Mathematical Modeling of Resistance Spot Welding

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***Abstract*—The paper presents a mathematical model for resistance spot welding (RSW) from the aspect of heat transfer. After physically analyzing the energy system of resistance spot welding process, the authors have constructed the heat transfer model in the form of Fourier equation, Maxwell-Cattaneo equation by implementing the law of conservation of energy. Finally, by applying the proper initial and boundary conditions, the mathematical model is developed to describe the distribution of temperature in the RSW system as a function of space coordinates and time. The significance of this model is that it can be implemented to control the weld temperature via monitoring the welding current, so as to achieve the welding nugget of optimum shape and size.**

Keywords—Resistance Spot Welding; Heat Transfer; Fourier equation; Maxwell-Cattaneo equation

I. INTRODUCTION

The process of joining two metal plates together by running a high current through them to form a welded nugget in-between metal sheets is called resistance spot welding (RSW). It is one of the oldest electrical welding techniques and is used to weld most of known metals [1]. The actual weld is made at the interface of the two workpieces. As shown in Figure 1, the regional electrical resistance of the metal sheets due to the surface roughness leads to a concentrated heating at the interfaces to be fastened. Resistance spot welder usually has two electrode tips of brass because of its low resistance. When current passes through the workpiece, it can generate massive amount of heat in short period of time and can heat up the workpiece up to 1000℃ [2]. A welding nugget is formed between the two workpieces and the two metal pieces are welded during the process.

Before the resistance spot welding process, the workpieces need to be filed to remove any coating or paint and make sure the welding surface is smooth. There are four steps in a RSW cycle. First, pressure need to be applied to the surface of the work pieces before the current is on, to make sure the electrodes are in good contact with the workpieces. Second, current

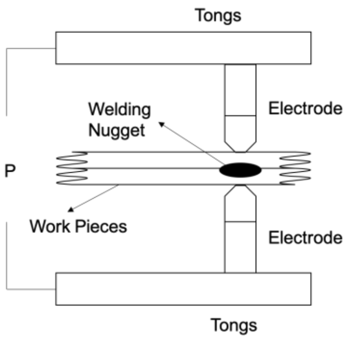


Fig. 1. *Resistance Spot Welding Machine with Workpiece*

will be applied and welded area will begin to heat up and welded nugget will be generated. This step will take

about ten cycles (1/60 of a second is one cycle) [2]. Third, after the current stops, the electrodes will keep pressing on the workpiece for a period of time to keep the molten nugget in shape and cooling off. Afterwards, the electrodes will be released and the welding process is done.

There are some factors that will influence the final welding product [3]. Before the actual process, the welding electrode tips need to be aligned properly to ensure the current flow through the workpiece is in an hourglass shape. If the welding tips are not aligned properly, the current will be distorted and final weld piece may be bent or deformed. The weld tip force or squeeze pressure also need to be properly adjusted to maintain the proper amount of resistance both between the workpieces and between the electrode and workpiece. If the force is too low, the resistance will be too high. Overheating the workpiece surfaces can lead to excessive indentation, surface expulsion, brassing and blow holes. If the force is too low, the resistance will decrease, causing the heat generated to be too low and the final weld will be weak and substandard.

Weld tip size also needs to be considered before the actual process. Weld tips need to be coordinated with the size of the metal plates so that the central spot between the two electrode tips is on the contact surface of two workpieces. This ensures the center of the weld nugget locate at the contact surface, leading to a tight and sturdy structure.

Weld current and weld time are important factors affecting the final weld product. There are usually 10 cycles of weld time. After the current flows, the first 4 to 5 cycles will heat up the metal and the tips will move closer. After the 6th cycle, the weld nugget will start forming and the next 5 cycles the nugget will begin to increase in size. The scale of the current need to be properly adjusted based on the materials and size of the workpieces.

