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Fractal Interpolation Surfaces with Function Vertical Scaling factors on Plural Domain

Congrong Yan
Faculty of Science
Jiangsu University,Jiangsu, 212013
Zhenjiang,China
E-mail: 1126093975@qq.com

Zhigang Feng
Faculty of Science
Jiangsu University,Jiangsu,
Zhenjiang,China
E-mail: zgfeng@ujs.edu.cn

***Abstract*—This paper mainly research a fractal interpolation function on the plural domain.By using function vertical scaling factors ,a method of construction for the iterated function systems(IFS) on the plural domain is proposed.**

Then it can be proved that the invariant set of the (IFS) is the interpolation surfaces which passes through interpolation nodes. And it can be proofed that the invariant is the graph of fractal interpolation function. And by using matlab software draw the projection of fractal interpolation surfaces .we get two figures of fractal interpolation functions ,it can be seen from the nature of the complex variable function that both the real part and the imaginary part function are fractal interpolation functions.

Keywords—plural domain; plural iterated function systems; invariant set

I. INTRODUCTION

Fractal interpolation[1-2] is a new method to fit data,which provides a new tool for data fitting,function approximation,and computer applications.such as fitting coastline,material fracture profile, electrocardiogram and other irregular objects.Document[3-4] proposed that the establishment of fractal interpolation surfaces on rectangular regions which made a foundation.Reference [5-6] refereed to the reflection transformation in the iterated function system, and gave the construction method of the fractal interpolation function of arbitrary interpolation nodes.

The previous research on fractal interpolation theories were on the real number domain, this paper mainly discusses the construction method of fractal interpolation surface for the given interpolation node and constant vertical scale factor in the plural domain.Firstly, an iterative function system with function vertical scaling factor is constructed on the complex field, and then it is proved that there exists an invariant set, and the invariant set is an image of a fractal interpolation function. Finally, using matlab to map, in order to analyze the fractal interpolation surface in the future. .

II. ITERATED FUNCTION SYSTEM AND ITS INVARIANT SET

In this section ,we construct IFS on the basis of a data on plural rectangular grids.Define E is rectangular grids of plural domain C , And the Uniform plots on $E \subset C$ are satisfied with follows conditions:

$$z_{\alpha\beta} = x_{\alpha} + iy_{\beta}, \quad \alpha = 1,\ldots,m; \beta = 0,1,\ldots,n.$$

And rectangular area

$$E = \{x + iy \mid x_1 < x < x_m, y_1 < y < y_n\}$$

Define mappings $L_{\alpha\beta}(z)$: $E \to E_{\alpha\beta} \subset E$ to be contraction homeomorphism satisfying the following conditions:

$$L_{\alpha\beta}(z) = k_{\alpha\beta} \cdot z + r_{\alpha\beta} \cdot \bar{z} + h_{\alpha\beta} \tag{1}$$

where $0 < k_{\alpha\beta} < 1/2$, $0 < r_{\alpha\beta} < 1/4$,The $L_{\alpha\beta}(z)$ maps the end points of E to the end points of $E_{\alpha\beta}$, i.e

$$L_{\alpha\beta}(z_{11}) = z_{\alpha\beta}, L_{\alpha\beta}(z_{1n}) = z_{\alpha,\beta+1},$$
$$L_{\alpha\beta}(z_{m1}) = z_{\alpha+1,\beta}, L_{\alpha\beta}(z_{mn}) = z_{\alpha+1,\beta+1}$$

Define mappings $F_{\alpha\beta}(z,w): E \times C \to C$ as follows:

$$F_{\alpha\beta}(z,w) = S_{\alpha\beta}(z) \cdot w + \varphi_{\alpha\beta}(z) \tag{2}$$

Where $0 < \left\| S_{\alpha\beta}(z) \right\| < 1$ and $\varphi_{\alpha\beta}(z)$ are continuous functions defined on C .

