instability D(2). Then both the system gain and phase margins are negative, being G_m = –1.56 dB and P_m = –5.65 rad/sec and confirming the results obtained from the Advanced D-Partitioning analysis that the closed-loop system is unstable. The step response of the closed-loop system consists of oscillations with continuously rising amplitude, indicating instability of the system in the region of D(2).

```
>> T3=0.7
>> Go04=tf([10],[0.28 1.31 2 1])
>> margin(Go04)
GM=-1.56, PM=-5.65
```

If the time-constant is set to T_3 = 10 sec, the system operates within the region of stability D2(0). Both the system gain and phase margins are positive, being G_m = 11.3 dB and P_m = 43.9 rad/sec and again confirming the D-Partitioning analysis results that the closed-loop system is stable.

```
>> T3=10
>> Go05=tf([10],[4 13.4 11.3 1])
>> margin(Go05)
GM=11.3, PM=43.9
```

III. SISTEM WITH SIMULTANEOUSLY VARIABLE GAIN AND TIME-CONSTANT

The Advanced D-Partitioning analysis in case of two simultaneously variable parameters [2], [4], [8] can be demonstrated again for the control system of the armature-controlled dc motor and a type-driving mechanism. The gain and one of the time-constants are uncertain and variable. The open-loop transfer function of the system is presented as:

$$G_{PO}(s) = \frac{K}{(1+Ts)(1+0.5s)(1+0.8s)} \qquad (6)$$

Then the characteristic equation of the unity feedback system is:

$$K + (1+Ts)(1+0.5s)(1+0.8s) = 0 \qquad (7)$$

By substituting $s = j\omega$ equation (7) is modified to:

$$K = -1 + (1.3T + 0.4)\omega^2 + j\omega(0.4T\omega^2 - 1.3 - T) \qquad (8)$$

Since the gain may have only real values, the imaginary term of equation (8) is set to zero. Then:

$$\omega^2 = \frac{1.3+T}{0.4T} \qquad (9)$$

The result of (9) is substituted into the real part of equation (8), from where:

$$K = \frac{1.3T^2 + 1.69T + 0.52}{0.4T} = 3.25T + 4.225 + \frac{1.3}{T} \qquad (10)$$

The D-Partitioning curve $K = f\ (T)$ is plotted with the aid of the following code:

```
>> T = 0:0.1:5;
>> K = 3.25.*T+4.225+1.3./T
K =
Columns 1 through 10
Inf   17.5500   11.3750   9.5333   8.7750   8.4500   8.3417
8.3571   8.4500   8.5944
Columns 11 through 20
8.7750   8.9818   9.2083   9.4500   9.7036   9.9667   10.2375
10.5147   10.7972   11.0842
Columns 21 through 30
11.3750   11.6690   11.9659   12.2652   12.5667   12.8700
13.1750   13.4815   13.7893   14.0983
Columns 31 through 40
14.4083   14.7194   15.0313   15.3439   15.6574   15.9714
16.2861   16.6014   16.9171   17.2333
Columns 41 through 50
17.5500   17.8671   18.1845   18.5023   18.8205   19.1389
19.4576   19.7766   20.0958   20.4153
Column 51
20.7350
>> plot(T,K)
```

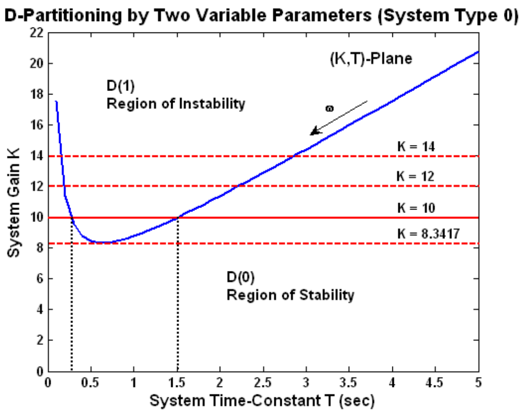

Figure 3. D-Partitioning in Terms of Two Variable Parameters

The D-Partitioning curve $K = f\ (T)$ defines the border between the region of stability D(0) and instability D(1) for case of simultaneous variation of the two system parameters. Each point of the D-Partitioning curve represents the marginal values of the two simultaneously variable parameters. This is a unique advancement and an innovation in the theory of control systems stability analysis. The demonstration of the system performance in case of variation of the time-constant T is done at gain set to K = 10. When $0 < T < 0.25$ sec and $T > 1.5$ sec the system is stable. But it becomes unstable in the range 0.25 sec $< T <$ 1.5 sec. It is also obvious that the system performance and stability depends on the interaction between the two simultaneously varying parameters. If $K < 8.3417$, the system is stable for any value of the T. Higher values of K (K = 12, K = 14), enlarge the range of T at which the system will fall into instability.