After the current is cut off, there will be a period of hold time where the electrode tips remain in contact with the welding area to absorb heat and hold the structure in place. There is cooling water pumping inside both tips to take out the heat from the welding area. The hold time is usually about 4-5 cycles. If the hold time is too short, weld nugget may not have enough time to solidify and may spring apart as the electrodes are removed. If the hold time is too long, excessive indentation may happen due to the weld nugget deformation. Cracking may also happen due to excessive cooling on the weld surface [4].

II. Physical Analysis of RSW

Resistance spot welding is a process involving metal transforming from solid state to semi-solid and liquid states. Three aspects of physical study can be introduced during RSW process: solid-liquid metal morphology, casting rheology and thermal expansion.

During the resistance spot welding process, as the welding interfaces heats up and begins to melt, semi-solid metal inside the welding area exhibits unique properties in the process [5]. The entire workpiece cannot be regarded as a simple Hookean solid or Newtonian liquid. Between the solid exterior and the liquid core, solid metal crystal clusters of metal floats around liquid metal in different shapes such as sphere, spike and dendrite. Under the clamping force of RSW electrodes, the special morphology of solid metal exterior containing a semi-solid layer and a liquid core exhibits rheological properties such as shear thinning and thixotropy. What's more, as diffusivity of most solid and liquid metals differs profoundly (D_s~10^{-5} cm^3/s, D_l~10^{-8} cm^3/s), solute inside liquid state diffuses much faster. Concentration gradient of solute between liquid and solid metal during RSW process can lead to an uneven phase distribution. With the proper model, these properties can be considered at refining the simulation of the final welding product.

In RSW, as heating processes, the asperities in the contact areas of two work pieces softens and eventually melt down [5]. In the meantime, the clamping force of the electrodes will plastically deform the molten welding area. The plastic deformation not only cause the resistivity between the workpieces to drop, it will also influence the microstructure of the welding nugget. As the deformation process happens in a sandwich structure, which contains a solid metal exterior with a semi-solid layer and a liquid core, it cannot be regarded as a simple solid-state deformation. Thus, final weld morphology has to take rheological properties of the material into account.

For resistance spot welding, thermal expansion and collapsing of the welding area both con-tribute to the final modeling of the RSW process and workpiece configuration variations. The model can be divided into three parts. First, the heat generated from electrical current will heat up the work piece, resulting into an initial thermal expansion of solid metal. The heat will also be conducted to the area surrounding it. The thermal expansion equation for solidus metal is as following:

$$\Delta L = L_f - L_0 = \alpha \Delta T L_0 \quad (1)$$

where ΔL is the expanded length, L_f and L_0 being the final and original lengths, ΔT the temperature change and α the thermal expansion coefficient.

Secondly, as temperature approaching the melting point of the metal, phase change will begin, a part of the heat generated will be absorbed for phase changing. [6] Once the metal has completed the phase change and becomes liquid state, the thermal expansion mechanism changes from interatomic interactions to an increase in the free volume holes [7]. As temperature rises, there will be initiation and growth of free volume holes in the liquid. Due to higher mobility of the atoms, higher thermal power will result in larger frequency and amplitude of atom. The change will lead to a higher thermal expansion coefficient and consequently a faster expansion rate of the welded area.

Finally, while the volume of the metal is growing, the shape of the kernel is also altering. The change in phase will result in change of mechanical structure. As the kernel is growing, the shape will change from a sphere to an ellipsoid due to mechanical collapse, resulting in a smaller distance between two electrodes.

In modern automotive industries, RSW has become one of the most important methods for welding thin metal sheets [3]. Conventional methods for testing the welding quality include destructive tests such as peel test, which involves pulling the welded coupons apart and measure the nugget diameters [8]. The other way to ensure the weld quality is to real time monitoring the resistance between the welded area, as resistance of it change dynamically in different stages of welding process. Sometimes it is also considered reasonable to generate a small amount of expulsion between the workpieces in order to ensure a tight and solid weld. However, all these methods compromise to either unsatisfying weld products or uncertainty to consistence of the weld quality.