Then we get an mappings $\Gamma_{\alpha\beta}(z,w)$ as follows:

$$\Gamma_{\alpha\beta}(z,w) = \begin{cases} L_{\alpha\beta}(z) = k_{\alpha\beta} \cdot z + r_{\alpha\beta} \cdot \bar{z} + h_{\alpha\beta} \\ F_{\alpha\beta}(z,w) = S_{\alpha\beta}(z) \cdot w + \varphi_{\alpha\beta}(z) \end{cases} \tag{3}$$

Where

$$\begin{cases} h_{\alpha\beta} = z_{\alpha\beta} - z_{11}\cdot(u_{\alpha\beta}+v_{\alpha\beta}) - \bar{z}_{11}\cdot(u_{\alpha\beta}-v_{\alpha\beta}) \\ r_{\alpha\beta} = u_{\alpha\beta} - v_{\alpha\beta} \\ r_{\alpha\beta} = u_{\alpha\beta} + v_{\alpha\beta} \end{cases}$$

$$u_{\alpha\beta} = \frac{1}{2}\cdot\frac{z_{\alpha+1,\beta}+\bar{z}_{\alpha+1,\beta}-z_{\alpha,\beta}-\bar{z}_{\alpha,\beta}}{z_{n1}+\bar{z}_{n1}-z_{11}-\bar{z}_{11}}$$

$$v_{\alpha\beta} = \frac{1}{2}\cdot\frac{z_{\alpha,\beta+1}+\bar{z}_{\alpha,\beta+1}-z_{\alpha,\beta}+\bar{z}_{\alpha,\beta}}{z_{1n}+\bar{z}_{1n}-z_{11}+\bar{z}_{11}}$$

Then we get Iterated function systems as follows:

$$\{E\times D\to C^2;\Gamma_{\alpha\beta}(z,w),\quad \alpha=1,\dots,m-1;\beta=1,\dots,n-1.\} \qquad (4)$$

Theorem1: If $0<\|S_{\alpha\beta}(z)\|<1$,there exists a distance ρ on the plural domain C such that the iterated function system (4) is hyperbolic,Therefore, there exists a unique non-empty set A ,such as

$$\bigcup_{\alpha=1}^{m}\bigcup_{\beta=1}^{n}\Gamma_{\alpha\beta}(A)=A.$$

Proof： $S_{\alpha\beta}(z)$, $\varphi_{\alpha\beta}(z)$ are Lipschitz functions defined on C ,for any $z,w\in C$,there are positive numbers L_j, $j=1,2,3$,such that

$$|S_{\alpha\beta}(z_1)w_1 - S_{\alpha\beta}(z_2)w_1| \le L_1|z_1 - z_2|,$$

$$|S_{\alpha\beta}(z_2)w_1 - S_{\alpha\beta}(z_2)w_2| \le L_2|w_1 - w_2|,$$

$$|\varphi_{\alpha\beta}(z_1)w - \varphi_{\alpha\beta}(z_2)w| \le L_3|z_1 - z_2|.$$

$\rho((z_1,w_1),(z_2,w_2)) = |z_1-z_2| + \varepsilon|w_1-w_2|$,It is obvious that ρ is a distance on the space C^2.