IV. DESIGN OF A ROBUST CONTROLLER

The system of armature-controlled dc motor and a type-driving mechanism is also considered for the design of a robust controller [8], [9]. The open-loop transfer function of the plant $G_{P0}(s)$ is modified and now presented in equation (11) considering the two variable parameters that are the system's gain K and time-constant T. Initially, it is suggested that the gain is set to K = 10, while T is variable [5], [9].

$$G_{PO}(s) = \frac{K}{(1+Ts)(1+0.5s)(1+0.8s)} = \frac{K}{0.4Ts^3 + (1.3T+0.4)s^2 + (1.3+T)s + 1} \qquad (11)$$

The robust controller consists of a series stage G_{S0}(s) and a forward stage G_{F0}(s). An integrating stage G_{I0}(s) is also included in the controller as seen from Figure 4.

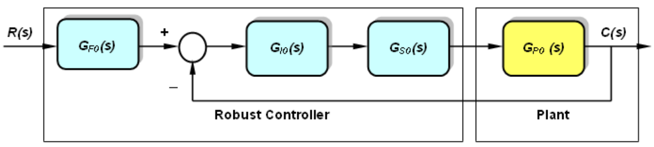

Figure 4. Robust Controller and Integration Incorporated into the Control System

The following steps are considered for the design procedure of the robust control:

Step 1: Initially, the plant transfer function G_{P0} (s), as a standalone block, is involved in a unity feedback system having a closed-loop transfer function presented as:

$$G_{CL}(s) = \frac{K}{(1+Ts)(1+0.5s)(1+0.8s)+K} = \frac{K}{0.4Ts^3+(1.3T+0.4)s^2+(1.3+T)s+1+K} \quad (12)$$

Equation (12) is used as a base in the design strategy for constructing the series stage of the robust controller. It has the task to place its two zeros near the desired dominant closed-loop poles, that satisfy the condition $\zeta = 0.707$.

These zeros will become the dominant poles of the unity negative feedback system, involving the cascade connection of the series controller stage $G_{S0}(s)$, the integrator $G_{I0}(s)$ and the plant $G_{P0}(s)$. If the gain is set to K = 10, the optimal value of the time-constant T, corresponding to the relative damping ratio ζ = 0,707 of the closed-loop system, is determined by the code:

```
>> T=[20:0.01:35];
>> for n=1:length(T)
G_array(:,:,n)=tf([10],[0.4*T(n) (1.3*T(n)+0.4) (1.3+T(n)) 11]);
end
>> [y,z]=damp(G_array);
>> [y,z]=damp(G_array);
>> plot(T,z(1,:))
```

As seen from Figure 5, the relative damping ratio is ζ = 0.707 for a time-constant of T = 27.06 sec.

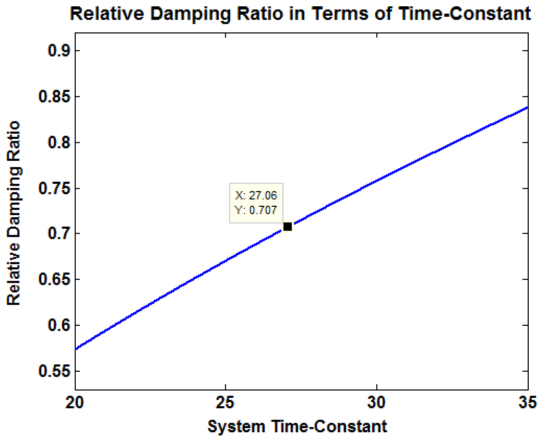

Figure 5. Time-Constant Corresponding to Relative Damping Ratio ζ = 0,707

By substituting the values T = 27.06 sec and K = 10 in equation (12), the transfer function of the closed-loop system becomes:

$$G_{CL}(s) = \frac{10}{10.824s^3+35.578s^2+28.36s+11} \quad (13)$$

The assessment of the system proves that the relative damping ratio becomes ζ = 0.707, when the time-constant is T = 27.06 sec, resulting in system's desired closed-loop poles –0.466 ± j0.466. These outcomes are determined from the code:

```
>> GCL0=tf([10],[10.824 35.578 28.36 11])
>> damp(GCL0)

 Eigenvalue               Damping     Freq. (rad/s)
-4.64e-001 + 4.64e-001i   7.07e-001   6.56e-001
 -4.64e-001 - 4.64e-001i  7.07e-001   6.56e-001
 -2.36e+000               1.00e+000   2.36e+000
```