Thus, simulation of the welding area is of great importance and convenience. It provides in time visual proof of the weld development without destructing the weld pieces or compromising to overly welded parts. However, there is been limited research on modeling of the resistance spot welding process. While previous researches have been performed to analyze the process, most of them are conducted from an experiment-observing point of view and sophisticated mathematical model for the welding process is lacking [9].

To make amends, the authors of this paper are proposing to analyze the heat transfer process of resistance spot welding and develop a complete mathematical model to describe its thermal properties.

III. Mathematical Modeling of RSW

A. Core Process of RSW

Resistance Spot Welding (RSW) could be modeled in the aspects of mechanical system, electrical circuit, heat transfer, or so. But intrinsically, RSW process is actually a problem of heat transfer while others are only conditions coupled with the thermal aspect. In terms of heat transfer, RSW is related more to heat conduction, very little to heat convection or heat radiation. And the metal sheets that undergoes the welding are only involved with single phase change, which is from solid phase to liquid phase.

In RSW, the electrical current flows throughout the whole cylindrical volume that is clamped and covered by electrodes, and generates heat according to Joule's First Law, i.e., the heat produced is proportional to the electrical resistance multiplied by the square of the current. Where the resistance is larger, more heat will be produced locally inside of the volume, rather than at the boundaries. This is the so-called 'Joule heating'. The metal sheets are the conductor that the current flows through. But the conductors always have resistance. For RSW, the largest resistance occurs between the metal sheets. Metal is also a temperature-sensitive resistor and its resistance increases with temperature. For instance, low carbon steel's resistance increases 5 to 10 times between room temperature to 100 ℃ [10]. As temperature at the metal sheet contact zone increases, resistance also goes up which leads to more heat being generated. Finally, as the center reaches melting temperature, the weld nugget will be initiated and grow into a mushy region where solid-liquid phases co-exist.

B. Fourier Equation and Maxwell-Cattaneo Equation

In general, for a fixed control volume, the law of conservation of energy can be written as [11]

$$R_{in} + R_{gen} = R_{out} + R_{stor} \qquad (2)$$

where

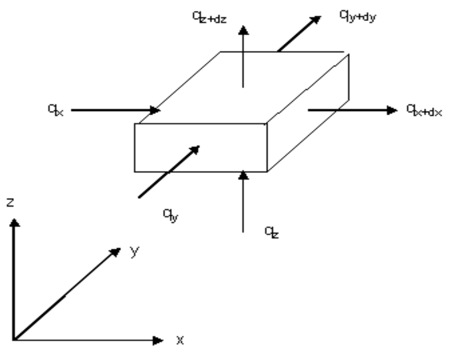
R_{in}= the rate at which energy of all forms enters the control volume,

R_{gen}= the rate at which energy is generated within the control volume itself,

R_{out}= the rate at which energy of all forms leaves the control volume,

R_{store}= the time rate of change of stored energy within the control volume itself,

In the case of $R_{gen} \neq 0$, the resistance dissipates the energy and generates heat. In the case of $R_{stor} \neq 0$,



the RSW is an unsteady process.

Fig. 2. *The differential element in a control volume with heat conduction rates*

From Figure 2, the following relationships are obvious:

$$R_{in} = q_x + q_y + q_z \qquad (3)$$

$$R_{out} = q_{x+dx} + q_{y+dy} + q_{z+dz} \qquad (4)$$

where

$$q_{x+dx} = R_{in} = q_x + \frac{\partial q_x}{\partial x}dx + \frac{\partial^2 q_x}{\partial x^2}\frac{dx^2}{21} + \frac{\partial^3 q_x}{\partial x^3}\frac{dx^3}{31} + \ldots \approx q_x + \frac{\partial q_x}{\partial x}dx$$

$$q_{y+dy} = q_y + \frac{\partial q_y}{\partial y}dy \qquad (5)$$

$$q_{z+dz} = q_z + \frac{\partial q_z}{\partial z}dz$$

Thus

$$R_{out} = q_x + q_y + q_z + \frac{\partial q_x}{\partial x}dx + \frac{\partial q_y}{\partial y}dy + \frac{\partial q_z}{\partial z}dz \qquad (6)$$