For any $(z_1,w_1),(z_2,w_2)\in E\times C,$

$$\begin{aligned}
&\rho(\Gamma_{\alpha\beta}(z_1,w_1),\Gamma_{\alpha\beta}(z_2,w_2)) \\
&= |L_{\alpha\beta}(z_1)-L_{\alpha\beta}(z_2)| + \varepsilon\cdot|F_{\alpha\beta}(z_1,w_1)-F_{\alpha\beta}(z_2,w_2)| \\
&\le k\cdot|z_1 - z_2| \\
&+\varepsilon\cdot[|S_{\alpha\beta}(z_1)w_1 - S_{\alpha\beta}(z_2)w_2| + |\varphi_{\alpha\beta}(z_1)-\varphi_{\alpha\beta}(z_2)|] \\
&= k\cdot|z_1 - z_2| + \varepsilon\cdot[|S_{\alpha\beta}(z_1)w_1 - S_{\alpha\beta}(z_2)w_1| \\
&\qquad + |\varphi_{\alpha\beta}(z_1)-\varphi_{\alpha\beta}(z_2)| + |S_{\alpha\beta}(z_2)w_1 - S_{\alpha\beta}(z_2)w_2|] \\
&\le k\cdot|z_1 - z_2| + \varepsilon\cdot[(L_1+L_3)|z_1 - z_2| + L_2|w_1 - w_2|] \\
&= [k+\varepsilon\cdot(L_1+L_3)]\cdot|z_1 - z_2| + \varepsilon\cdot L_2|w_1 - w_2|
\end{aligned}$$

Where

$\varepsilon = (1-k)/2(L_1+L_3)$, $\bar{\alpha} = k+\varepsilon\cdot(L_1+L_3)$

then $\bar{\alpha} = (1+k)/2 < 1$. $0<k<1$.

We have

$$\begin{aligned}
&\rho(\Gamma_{\alpha\beta}(z_1,w_1),\Gamma_{\alpha\beta}(z_2,w_2)) \\
&\le \max\{\bar{\alpha},L_2\}(|z_1 - z_2| + \varepsilon|w_1 - w_2|) \\
&= \max\{\bar{\alpha},L_2\}\rho((z_1,w_1),(z_2,w_2))
\end{aligned}$$

It is obvious that $\max\{\bar{\alpha},L_2\}<1$, then Γ is a contraction mappings on ρ .

According to the iterative function system in the complete metric space is compressed,we get a hyperbolic iterative function system Γ , therefore there exists a unique non-empty invariant set A which satisfied

$$\bigcup_{\alpha=1}^{m}\bigcup_{\beta=1}^{n}\Gamma_{\alpha\beta}(A)=A$$

III. Plural Fractal Iterated Surfaces

Define mappings $E\times C\to C$ as follows:

$$F_{\alpha\beta}(z,w) = S_{\alpha\beta}(z)\cdot w + \varphi_{\alpha\beta}(z)$$

$$S_{\alpha\beta}(z) = \lambda_{\alpha\beta}\cdot(z-a)\cdot(z-b)\cdot(z-c)\cdot(z-d)$$

$$\varphi_{\alpha\beta}(z) = a_{\alpha\beta}(z^2-\bar{z}^2) + b_{\alpha\beta}z + c_{\alpha\beta}\bar{z} + d_{\alpha\beta}$$

Where

$$\begin{cases} F_{\alpha\beta}(z_{11},w_{11}) = w_{\alpha\beta} \\ F_{\alpha\beta}(z_{1n},w_{1n}) = w_{\alpha,\beta+1} \\ F_{\alpha\beta}(z_{m1},w_{m1}) = w_{\alpha+1,\beta} \\ F_{\alpha\beta}(z_{mn},w_{mn}) = w_{\alpha+1,\beta+1} \end{cases} \qquad (5)$$

And $a_{\alpha\beta}, b_{\alpha\beta}, c_{\alpha\beta}, d_{\alpha\beta}$ are all constants.