Step 2: The series robust controller zeros can be placed at the approximated values –0.5 ± j0.5. Therefore, the transfer function of the series robust controller $G_{S0}(s)$ is:

$$G_{S0}(s) = \frac{(s+0.5+j0.5)(s+0.5-j0.5)}{0.5} = \frac{s^2+s+0.5}{0.5} \quad (14)$$

An integrating stage G_{I0}(s) is added to eliminate the steady-state error of the system. It is connected in cascade with the series controller. Then, the transfer function of the compensated open-loop control system will be as follows:

$$G_{OL}(s) = G_{I0}(s)G_{S0}(s)G_{P0}(s) = \frac{K(s^2+s+0.5)}{0.5s(1+Ts)(1+0.5s)(1+0.8s)} \quad (15)$$

When G_{OL}(s) is involved in a unity feedback, its closed-loop transfer function is determined as:

$$G_{CL}(s) = \frac{K(s^2+s+0.5)}{\left[\begin{array}{l}0.5s(1+Ts)(1+0.5s)(1+0.8s)+\\+K(s^2+s+0.5)\end{array}\right]} \quad (16)$$

Step 3: It is seen from the equation (16) that the closed-loop zeros will attempt to cancel the closed loop poles of the system, being in their area. This problem can be avoided if a forward controller G_{F0}(s) is added to the closed-loop system, as shown in Figure 4. The poles of G_{F0}(s) are designed to cancel the zeros of the closed-loop transfer function G_{CL}(s), as shown in equation (17):

$$G_{F0}(s) = \frac{0.5}{s^2+s+0.5} \quad (17)$$

Step 4: Finally, the transfer function of the total compensated system is derived considering the block diagram in Figure 4.

$$G_{T0}(s) = G_{F0}G_{CL0S}(s) = \\ = \frac{K}{\left[\begin{array}{l} 0.5s(1+Ts)(1+0.5s)\times \\ \times(1+0.8s)+K(s^2+s+0.5) \end{array}\right]} = \\ = \frac{K}{\left[\begin{array}{l} 0.5s[0.4Ts^3+(1.3T+0.4)s^2+ \\ +(1.3+T)s+1]+K(s^2+s+0.5) \end{array}\right]} \\ = \frac{K}{\left[\begin{array}{l} 0.2Ts^4+(0.65T+0.2)s^3+ \\ +(0.65+0.5T)s^2+0.5s+K(s^2+s+0.5) \end{array}\right]} \quad (18)$$

V. Robust Performance Assessment of the Compensated System

The system's insensitivity to variations of the system's gain *K* and time-constant *T* is compared before and after applying the robust compensation. Initially at system gain is set to *K* = 10 and three different values of the time-constants are set successively to *T* = 0.1 sec, *T* = 0.8 sec and *T* = 2 sec and are substituted in equation (11).

The case of *T* = 0.8 sec, corresponds to the region D(2) and definitely to an unstable control system. The cases of *T* = 0.1 sec and *T* = 2 sec, reflect regions D1(0) and D2(0) accordingly and are related to a stable control system.

The transient responses of the system before applying the robust compensation are illustrated in Figure 6 and Figure 7 and are achieved by the following code:

```
>> Gp001=tf([0 10],[0.04 0.53 1.4 1])
>> Gp008=tf([0 10],[0.32 1.44 2.1 1])
>> Gp02=tf([0 10],[0.8 3 3.3 1])
>> Gp0fb001=feedback(Gp001,1)
>> Gp0fb02=feedback(Gp02,1)
>> step(Gp0fb001,Gp0fb02)
>> step(Gp0fb008)
```

