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The MATLAB Program of Hidden Variable Fractal Interpolation Curve

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***Abstract*—In this paper, the mathematical model of hidden variable fractal interpolation function and the MATLAB program of hidden variable fractal interpolation curve are given. A few data points are used to simulate rough and irregular curves by the method of hidden variable fractal interpolation, which is of great significance and practical value for fitting the complex objects and studying fractal figures.**

Keywords—fractal geometry; hidden variable fractal interpolation; MATLAB language; irregular curve

I. INTRODUCTION

Fractal interpolation[1-3] is a new metho-d to fit data, which provides powerful theoreti-cal basis in many fields[4-5]. For example, fitti-ing coastline,material fracture profile, electroca-rdiogram and other irregular objects. In order to improve the flexibility of fractal interpolatio-n, hidden variable fractal interpolation functio-n(HVFIF) is introduced. It is firstly mention-ed by Barnsley[7]. Since then, HVFIF have b-een widely studied in many papers [8-13]. H-VFIF is the projection of vector-valued FIF on the plane, which picture passes given datas. Additional freedom degrees are hidden vari-able used to ajust the shape and dimension of HVFIF.

MATLAB language is a high-level langu-age for scientific compute, which have stro-ng function,simple language and high efficie-cy. It is convenient for us to use. This paper will give MATLAB program of hidden variable fractal interpolation curve. Examples are giv-en with data, which shows the theory of hid-den variable fractal interpolation is practical. Complicated things also can be simulated b-etter by HVFIF.

II. THE MODEL OF HIDDEN VARIABLE FRACTAL INTERPOLATION CURVE

Assuming $\{(x_i, y_i) \in I \times J_1 : \mathrm{i} = 0,1,\ldots,\mathrm{N}\}$ is a set of given points in R^2.Here, $I = [0,1]$, $0 = x_0 < x_1 < \ldots x_N = 1$, $N > 1$ and N is a positve integer. J_1 is a closed interval. Giving a set of real number $\{\mathrm{z_i} : \mathrm{i} = 0,1,\ldots,\mathrm{N}\}$, we can get a generalized data set

$$\{(x_i, y_i, z_i) \in I \times J_1 \times J_2 : i = 0,1,\ldots,N\} \subset R^3$$

Where J_2 including z_i is a proper closed interval. We denote $I_i = [x_{i-1}, x_i]$, $|I_i| = x_i - x_{i-1}$, $i = 0,1,\ldots,N$, $D = J_1 \times J_2$, $K = I \times D$.

Define mappings $L_i : I \to I_i$ for satisfying the conditi-on:

$$L_i(x) = |I_i| x + x_{i-1}$$

Next,we give continuous mappings $F_i : K \to D, i = 1,2,\ldots, N$ as follows:

$$F_i(x, y, z) = \begin{pmatrix} a_i & b_i \\ 0 & d_i \end{pmatrix} \begin{pmatrix} y \\ z \end{pmatrix} + \begin{pmatrix} p_i(x) \\ q_i(x) \end{pmatrix}$$

Where a_i, b_i, d_i are free parameters whose absolute values satisfy $|a_i| < 1, |b_i| + |d_i| < 1$ and $p_i(x), q_i(x)$ are Lipschitz functions. For the continuity of interpolation, the following condition is set up:

$$\begin{cases} F_i(x_0, y_0, z_0) = (y_{i-1}, z_{i-1})^{\mathrm{T}} \\ F_i(x_N, y_N, z_N) = (y_i, z_i)^{\mathrm{T}} \end{cases} \quad i = 1,2,\ldots,N$$

So,we can compute the endpoint conditions of $p_i(0)$, $p_i(1), q_i(0), q_i(1)$.

Define the metric in R^2,

$$\rho((y_1, z_1), (y_2, z_2)) = |y_1 - y_2| + |z_1 - z_2|$$

for any $(y_1, z_1), (y_2, z_2) \in R^3$ .Therefore, for arbitrary $(x, y, z), (x', y, z), (x, y', z') \in \mathrm{K}$, there exist a positive number c ,so that the following is found,

$$\begin{cases} \rho(F_i(x, y, z),\ F_i(x', y, z)) \le c|x - x'| \\ \rho(F_i(x, y, z), F_i(x, y', z')) \le s'\rho((y, z), (y', z')) \end{cases}.$$

Here, $s' = \max_{1 \le i \le k}\{|a_i|, |b_i| + |d_i|\} < 1$.