The energy generated in the control volume is represented with the rate R_{gen}.

$$R_{gen} = q'''dxdydz \qquad (7)$$

and the rate R_{stor} reflects the energy stored in a solid with its molecular arrangement and motion, which is normally called "internal energy" per unit mass, denoted by u. Thus, (8) can be drawn

$$R_{stor} = \frac{\partial}{\partial t}[\rho(dxdydz)] \qquad (8)$$

Equation (2) can be rewritten and simplified as

$$q'''dxdydz = \frac{\partial q_x}{\partial x}dx + \frac{\partial q_y}{\partial y}dy + \frac{\partial q_z}{\partial z}dz + \rho\frac{\partial u}{\partial t}dxdydz \qquad (9)$$

For a rigid solid, the internal energy u, related to the pressure p and temperature T_0 for the particle substances, can be expressed as

$$u = u_0 + c_p(T - T_0) \qquad (10)$$

where c_p is the constant pressure specific heat in unit $J/(kg℃)$. The derivative of (10) leads to

$$\frac{\partial u}{\partial t} = c_p\frac{\partial T}{\partial t} \qquad (11)$$

According to Fourier's Law,

$$q''' = -k\frac{\partial T}{\partial n} \quad (12)$$

where k is the thermal conductivity in unit $Watt/(m℃)$, thus

$$q = dA_n q''' = -k\frac{\partial T}{\partial n}dA_n \quad (13)$$

Its components can be expressed as

$$q_x = -k\frac{\partial T}{\partial x}dydz$$
$$q_y = -k\frac{\partial T}{\partial y}dxdz \quad (14)$$
$$q_z = -k\frac{\partial T}{\partial z}dxdy$$

and

$$\frac{\partial q_x}{\partial x} = -\frac{\partial}{\partial x}(k\frac{\partial T}{\partial x})dydz$$
$$\frac{\partial q_y}{\partial y} = -\frac{\partial}{\partial y}(k\frac{\partial T}{\partial y})dxdz \quad (15)$$
$$\frac{\partial q_z}{\partial z} = -\frac{\partial}{\partial z}(k\frac{\partial T}{\partial z})dxdy$$

substituting (11) & (15) into (9) and canceling the common terms result in a partial differential equation:

$$q''' + \frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}(k\frac{\partial T}{\partial z}) = \rho c_p\frac{\partial T}{\partial t} \quad (16)$$

This is the mathematical expression of the law of conservation of energy. Its solution with the proper initial conditions and boundary conditions describes the distribution of the temperature as a function of space coordinates and time in the control volume.

When the thermal conductivity k is a constant, (16) can be expressed as

$$\frac{q'''}{k} + \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t} \quad (17a)$$

or

$$\frac{q'''}{k} + \nabla^2 T = \frac{1}{\alpha}\frac{\partial T}{\partial t} \quad (187b)$$

where $\alpha = \frac{k}{\rho c_p}$ is the thermal diffusivity. This is the so-called "Fourier Equation".

Equation (17) can be expressed in other coordinate systems. In the circular cylindrical coordinate system, it becomes

$$\frac{q'''}{k} + \frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2} + \frac{\partial^2 T}{\partial z^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t} \quad (18)$$

where θ is the angle in xy plane.

In the spherical coordinate system, it is

$$\frac{q'''}{k} + + \frac{2}{r}\frac{\partial T}{\partial r} + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}(\sin\theta\frac{\partial T}{\partial \theta}) + \frac{1}{r^2 \sin^2\theta}\frac{\partial^2 T}{\partial \emptyset^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t} (19)$$

According to [12], Fourier Equation (17) is nonphysical, because the temperature calculated is under this assumption: a temperature change in one part of the body causes an immediate change in temperature throughout the body. This situation is even more significant when there is no heat generation in the control volume.