Where

$a = x_1$, $b = x_n$, $c = iy_1$, $d = iy_m$

According to the condition,we have

$$\begin{cases} a_{\alpha\beta} = \dfrac{(w_{\alpha+1,\beta+1}-w_{\alpha,\beta+1})-(w_{\alpha+1,\beta}-w_{\alpha\beta})}{4(b-a)(d-c)} \\ b_{\alpha\beta} = \dfrac{[-(a+c)\cdot w_{\alpha+1,\beta+1}+(d+a)\cdot w_{\alpha+1,\beta}+(c+b)\cdot w_{\alpha,\beta+1}-(b+d)\cdot w_{\alpha\beta}]}{(b-a)(d-c)} \\ c_{\alpha\beta} = \dfrac{[(a-c)\cdot w_{\alpha+1,\beta+1}+(d-a)\cdot w_{\alpha+1,\beta}+(c-b)\cdot w_{\alpha,\beta+1}+(b-d)\cdot w_{\alpha\beta}]}{(b-a)(d-c)} \\ d_{\alpha\beta} = \dfrac{[ca\cdot w_{\alpha+1,\beta+1}-da\cdot w_{\alpha+1,\beta}-bc\cdot w_{\alpha,\beta+1}+bd\cdot w_{\alpha\beta}]}{(b-a)(d-c)} \end{cases}$$

Then we get the special mappings $\Gamma_{\alpha\beta}(z,w)$ which satisfied the following condition

$$\begin{cases}\Gamma_{\alpha\beta}(z_{11},w_{11})=(z_{\alpha\beta},w_{\alpha\beta})\\ \Gamma_{\alpha\beta}(z_{1n},w_{1n})=(z_{\alpha,\beta+1},w_{\alpha,\beta+1})\\ \Gamma_{\alpha\beta}(z_{m1},w_{m1})=(z_{\alpha+1,\beta},w_{\alpha+1,\beta})\\ \Gamma_{\alpha\beta}(z_{nn},w_{nn})=(z_{\alpha+1,\beta+1},w_{\alpha+1,\beta+1})\end{cases} \quad (6)$$

Theorem 2: while $|\lambda_{\alpha\beta}|<16/[(b-a)^2(c-d)^2]$, there exists a continuous function f on E which interpolates the data set,i.e. $f(z_{\alpha\beta})=w_{\alpha\beta},\alpha=1,\dots,n;\beta=1,\dots,m$,and the graph G of f:

$$G=Graph(f)=\{(z,f(z))\,|\,z\in E\}$$

is the invariant set of iterative function system (4),if and only if f satisfies the equations

$f(L_{\alpha\beta}(z))=F_{\alpha\beta}(z,f(z)),z\in E,\quad \alpha=1,2,..n;\beta=1,2,\dots,n$

Proof:

Let $C(E)$ are the set of continuous functions on E, for any $f\in C(E)$,

$$\|f\|_\infty=\max\{|f(z)|:z\in E\},$$

then $(C(E);|\cdot|_\infty)$ constitutes a complete metric space,

Let $C_0(E)=\{f\in C(E)\,|\,f(z_{11})=w_{11};$

$f(z_{1n})=w_{1n};f(z_{m1})=w_{m1};f(z_{mn})=w_{mn}\}$

it is obviously a closed subspace of $C(E)$,Therefore $(C_0(E);\|\cdot\|_\infty)$ is also a complete metric space

Define a mapping T on the space $C_0(E)$:

$$Tf(z)=F_{\alpha\beta}(L_{\alpha\beta}^{-1}(z),f(L_{\alpha\beta}^{-1}(z)))$$

where $z\in E_{\alpha\beta},\alpha=1,\dots,m-1;\beta=1,\dots,n-1.$

Because $L_{\alpha\beta}^{-1}(z),\ S_{\alpha\beta}(z),\varphi_{\alpha\beta}(z),f$ are all continuous functions on $E_{\alpha\beta}$, it is easy to verify that

$$F_{\alpha\beta}(L_{\alpha\beta}^{-1}(z),f(L_{\alpha\beta}^{-1}(z)))$$

is continuous functions on $E_{\alpha\beta}$.

