Strong stabilization of uncertain generalized Rossler chaotic control systems

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***Abstract*—In this paper, the concept of strong stabilization is introduced and the chaos stabilization of uncertain generalized Rossler chaotic systems is explored. Based on the differential and integral inequalities approach, a suitable control is proposed to realize strong stabilization for the uncertain generalized Rossler chaotic systems with any pre-specified exponential convergence rate. The critical time can also be correctly estimated. Finally, numerical simulations are offered to demonstrate the feasibility and effectiveness of the obtained results.**

Keywords—Generalized Rossler chaotic systems; strong stabilization; uncertain chaotic systems; exponential convergence rate

I. INTRODUCTION

In recent years, various chaotic systems have been widely investigated; see, for example, [1-9] and the references therein. Chaotic systems not only have high sensitivity to the initial value, but also the unpredictability of their output signals, which is often the cause of system instability and the generation of oscillation. Therefore, the design of the stability controller of the chaotic system is of absolute importance in the practical application of various types of engineering.

In this paper, the concept of strong synchronization is introduced and the chaos stabilization of uncertain generalized Rossler chaotic systems is investigated. Using the differential and integral inequalities approach, a suitable control is proposed to realize strong stabilization for the uncertain generalized Rossler chaotic systems with any pre-specified exponential convergence rate. Moreover, the critical time can be precisely calculated. Numerical simulations are also provided to demonstrate the feasibility and effectiveness of the obtained results.

The layout of the rest of this paper is organized as follows. The problem formulation and main result are presented in Section 2. Numerical simulations are given in Section 3 to illustrate the main result. Finally, conclusion is made in Section 4. Throughout this paper, $\Re^n$ denotes the n-dimensional Euclidean space, $\|x\| := \sqrt{x^T \cdot x}$ denotes the Euclidean norm of the column vector *x*, and A^T denotes the transport of the matrix *A*.

II. PROBLEM FORMULATION AND MAIN RESULTS

In this paper, we consider the following uncertain generalized Rossler chaotic systems:

$$\dot{x}_1(t) = \Delta a_1 x_1(t) + b_1[x_2(t) + x_3(t)] + u_1(t), \quad (1a)$$

$$\dot{x}_2(t) = b_2 x_1(t) + \Delta a_2 x_2(t) + u_2(t), \quad (1b)$$

$$\dot{x}_3(t) = \Delta a_3 x_3(t) + b_3 + b_4 x_1(t) x_3(t) + u_3(t), \quad (1c)$$

$$[x_1(0) \quad x_2(0) \quad x_3(0)]^T = [x_{10} \quad x_{20} \quad x_{30}]^T, \quad (1d)$$

where $x(t) := [x_1(t) \quad x_2(t) \quad x_3(t)]^T \in \Re^3$ is the state vector, $u(t) := [u_1(t) \quad u_2(t) \quad u_3(t)]^T \in \Re^3$ is the system control, $[x_{10} \quad x_{20} \quad x_{30}]^T$ is the initial value, and $\Delta a_i, b_i \in \Re$ represent the parameters of the system. The original Rossler chaotic system is a special case of system (1). It is well known that the system (1) without any control (i.e., $u(t) = 0$) displays chaotic behavior for certain values of the parameters [1-2]. The objective of this paper is to search a new control for the system (1) such that the strong stability of the feedback-controlled system can be guaranteed.

Throughout this paper, the following assumption is made:

(A1) There exist constants $\underline{a}_i$ and $\bar{a}_i$ such that $\underline{a}_i \le \Delta a_i \le \bar{a}_i, \quad \forall\, i \in \{1,2,3\}$.

Before presenting the main result, we provide a definition as follows.

Definition 1. The uncertain systems (1) is said to realize the strong stabilization if there exist a suitable control u and three positive numbers *k*, *b*, and t_c, such that the following conditons are satisfied.

(i) $\|x(t)\| \le k \cdot e^{-bt}, \quad \forall t \ge 0$,

(ii) $x(t) = 0, \quad \forall\, t \ge t_c$.

In this case, the positive number b is called the exponential convergence rate and the positive number t_c is called the critical time.

Now, we are in a position to present the main results for the strong stabilization of the uncertain systems (1).