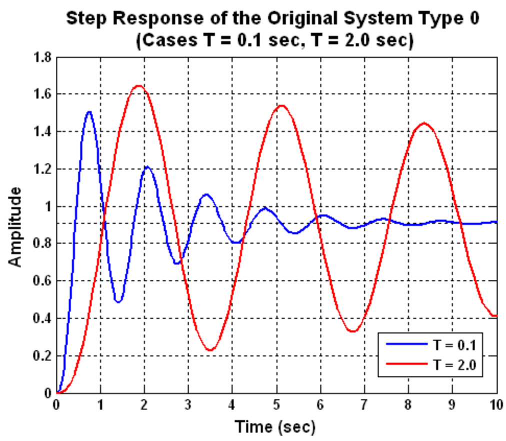

Figure 6. Step Responses of the Original Control System (T = 0.1sec, T = 2sec at K = 10)

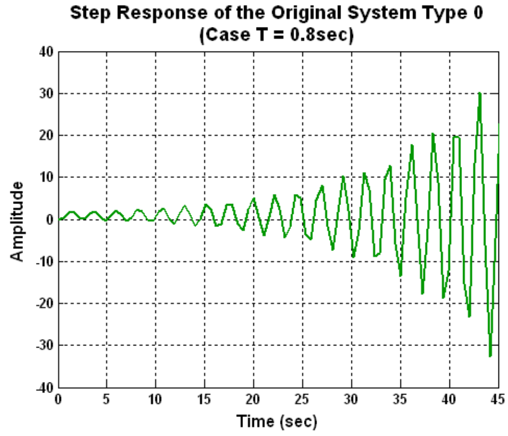

Figure 7. Step Responses of the Original Control System (T = 0.8sec, at K = 10)

Further, the compensated system is examined for robustness in the time-domain. Considering the same values of the time-constant, as those used for the assessment of the original system, *T* = 0.1 sec, *T* = 0.8 sec and *T* = 2 sec at system gain *K* = 10 and substituting them in equation (18), the following transfer functions are obtained:

$$G_{T0}(s)\big|_{T=0.1} = \frac{10}{0.04s^4+0.53s^3+21.4s^2+21s+10} \quad (19)$$

$$G_{T0}(s)\big|_{T=0.8} = \frac{10}{0.32s^4+1.44s^3+22.1s^2+21s+10} \quad (20)$$

$$G_{T0}(s)\big|_{T=2} = \frac{10}{0.8s^4+3s^3+23.3s^2+21s+10} \quad (21)$$

The step responses, representing the time-constant variation of the robust system Type 0, shown in Figure 8, are determined by the code:

```
>> GT01=tf([10],
        [0.04 0.53 21.4 21 10])
>> GT08=tf([10],
        [0.32 1.44 22.1 21 10])
>> GT20=tf([10],
        [0.8 3 23.3 21 10])
>> step(GT01,GT08,GT20)
```

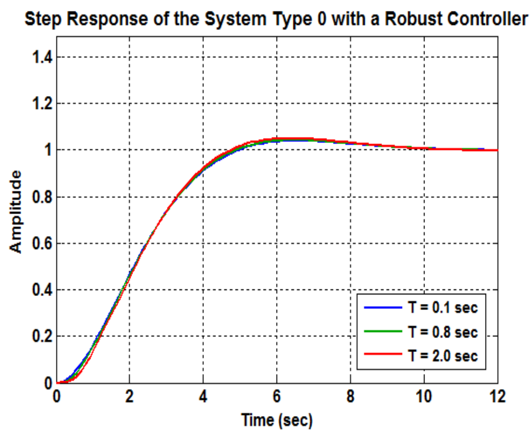

Figure 8. Step Responses of the System with a Robust Controller (T = 0. 1 sec, T = 0. 8 sec, T = 2 sec at K = 10)

As seen from Figure 8, due to the applied robust controller, the control system becomes quite insensitive to variation of the time-constant *T*. The step responses for *T* = 0. 1 sec, *T* = 0. 8 sec and *T* = 2 sec coincide. Additional zero included into the series robust controller stage can further improve the rise time of the system's step response. Since the discussed system is with two variable parameters, now the gain will be changed, applying: *K* = 5, *K* = 10, *K* = 20, keeping the time-constant at *T* = 0.8 sec.