Now there exists two situations:

(1) $z\in E_{\alpha\beta}\cap E_{\alpha+1,\beta}$ (2) $z\in E_{\alpha\beta}\cap E_{\alpha,\beta+1}$

Proof of the first case here, other cases are equally plausible, While $z\in E_{\alpha\beta}\cap E_{\alpha+1,\beta}$

$z=x_{\alpha+1}+iy,y\in[y_\beta,y_{\beta+1}]$;

$$L_{\alpha\beta}^{-1}(x_{\alpha+1}+iy)=x_n+iY,Y\in[y_1,y_n]$$

$$F_{\alpha\beta}(L_{\alpha\beta}^{-1}(x_{\alpha+1}+iy),f(L_{\alpha\beta}^{-1}(x_{\alpha+1}+iy)))$$
$$=F_{\alpha\beta}(x_n+iY,f(x_n+iY))=F_{\alpha\beta}(b+iY,f(b+iY))$$
$$=a_{\alpha\beta}[(x_n+iY)^2-(x_n-iY)^2]+b_{\alpha\beta}(x_n+iY)$$
$$+c_{\alpha\beta}(x_n-iY)+d_{\alpha\beta}$$
$$=\frac{d\cdot w_{\alpha+1,\beta}}{d-c}-\frac{c\cdot w_{\alpha+1,\beta+1}}{d-c}$$

Because of

$$L_{\alpha+1,\beta}^{-1}(x_{\alpha+1}+iy)=x_1+iY,Y\in[y_1,y_n]$$

The same reason ,we have

$$F_{\alpha+1,\beta}\left(L_{\alpha+1,\beta}^{-1}(x_{\alpha+1}+iy),f\left(L_{\alpha+1,\beta}^{-1}(x_{\alpha+1}+iy)\right)\right)$$
$$=F_{\alpha+1,\beta}(x_1+iY,f(x_1+iY))=F_{\alpha+1,\beta}(a+iY,f(a+iY))$$
$$=\frac{d\cdot w_{\alpha+1,\beta}}{d-c}-\frac{c\cdot w_{\alpha+1,\beta+1}}{d-c}$$

Then we get

$$F_{\alpha\beta}(L_{\alpha\beta}^{-1}(x_{\alpha+1}+iy),f(L_{\alpha\beta}^{-1}(x_{\alpha+1}+iy)))$$
$$=F_{\alpha+1,\beta}(L_{\alpha+1,\beta}^{-1}(x_{\alpha+1}+iy),f(L_{\alpha+1,\beta}^{-1}(x_{\alpha+1}+iy)))$$

The same can be proved while $z\in E_{\alpha\beta}\cap E_{\alpha,\beta+1}$,

$$F_{\alpha\beta}(L_{\alpha\beta}^{-1}(z),f(L_{\alpha\beta}^{-1}(z)))=F_{\alpha,\beta+1}(L_{\alpha,\beta+1}^{-1}(z),f(L_{\alpha,\beta+1}^{-1}(z)))$$

Therefore we get Tf is also a continuous functions on E. $\because z_{11}\in E_{11},\therefore(Tf)(z_{11})=F_{11}(z_{11},w_{11})=w_{11}$,

The same reason

$(Tf)(z_{1n})=w_{1n},(Tf)(z_{mn})=w_{mn},(Tf)(z_{m1})=w_{m1}$,

$Tf\in C_0(E)$,

T is the mapping from $C_0(E)$ into $C_0(E)$.

For any $f_1,f_2\in C_0(E)$,

$$\|Tf_1(z)-Tf_2(z)\|_\infty$$
$$=\|F_{\alpha\beta}(L_{\alpha\beta}^{-1}(z),f_1(L_{\alpha\beta}^{-1}(z)))-F_{\alpha\beta}(L_{\alpha\beta}^{-1}(z),f_2(L_{\alpha\beta}^{-1}(z)))\|$$
$$=\|S_{\alpha\beta}(z)\cdot f_1(L_{\alpha\beta}^{-1}(z))+\varphi_{\alpha\beta}(L_{\alpha\beta}^{-1}(z))$$
$$-S_{\alpha\beta}(z)\cdot f_2(L_{\alpha\beta}^{-1}(z))-\varphi_{\alpha\beta}(L_{\alpha\beta}^{-1}(z))\|$$
$$\le\|S_{\alpha\beta}(z)\|\cdot\|f_1(L_{\alpha\beta}^{-1}(z))-f_2(L_{\alpha\beta}^{-1}(z))\|$$
$$\le S\cdot\|f_1(L_{\alpha\beta}^{-1}(z))-f_2(L_{\alpha\beta}^{-1}(z))\|_\infty$$

Where $S=\max_{\alpha\beta}\sup|S_{\alpha\beta}(z)|<1,z\in E_{\alpha\beta}$, therefore T is a contraction operator.