In this case, Fourier Equation can be substituted by Maxwell-Cattaneo Equation:

$$\frac{q'''}{k} + \nabla^2 T = \frac{1}{\alpha}\frac{\partial T}{\partial t} + \frac{1}{c^2}\frac{\partial^2 T}{\partial t^2} \quad (20)$$

where c has unit of velocity. If c is approximated as the speed of sound in the body, for good conductor like low carbon steel, the ratio of the coefficients is $\alpha/c = 10^{-11}$ second [8, p 120]. In this case, the RSW is more unsteady than the situation described in (17).

C. Side Conditions

The integration constants need to be determined by the initial conditions (I.C.) and boundary conditions (B.C.). The highest derivatives in the partial differential equation determine the number of integration constants, thus the number of the initial conditions and boundary conditions.

If Fourier Equation (23) is used for temperature rising, nugget formation, one initial condition as well as two boundary conditions for each space coordinates are needed.

If Maxwell-Cattaneo Equation (26) is used for the later phase of the nugget formation, two initial conditions as well as two boundary conditions for each space coordinates are needed.

In the case of Fourier Equation [11]:

I.C.:

The one initial condition needed is when t = 0. In this case, the temperature distribution

$$T(x,y,z,t)|_{t=0} = T(x,y,z,0) = T(x,y,z) \quad (21)$$

In many cases, $T(x,y,z,0) = T_0$ is simply a constant. For the simplicity, we have

$$T(x,y,z,0) = 0 \quad (22)$$

B.C.:

(1) Boundary Condition of the first kind: specific temperature

For $z = z_0$, temperature is known on the bounding surface. For all x and y on the boundary,

$$T(x,y,z,t) = T_w = G(x,y,z_0,t), for\ \forall t > 0 \quad (23)$$

where G is a known function, often a constant.

For RSW, the temperature of electrode surface is the boundary condition of the first kind, i.e.,

$$T(x,y,z_0,t) = T_{electrode} \quad (24)$$

(2) Boundary Condition of the second kind: specific flux

During the heat conduction process of RSW, the internal energy dissipation on the resistance is mainly in the direction of axis of electrodes. In this direction, the nugget grows. 80% heating was taken away by the electrodes at holding (see Figure 3).

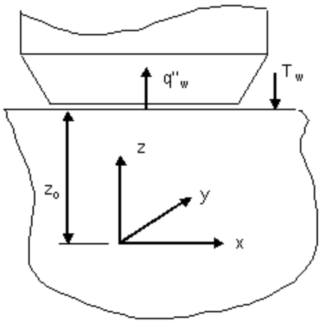


Fig. 3. *B.C. of the Second Kind: Specific Flux*

The known flux q_w'' at z_0 , for $t > 0$,

$$-k\frac{\partial T}{\partial t} = q_w'' = F(r,t) \quad (25)$$

It could be a function varying with time and location, or simply a constant. For RSW, it will be $-k\frac{\partial T}{\partial z} = q_w'' = F(r,t)$. But at the start of the analysis, flux q_w'' is often unknown, which makes this boundary condition unusable.

(3) Boundary Condition of the third kind: convective-type Boundary Condition

Due to the high effectiveness of thermal conductivity of the electrode material (brass), for the simplicity, the temperature drop between the exterior surface of the electrode and the internal surface will be ignored so that the electrode cooling could be simply modeled as a fluid convection (Figure 4).

$$-k\frac{\partial T}{\partial z} = h(T - T_s) \quad (26)$$

where h is the surface coefficient of heat transfer in unit $J/(sec\, m^2\, °C)$ and Ts is the temperature of the cooling fluid.

This could be used in simplified situation. However, for electrode vs. metal sheet, the fourth kind boundary condition is a better choice. But for the situation of the cooling water vs. electrode, the third boundary condition is feasible.