We denote

$$W_i(x, y, z) = (L_i(x), F_i(x, y, z)), i = 1,2,\ldots,N$$

Then $\{\mathrm{k}; \mathrm{w_i}(\mathrm{x}, \mathrm{y}, \mathrm{z}), \mathrm{i} = 1,2,\ldots,\mathrm{N}\}$ is IFS in R^3. That a metric exsits in R^3 makes W_i become contraction transformation. This IFS has an unique attraction[7] G, which is a continuous graph of vector-valued fractal interpolation function written as $V : I \to D$. V satisfies $V(x_i) = (y_i, z_i), y = 1,2\ldots,N$ and has fixed point equation

$$V(x) = F_i(L_i^{-1}(x), V(L_i^{-1}(x))), \forall x \in I_i, i = 1,2,\ldots,N$$

Denote

$$V(x) = (f(x), g(x)), x \in I$$

$f : I \to R$ is the projection of attractor G on xy-plane with interpolating the data $\{(x_i, y_i) \in I \times J_1 : i = 0,1,\ldots,N\}$, which is called hidden variable fractal interpolation function abbreviated as HVFIF. $g:I\to R$ is the project-on of attractor G on xz-plane.

III. The MATLAB program of hidden variable fractal interpolation curve

According to the construction of hidden variable fractal interpolation function, the corresponding MATLAB language is given below. Rough and irregular curve is realized by computer . The program here takes four data points as example, and more points is similar.

A. Source program

% This is a program of $V(x)$ and its projection $f(x), g(x)$.

```
x0=[Input the first point];
N=Input the times of iteration;
x=x0(1);y=x0(2);z=x0(3);
plot3(x,y,z,'b*');
hold on;
M=x0;
for i=1:N
   [c,r]=size(M);
  for   j=1:r
        x0=M(:,j);
        x=x0(1);
         y=x0(2);
         z=x0(3);
[w1,w2,w3]=fund(x0);
M=[M,w1,w2,w3];
plot3(x,0,z,'g*');
plot3(x,y,0,'r*');
   end
end
x0=[Input the second point];
x=x0(1);y=x0(2);z=x0(3);
plot3(x,y,z,'b*');
hold on;
M=x0;
for i=1:N
   [c,r]=size(M);
   for   j=1:r
        x0=M(:,j);
        x=x0(1);
         y=x0(2);
         z=x0(3);
[w1,w2,w3]=fund(x0);
M=[M,w1,w2,w3];
plot3(x,0,z,'g*');
plot3(x,y,0,'r*');
     end
end
x0=[Input the third point];
x=x0(1);y=x0(2);z=x0(3);
plot3(x,y,z,'b*');
hold on;
M=x0;
for i=1:N
     [c,r]=size(M);
     for   j=1:r
              x0=M(:,j);
              x=x0(1);
              y=x0(2);
              z=x0(3);
   [w1,w2,w3]=fund(x0);
   M=[M,w1,w2,w3];
   plot3(x,0,z,'g*');
   plot3(x,y,0,'r*');
       end
   end
   x0=[Input the fourth point];
   x=x0(1);y=x0(2);z=x0(3);
   plot3(x,y,z,'b*');
   hold on;
   M=x0;
   for i=1:N
        [c,r]=size(M);
        for   j=1:r
              x0=M(:,j);
              x=x0(1);
             y=x0(2);
             z=x0(3);
[w1,w2,w3]=fund(x0);
M=[M,w1,w2,w3];
plot3(x,0,z,'g*');
plot3(x,y,0,'r*');
grid on;
xlabel('x axis')
ylabel('y axis')
zlabel('z axis')
box on;
      end
end
```

Illustration: 'b' represents the image color of $V(x)$.'r' represents the image color of $f(x)$,'g' represents the image color of $g(x)$.

B. Invoking function

```
function [w1,w2,w3]=fund(x0);
w1=[Input 3×3 matrix ]*x0+[Input 1×3 matrix];
w2=[Input 3×3 matrix]*x0+[Input 1×3 matrix];
w3=[Input 3×3 matrix]*x0+[Input 1×3 matrix];
plot3(w1(1),w1(2),w1(3),'b*');
hold on;
plot3(w2(1),w2(2),w2(3),'b*');
hold on;
plot3(w3(1),w3(2),w3(3),'b*');
end
```

Illustration: Where '3×3 matrix' and '1×3 matrix' are determined by IFS.

IV. Research examples

Example 1: Let $I=[0,1]$,N=4, The original data points are

$(x_0, y_0, z_0)=(0,0,0)$, $(x_1, y_1, z_1)=(1/3,1/2,1)$,

$(x_3, y_3, z_3)=(2/3,1,1/2)$, $(x_4, y_4, z_4)=(1,0,0)$.