According to the Banach fixed point theorem,there exists a function $f \in C_0(E)$, such that $Tf = f$,i.e.

$$F_{\alpha\beta}(L_{\alpha\beta}^{-1}(z), f(L_{\alpha\beta}^{-1}(z))) = f(z),$$
$$z \in E_{\alpha\beta} \subset E, \alpha = 1,2,\ldots,m-1; \beta = 1,2,\ldots,n-1$$

Let $G = Graph(f) = \{(z, f(z)) \mid z \in E\}$ be the graph of the function f .

We have

$$\bigcup_{\alpha=1}^{n} \bigcup_{\beta=1}^{n} \Gamma_{\alpha\beta}(G)$$
$$= \bigcup_{\alpha=1}^{n} \bigcup_{\beta=1}^{n} \{(L_{\alpha\beta}(z), F_{\alpha\beta}(z, f(z))) \mid z \in E\}$$
$$= \bigcup_{\alpha=1}^{n} \bigcup_{\beta=1}^{n} \{(z, F_{\alpha\beta}(L_{\alpha\beta}^{-1}(z), f(L_{\alpha\beta}^{-1}(z)))) \mid z \in E_{\alpha\beta}\}$$
$$= \bigcup_{\alpha=1}^{n} \bigcup_{\beta=1}^{n} \{(z, f(z)) \mid z \in E_{\alpha\beta}\}$$
$$= G$$

It is said that G is the unique invariant set of iterated function system(4)

Because $f \in C_0(E)$,

$$f(z_{11}) = w_{11}, f(z_{1n}) = w_{1n}, f(z_{n1}) = w_{n1}, f(z_{nn}) = w_{nn}$$

it can be deserved with the condition(5),that

$$f(z_{\alpha\beta}) = F_{\alpha\beta}(L_{\alpha\beta}^{-1}(z_{\alpha\beta}), f(L_{\alpha\beta}^{-1}(z_{\alpha\beta}))) = F_{\alpha\beta}(z_{11}, f(z_{11})) = w_{\alpha\beta}$$

Therefore ,the function f interpolates the data set Δ ,and the graph of f is the invariant set of iterated function system(4).

IV. EXAMPLE

The points set on $E \times D \subset C \times C$ as follows:

(1+2i,1+i) (2+2i,1+2i) (3+2i,2+i) (4+2i,1+i)

(1+3i,2+i) (2+3i,2+2i) (3+3i,3+3i) (4+3i,2+i)

(1+4i,1+2i) (2+4i,1+i) (3+4i,2+2i) (4+4i,2+2i)

The imaginary and real map of interpolation nodes as Fig 1 and Fig 2.