(4) Boundary Condition of the fourth kind: conjugation conditions

As shown in Figure 4, due to the high effectiveness of thermal conductivity of the electrode (brass), the heat flux is homogeneous at the interface between the

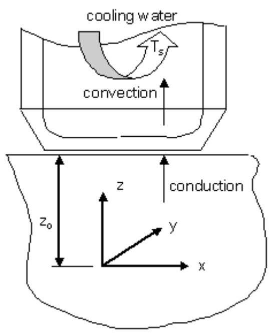


Fig. 4. *B.C. of the third kind: convective type boundary condition*

electrode and the metal sheet. The following boundary condition can be drawn:

$$\frac{q'''}{k} + \nabla^2 T = \frac{1}{\alpha}\frac{\partial T}{\partial t} + \frac{1}{c^2}\frac{\partial^2 T}{\partial t^2} \quad (27)$$

In the case of Maxwell-Cattaneo Equation:

I.C.:

The one initial condition needed is when t = 0.

In this case, the temperature distribution

$$\frac{q'''}{k} + \nabla^2 T = \frac{1}{\alpha}\frac{\partial T}{\partial t} + \frac{1}{c^2}\frac{\partial^2 T}{\partial t^2} \quad (28)$$

In many cases, $T(x, y, z, 0) = T_0$ is a constant. For simplicity,

$$\frac{q'''}{k} + \nabla^2 T = \frac{1}{\alpha}\frac{\partial T}{\partial t} + \frac{1}{c^2}\frac{\partial^2 T}{\partial t^2} \quad (29)$$

Also, it can be drawn that,

$$\frac{q'''}{k} + \nabla^2 T = \frac{1}{\alpha}\frac{\partial T}{\partial t} + \frac{1}{c^2}\frac{\partial^2 T}{\partial t^2} \quad (30)$$

Its boundary conditions will be the same as discussed above.

D. Modeling the RSW with Side Conditions

In the six phases of the RSW, Temperature rising (electrode load holds, electrical cycles ON) can be described by (17) or (20). Both are valid.

It is noticed that temperature derivatives in x-y plane are fairly small so that the average temperatures in these two directions can be at the same point of z-direction. This limped temperature distribution in z coordinate is the result of a quasi-one-dimensional conduction. (Figure 5)

In order to build a mathematical model to describe the transient quasi-one-dimensional conduction, the following assumptions are set based on observations: (1) the heat in the process is only supplied by heat conduction; (2) there are two phases involved in this process: solid phase, with its temperature, coefficient of heat conduction, specific heat, and density. The transition from solid phase to liquid phase is caused by absorption of latent heat, and the transition from liquid phase to solid phase is caused by liberation of latent heat. (3) The nugget is a disk shape, and its growth is only in one dimension: the disk becomes thicker and thicker until the full size is formed. The diameter of the nugget is basically a constant during the RSW process, if the welding parameters are controlled well.

1) Temperature rising before fusion

For RSW fixed control volume, the law of conservation of energy can be written as[11]

$$R_{in} + R_{gen} = R_{out} + R_{stor} \quad (31)$$

The following relationships are obvious:

$$R_{in} = q_z \quad (32)$$

$$R_{out} = q_{z+dz} \approx q_z + \frac{\partial q_z}{\partial z}dz \quad (33)$$

The energy generated in the control volume is represented with the rate R_{gen}

$$R_{gen} = q'dz \quad (34)$$

where $q' = q'''A$, and the rate R_{stor} reflects the energy stored in a solid with its molecular arrangement and motion, which is normally called as "internal energy" per unit mass, denoted by u. Thus, the following equations can be drawn.

$$R_{stor} = \frac{\partial}{\partial t}[\rho(Adz)u] \quad (35)$$

Thus, it can be drawn that

$$q'dz = \frac{\partial q_z}{\partial z}dz + \rho A\frac{\partial u}{\partial t}dz \quad (36)$$