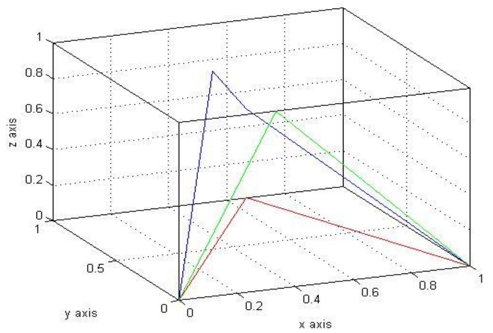

Fig 1: Curve of original data and its projection

Free parameters are as follows,

$$\begin{pmatrix} a_1 & b_1 \\ 0 & d_1 \end{pmatrix} = \begin{pmatrix} -0.4 & 0.6 \\ 0 & 0.3 \end{pmatrix}, \begin{pmatrix} a_2 & b_2 \\ 0 & d_2 \end{pmatrix} = \begin{pmatrix} 0.3 & -0.2 \\ 0 & 0.5 \end{pmatrix}$$

$$\begin{pmatrix} a_3 & b_3 \\ 0 & d_3 \end{pmatrix} = \begin{pmatrix} 0.5 & 0.4 \\ 0 & -0.3 \end{pmatrix}.$$

Let $p_i(x) = e_i x + l_i,\ q_i(x) = t_i x + m_i,\ i = 1,2,3.$

e_i, l_i, t_i, m_i can be computed by endpoint condition. So, we have

$$w_1(x,y,z) = \begin{pmatrix} 1/3 & 0 & 0 \\ 1/2 & -0.4 & 0.6 \\ 1 & 0 & 0.3 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

$$w_2(x,y,z) = \begin{pmatrix} 1/3 & 0 & 0 \\ 1/2 & 0.3 & -0.2 \\ -1/2 & 0 & 0.5 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} + \begin{pmatrix} 1/3 \\ 1/2 \\ 1 \end{pmatrix}$$

$$w_3(x,y,z) = \begin{pmatrix} 1/3 & 0 & 0 \\ -1 & 0.5 & 0.4 \\ -1/2 & 0 & -0.3 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} + \begin{pmatrix} 2/3 \\ 1 \\ 1/2 \end{pmatrix}.$$

The iteration graph is obtained by MATLAB program,

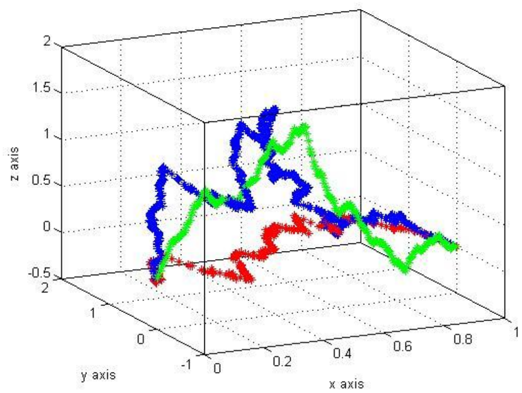

Fig 2: Iteration 5 times

The red curve is the graph of hidden variable fractal interpolation function f. It is shown Fig3.

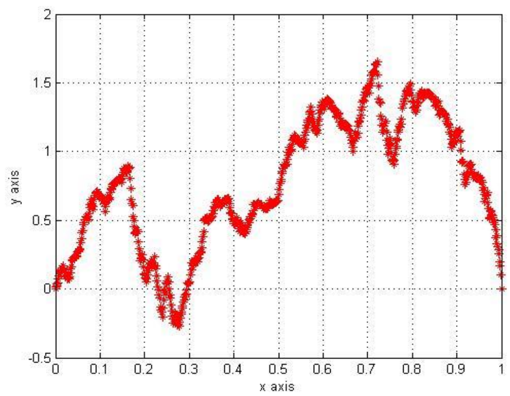

Fig 3: Hidden variable fractal curve

Example 2: The set of points is the same as in example 1.Change the free parameters as follows,

$$\begin{pmatrix} a_1 & b_1 \\ 0 & d_1 \end{pmatrix} = \begin{pmatrix} 0.3 & 0.7 \\ 0 & 0.1 \end{pmatrix}, \begin{pmatrix} a_2 & b_2 \\ 0 & d_2 \end{pmatrix} = \begin{pmatrix} -0.5 & 0.1 \\ 0 & 0.6 \end{pmatrix}$$

$$\begin{pmatrix} a_3 & b_3 \\ 0 & d_3 \end{pmatrix} = \begin{pmatrix} 0.4 & -0.1 \\ 0 & 0.5 \end{pmatrix}.$$