For comparison of the system's insensitivity to the gain variation before and after applying the robust compensation, initially the suggested values as shown above are substituted in equation (11). The transient responses of the original system are illustrated in Figure 9 and 10 and are achieved by the following code:

```
>> Gp05=tf([0 5],[0.32 1.44 2.1 1])
>> Gp010=tf([0 10],[0.32 1.44 2.1 1])
>> Gp020=tf([0 20],[0.32 1.44 2.1 1])
>> Gp0fb5=feedback(Gp05,1)
>> Gp0fb10=feedback(Gp010,1)
>> Gp0fb20=feedback(Gp020,1)
>> step(Gp0fb5,Gp0fb10,Gp0fb20)
>> step(Gp0fb5,Gp0fb10)
>> step(Gp0fb20)
```

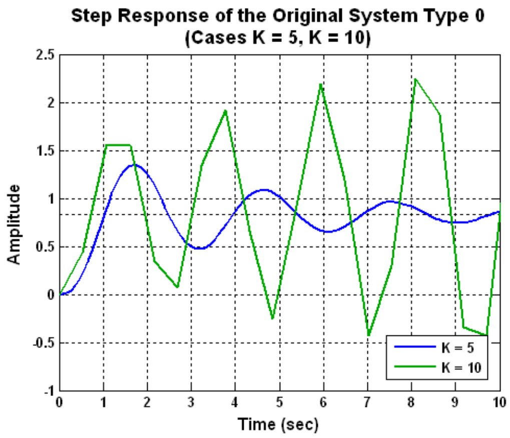

Figure 9. Step responses of the Original Control System (K = 5, K = 10 at T = 0.8sec)

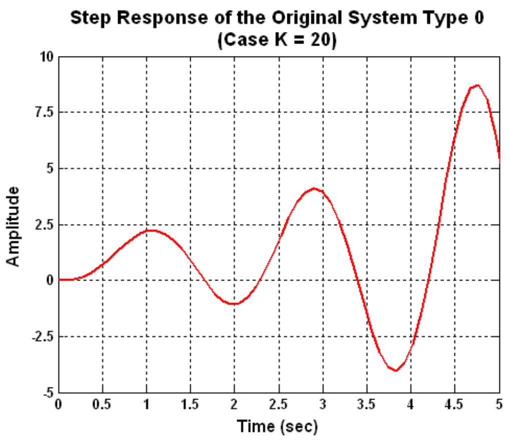

Figure 10. Step Response of the Original Control System (K = 20 at T = 0.8sec)

It is obvious that the cases of *K* = 10 and *K* = 20, correspond to an unstable original control system.

Next, the variable gain *K* = 5, *K* = 10, *K* = 20 will be applying to the robust compensated system, keeping the system's time-constant at *T* = 0.8 sec. These values are substituted in equation (18). As a result, the following outcomes are delivered:

$$G_{T0}(s)_{K=5} = \frac{5}{0.32s^4 + 1.44s^3 + 12.1s^2 + 11s + 5} \quad (22)$$

$$G_{T0}(s)_{K=10} = \frac{10}{0.32s^4 + 1.44s^3 + 22.1s^2 + 21s + 10} \quad (23)$$

$$G_{T0}(s)_{K=20} = \frac{20}{0.32s^4 + 1.44s^3 + 42.1s^2 + 41s + 20} \quad (24)$$

To compare the system robustness before and after the robust compensation, the step responses for the three different cases, representing the gain variation of the robust system, are plotted in Figure 11 with the aid of the code as shown below:

```
>> GTK5=tf([2.5],
          [0.16 0.72 6.05 5.5 2.5])
>> GTK10=tf([5],
          [0.16 0.72 11.05 10.5 5])
>> GTK20=tf([10],
          [0.16 0.72 21.05 20.5 10])
>> step(GTK5,GTK10,GTK20)
```

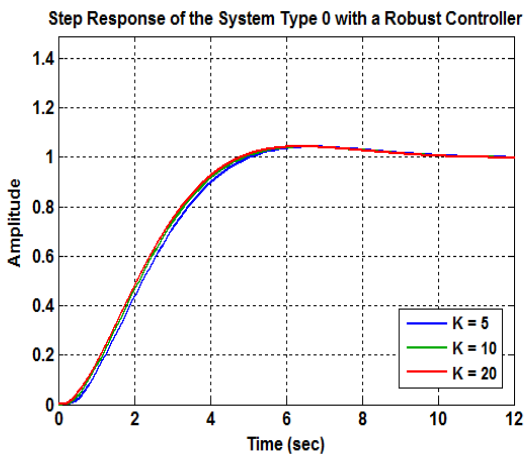

Figure 11. System's Step Responses with the Robust Controller (K = 5, K = 10, K = 20 at T = 0.8 sec)

Again, an additional zero included into the series robust controller stage can further improve the rise time of the system's step response.