For a rigid solid, the internal energy u, related to the pressure p and temperature T_0 for the particle substances, can be expressed as

$$u = u_0 + c_p(T - T_0) \quad (37)$$

where c_p is the constant pressure specific heat in unit $J/(kg\ ℃)$. The derivative of u is

$$\frac{\partial u}{\partial t} = c_p\frac{\partial T}{\partial t} \quad (38)$$

According to Fourier's Law,

$$q_z = -k\frac{\partial T}{\partial z}A \quad (39)$$

We have

$$q' + \frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)A = \rho A c_p\frac{\partial T}{\partial t} \quad (40a)$$

or

$$\frac{q'}{kA} + \frac{\partial^2 T}{\partial z^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t} \quad (40b)$$

where $\alpha = \frac{k}{\rho c_p}$ is the thermal diffusivity, and $q = \frac{IR(t)^2}{AL}$ in $Watt/m^3$, A is the electrode area, L is the total thickness of the metal sheets under electrodes, thus $q' = q'''A = \frac{IR(t)^2}{L}$ Watt/m and

$$\frac{1}{kAL}IR(t)^2 + \frac{\partial^2 T}{\partial z^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t} \quad (40c)$$

This is the conservation law of energy for RSW. The above Fourier Equation can be also substituted by Maxwell-Cattaneo Equation:

$$\frac{q'}{kA} + \frac{\partial^2 T}{\partial z^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t} + \frac{1}{c^2}\frac{\partial^2 T}{\partial z^2} \quad (41)$$

or

$$\frac{1}{kAL}IR(t)^2 + \frac{\partial^2 T}{\partial z^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t} + \frac{1}{c^2}\frac{\partial^2 T}{\partial z^2} \quad (41b)$$

It is assumed the sheet metal temperature and electrode temperature is the same T_0, and for simplicity and without loss of generality, $T_0 = 0$.

I.C.:

$$T(z,0) = 0 \text{ at } t = 0 \quad (42)$$

B.C.:

$$T(0,t) = 0 \text{ for } 0 < t < t_m \quad (43)$$

$$T(L,t) = 0 \text{ for } 0 < t < t_m \quad (44)$$

$$T(z,t) \text{ is infinite for } 0 < t < t_m \quad (45)$$

Equations (40) & (42)-(45) or, (41) & (42)-(45) describe the model of RSW before $T(z,t)$ reaches the melting temperature T_m. The time it takes to reach T_m is t_m.

2) Nugget Forming and Growth

During the period of nugget forming and growth, there are solid and liquid in the control volume, and the solid-liquid interface is expanding. The temperature of liquid is noted as $T_l(z,t)$, the temperature of solid is noted as $T_s(z,t)$, and the thickness of nugget is noted as $\xi(t)$, $0 < \xi(t) < 0.7L$. Assume the nugget starts at the middle of metal sheets, and the metal sheets have the same thickness.

Then the RSW model is composed of the following equations:

For the solid portion (outside of the nugget),

$$\frac{1}{kAL}IR(t)^2 + \frac{\partial^2 T}{\partial z^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t} \quad (46)$$

in $0 < z < 0.5(L - \xi(t)), \& \ 0.5(L + \xi(t)) < z < L$

and for the liquid portion (inside the nugget),

$$\frac{1}{kAL}IR(t)^2 + \frac{\partial^2 T}{\partial z^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t} \quad (47)$$

in $0.5(L - \xi(t)) < z < 0.5(L + \xi(t))$

subject to

I.C.:

$$T(z,t_m) = T_m \text{ at } t = t_m \quad (48)$$

B.C.:

$$T(0,t) = 0 \text{ for } 0 < t < t_M \quad (49)$$

$$T(L,t) = 0 \text{ for } t_m < t < t_M \quad (50)$$

$$T(z,t) \text{ is infinite for } 0 < t < t_m \quad (51)$$

$$T_s(\xi,t) = T_l(\xi,t) = T_M, 0 < \xi(t) < 0.7L \quad (52)$$

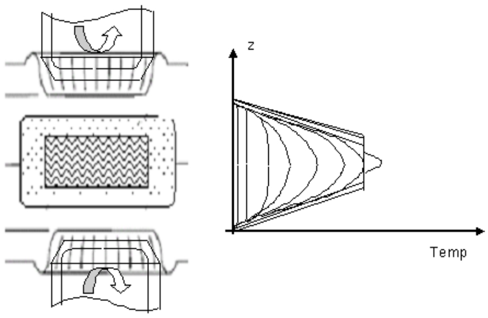


Fig. 5. *Temperature Distribution*

Stefan condition:

$$k_s \frac{\partial T_s(\xi,t)}{\partial z} - k_l \frac{\partial T_l(\xi,t)}{\partial z} = pL\frac{d\xi(t)}{\partial t} = \rho L v(t) \quad (53)$$

where v(t) is the nugget growth velocity.

Equations (46) & (47) with (48)-(53) describe the model of RSW during nugget forming and growth. t_M is the time that welding ends.

3) Nugget Cool Down during Hold

During the period of electrodes holding the welt metal sheets, there is no heat generation in this period. So, the RSW model is composed of the following equations:

For the solid portion (outside of the nugget),

$$\frac{\partial^2 T_s}{\partial z^2} = \frac{1}{\alpha}\frac{\partial T_s}{\partial t} \quad (54)$$

in $0 < z < 0.5\big(L - \xi(t)\big), \& \ 0.5\big(L + \xi(t)\big) < z < L$

and for the liquid portion (inside the nugget),

$$\frac{\partial^2 T_l}{\partial z^2} = \frac{1}{\alpha}\frac{\partial T_l}{\partial t} \quad (55)$$

in $0.5\big(L - \xi(t)\big) < z < 0.5\big(L + \xi(t)\big)$

subject to

I.C.:

$T(z, t_m) = T_i$ at $t = t_M, T_i > T_M, and\ , T_i \approx T_M,$ (56)

B.C.:

$T(0,t) = 0$ for $t_M < t < t_{RSW}$ (57)

$T(L,t) = 0$ for $t_M < t < t_{RSW}$ (58)

$T(z,t)$ is infinite for $t_M < t < t_{RSW}$ (59)

$T_s(\xi,t) = T_l(\xi,t) = T_M$ (60)

Stefan condition:

$$k_s \frac{\partial T_s(\xi,t)}{\partial z} - k_l \frac{\partial T_l(\xi,t)}{\partial z} = pL\frac{d\xi(t)}{\partial t} = \rho L v(t) \quad (61)$$

where v(t) is the nugget consolidation velocity.

Equations (51) & (52) with (53) - (58) describe the model of RSW during hold period, and $t_{RSW} = t_M + t_{hold}$.

E. Analytical Solutions of RSW Models

Solving the model of RSW described in section 3.4.1 gives the temperature distribution at given points of z axis. Actually, since the solid density is smaller than the solid density, the same $\alpha = \frac{k}{\rho c_p}$ is used in sections 3.4.1 and 3.4.2, even in 3.4.3.

Solving the RSW model in section 4.2, nugget thickness can be calculated as

$$\xi(t) = 2\lambda\sqrt{\alpha t} \quad (62)$$

where λ is the solution of

$$\lambda \exp(\lambda^2)\,\mathrm{erf}(\lambda) = \frac{c_p(T_i - T_M)}{HL\sqrt{\pi}} \quad (63)$$

The model in section 3.4.3 is not essential for RSW control, thus it is not of interest.

IV. Results and Discussion

In our study, the basics of the resistance spot welding has been covered including the welding process, operation techniques and specifications. Physical models regarding the aspects of solid-liquid metal morphology, casting rheology and thermal expansion have been developed for a better understanding of the RSW process. Afterwards, the mathematical model has been derived based on the conservation law of energy. The model described the course of heat transfer for the normal closed-system RSW process in the form of Fourier equation, Maxwell-Cattaneo equation.

Through the application of the model, the distribution of temperature in the RSW system can be predicted as a function of space coordinates and time. Thus, this model can be implemented into the real time monitoring of RSW without destructing the weld pieces or disrupting the welding process.

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