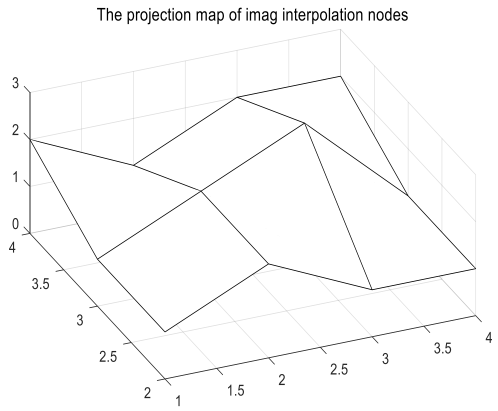

Fig 1: Imaginary interpolation nodes

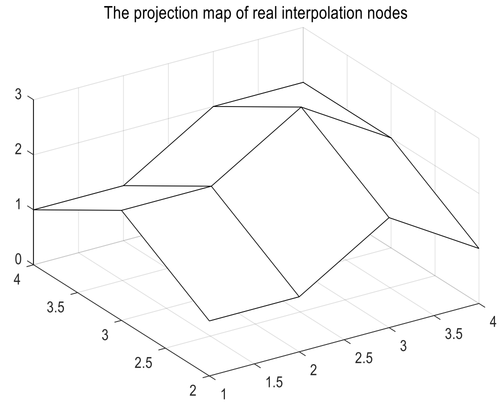

Fig2: Real interpolation nodes

Using the above formula to obtain:

$$\lambda_{\alpha\beta} = [1/32 + i/32, 1/32 + i/32; 1/32 + i/32, 1/32 + i/32; 1/32 + i/32, 1/32 + i/32]$$

$$\varphi_{11}(z) = -1/12 \cdot z + 5i/12 \cdot \bar{z} + 2i/3$$

$$\varphi_{12}(z) = -1/12 \cdot (z^2 - \bar{z}^2) + (5/12 + 3i/4) \cdot z + (-5/12 + i/4) \cdot \bar{z} + 3 - i$$

$$\varphi_{21}(z) = 1/12 \cdot (z^2 - \bar{z}^2) - 3i/4 \cdot z + (1/3 - i/4) \cdot \bar{z} + 3i - 1/3$$

$$\varphi_{22}(z) = (-1/12 + 5i/12) \cdot z + (5/12 - i/12) \cdot \bar{z} + 8/3 - 8i/3$$

$$\varphi_{31}(z) = -1/12 \cdot (z^2 - \bar{z}^2) + (1/2 + i/12) \cdot z + (-5/6 + 7i/12) \cdot \bar{z} + 4/3 - 5i/3$$

$$\varphi_{32}(z) = (1/12 - i/24) \cdot (z^2 - \bar{z}^2) + (-3/4 - i/3) \cdot z + (1/12 + i) \cdot \bar{z} + 14/3 + 16i/3$$

Then we get the real and imaginary of fractal interpolation surfaces as Fig 3 and Fig 4.

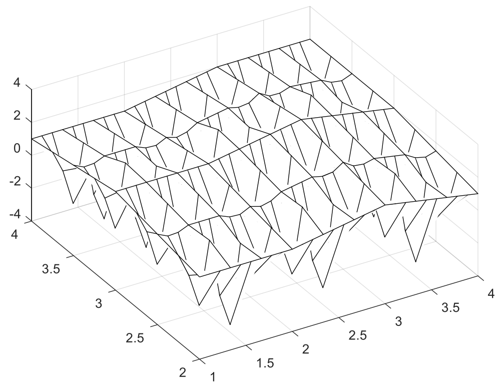

Fig 3: Real fractal interpolation surfaces

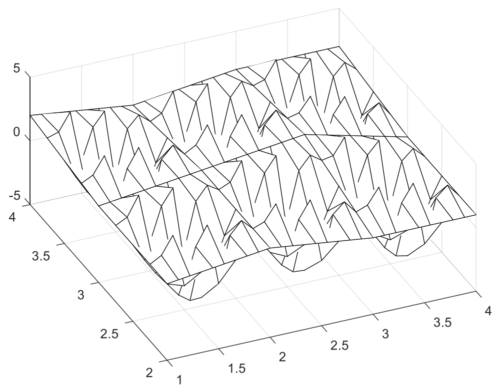

Fig 4: Imaginary fractal interpolation surfaces

V. Conclusion

In this paper, the fractal interpolation surface is constructed on the complex domain, and the feasibility of the theory is proved by examples. The fractal interpolation theory is extended to the complex domain, which enriches the fractal theory.

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