An average case is chosen with *K* = 10 and *T* = 0.8 sec for the assessment of the system's performance after the application of the robust controller. This case will differ insignificantly from the other cases of the discussed variable *K* and *T*. The performance evaluation is achieved by following code:

```
>> GT10=tf([10],[0.32 1.44 22.1 21 10])
>> damp(GT10)
```

Eigenvalue	Damping	Freq. (rad/s)
-4.91e-001 + 4.89e-001i	7.09e-001	6.93e-001
-4.91e-001 - 4.89e-001i	7.09e-001	6.93e-001
-1.76e+000 + 7.88e+000i	2.18e-001	8.07e+000
-1.76e+000 - 7.88e+000i	2.18e-001	8.07e+000

It is seen that the relative damping ratio enforced by the system's dominant poles is ζ = 0.709, being very close to the objective value of ζ = 0.707. This insignificant difference is due to the rounding of the desired system's poles to –0.5 ± j0.5, during the design of the series robust controller stage.

The Percentage Maximum Overshoot (PMO) is determined by substituting the relative damping ratio ζ = 0.709 in the following equation:

$$PMO = 100e^{-\pi\varsigma/\sqrt{1-\varsigma^2}} = 3.98\% \quad (25)$$

The settling time t_S for specified limit of ±5% is determined by substituting the obtained values of the relative damping ratio ζ = 0.709 and the natural frequency ω_n = 0.693 rad/sec as follows:

$$t_{S(5\%)} = \frac{4.6}{\varsigma\omega_n} = 9.362 \text{ sec} \quad (26)$$

The system's time to maximum overshoot t_m is determined similarly as follows:

$$t_m = \frac{\pi}{\omega_n\sqrt{1-\varsigma^2}} = 6.425 \text{ sec} \quad (27)$$

The time ratio is one of the objectives:

$$t_{S(5\%)} / t_m = 1.457 \quad (28)$$

As seen from Table 1, all the achieved results are meeting the ITAE criterion, since they either match or have values lower than the targeted objectives.

TABLE I. COMPARISON BETWEEN OBJECTIVES AND RESULTS

Specifications	Objectives	Results	Consideration
ζ	= 0.707	= 0.709	Close Match
PMO	≤ 4%	= 3.98%	Better
$t_{s(5\%)}/t_m$	≤ 2.5	= 1.457	Better

VI. CONCLUSION

Further advancement of the D-Partitioning method is achieved in terms of applying this analysis by one and two variable system parameters. In case of one variable parameter, the D-Partitioning curve, plotted on the complex plane of this parameter. This develops regions of stability and instability, showing a clear picture of the parameter limits of variation to keep the system stable.

The D-Partitioning analysis is further advanced for systems with multivariable parameters [8], [9], [10]. The Advanced D-partitioning in case two variable parameters is demonstrating the strong interaction between the variable parameters. Each point of the D-Partitioning curve represents the marginal values of the two simultaneously variable parameters, being a unique advancement and an innovation in the theory of control systems stability analysis.

The design strategy of a robust controller for linear control systems proves that by implementing desired dominant system poles, the controller enforces the required relative damping ratio and system performance. For systems Type 0, an additional integrating stage ensures a steady-state error equal to zero.

The robust controller has an effect of bringing the system to a state of insensitivity to the variation of its parameters within specific limits of the parameter variations. The experiments in the time-domain with variation of different parameters show only insignificant difference in performance for the different system conditions [10], [11], [12].

In the case of the discussed robust control system, experiments with variation of the time constant within the limits 0.1*T* < *T* < 10*T*, prove that the system becomes quite insensitive to these variations. Testing the robust control system with the gain variation within the limits 0.5*K* < *K* < 5*K*, makes obvious that the system is quite insensitive to variations of the gain *K*. Insignificant step response difference is observed also if the experiment is repeated with the same variation of the gain, but with different time-constants values. Similarly the strategy of the robust controller design can be applied with the same effect, if initially the time-constant is fixed and the gain considered as a variable.

Since the design of the robust controller is based on the desired system performance in terms of relative damping, its contribution and its unique property is that it can operate effectively for any of the system's parameter variations or simultaneous variation of a number of parameters. This property is demonstrated by the comparison of the system's performance before and after the application of the robust controller.

Tests demonstrate that the system performance in terms of damping, stability and time response remains robust and insensitive in case of any simultaneous variations of the gain and the time-constant within specific limits [13], [14]. The suggested analysis and design is beneficial for further advancement of control theory in this field.

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