Analogously, we can get the iterative graph,

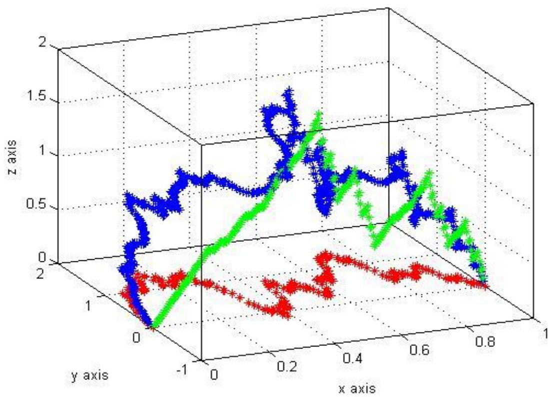

Fig 4: Iteration 5 times

The following is the graph of hidden variable fractal interpolation function f.

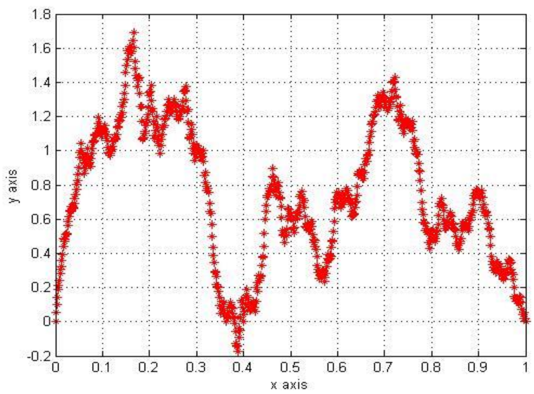

Fig 5: Hidden variable fractal curve

Another, if

$$\begin{pmatrix} a_1 & b_1 \\ 0 & d_1 \end{pmatrix} = \begin{pmatrix} 0.2 & -0.1 \\ 0 & 0.4 \end{pmatrix}, \begin{pmatrix} a_2 & b_2 \\ 0 & d_2 \end{pmatrix} = \begin{pmatrix} 0.5 & 0.2 \\ 0 & -0.5 \end{pmatrix}$$

$$\begin{pmatrix} a_3 & b_3 \\ 0 & d_3 \end{pmatrix} = \begin{pmatrix} 0.4 & -0.1 \\ 0 & 0.2 \end{pmatrix}.$$

Then, the graph of space iteration and hidden variable fractal interpolation function f are got respectively in Fig6, Fig7.

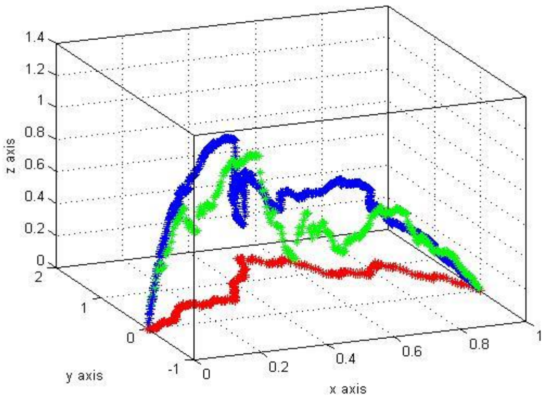

Fig 6: Iteration 5 times

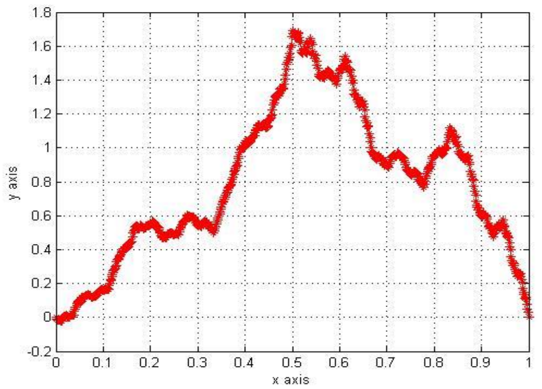

Fig 7: Hidden variable fractal curve

From the above examples, we can find that the curve of hidden variable fractal interpolation will change as long as the value of free parameter is changed.

V. CONCLUSION

Fractal interpolation use the principle of self-affine to obtain various curve that ups and downs. While the hidden variable fractal interpolation curve is non-self-affine. It involves more free variables, we could obtain different curves by changing those free parameters. Therefore, the fractal interpolation function of hidden variable is more flexible and more accurate, which provides a strong theoretical basis for simulating many objects and phenomena in nature. The realization of MATLAB also makes us deeply feel the beauty of mathematics.

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