Theorem 1. The uncertain systems (1) realizes the strong stabilization under the following control

$$u_1(t) = -b_1[x_2(t) + x_3(t)] - (\bar{a}_1 + b)x_1(t) - a x_1^{2\alpha-1}(t), \tag{2a}$$

$$u_2(t) = -b_2 x_1(t) - (\bar{a}_2 + b)x_2(t) - a x_2^{2\alpha-1}(t), \tag{2b}$$

$$u_3(t) = -b_3 - b_4 x_1(t)x_3(t) - (\bar{a}_3 + b)x_3(t) - a x_3^{2\alpha-1}(t), \tag{2c}$$

where $a > 0, b > 0,$ $\alpha := \dfrac{p+q-1}{2p-1}$, with $p, q \in N$ and $p > q$. In this case, the pre-specified exponential convergence rate and the guaranteed critical time are given by b and

$$t_c = \frac{\ln\left[\dfrac{\frac{a}{b}}{\left[x_1^2(0) + x_2^2(0) + x_3^2(0)\right]^{1-\alpha} + \frac{a}{b}}\right]}{-2(1-\alpha)b}, \tag{3}$$

respectively.

Proof. From (1)-(2), the feedback-controlled system can be performed

$$\dot{x}_1 = -(\bar{a}_1 - \Delta a_1)x_1 - b x_1 - a(x_1)^{2\alpha-1}, \tag{4a}$$

$$\dot{x}_2 = -(\bar{a}_2 - \Delta a_2)x_2 - b x_2 - a(x_2)^{2\alpha-1}, \tag{4b}$$

$$\dot{x}_3 = -(\bar{a}_3 - \Delta a_3)x_3 - b x_3 - a(x_3)^{2\alpha-1}. \tag{4c}$$

Obviously, one has $\bar{a}_i - \Delta a_i > 0, \forall\, i \in \{1,2,3\}$, in view of (A1). Let

$$W(x(t)) = x^T(t)x(t). \tag{5}$$

The time derivative of $W(x(t))$ along the trajectories of feedback-controlled systems is given by

$$\begin{aligned}
\dot{W} &= 2x_1 \cdot \dot{x}_1 + 2x_2 \cdot \dot{x}_2 + 2x_3 \cdot \dot{x}_3 \\
&= -\left[2\sum_{i=1}^{3} (\bar{a}_i - \Delta a_i)x_i^2\right] - 2b \cdot x_1^2 - 2b \cdot x_2^2 - 2b \cdot x_3^2 \\
&\quad - 2a \cdot x_1^{2\alpha} - 2a \cdot x_2^{2\alpha} - 2a \cdot x_3^{2\alpha} \\
&\le -2b \cdot x_1^2 - 2b \cdot x_2^2 - 2b \cdot x_3^2 - 2a \cdot x_1^{2\alpha} - 2a \cdot x_2^{2\alpha} \\
&\quad - 2a \cdot x_3^{2\alpha} \\
&= -2b\left(x_1^2 + x_2^2 + x_3^2\right) - 2a\left(x_1^{2\alpha} + x_2^{2\alpha} + x_3^{2\alpha}\right) \\
&= -2b \cdot W - 2a\left(x_1^{2\alpha} + x_2^{2\alpha} + x_3^{2\alpha}\right) \\
&\le -2b \cdot W - 2a \cdot W^{\alpha}, \quad \forall\, t \ge 0.
\end{aligned}$$

It follows that

$$(1-\alpha)W^{-\alpha}\dot{W} + 2(1-\alpha)bW^{1-\alpha} \le -2a(1-\alpha), \quad \forall\, t \ge 0. \tag{6}$$

Define

$$Q(t) := W(x(t))^{1-\alpha}, \quad \forall\, t \ge 0. \tag{7}$$

From (6) and (7), it can be readily obtained that

$$\dot{Q} + 2(1-\alpha)bQ \le -2a(1-\alpha), \quad \forall\, t \ge 0.$$

It is easy to deduce that

$$\begin{aligned}
&e^{2(1-\alpha)bt} \cdot \dot{Q}(t) + e^{2(1-\alpha)bt} \cdot 2(1-\alpha)bQ(t) \\
&= \frac{d}{dt}\left[e^{2(1-\alpha)bt} \cdot Q(t)\right] \\
&\le -2a(1-\alpha)e^{2(1-\alpha)bt}, \quad \forall\, t \ge 0.
\end{aligned}$$

It follows that

$$\begin{aligned}
\int_0^t \frac{d}{dt}\left[e^{2(1-\alpha)bt} \cdot Q(t)\right]dt &= e^{2(1-\alpha)bt} \cdot Q(t) - Q(0) \\
&\le \int_0^t -2a(1-\alpha)e^{2(1-\alpha)bt}\, dt \\
&= \frac{-a}{b}\left(e^{2(1-\alpha)bt} - 1\right), \quad \forall\, t \ge 0.
\end{aligned}$$

Consequently, we have

$$Q(t) \le \left[Q(0) + \frac{a}{b}\right] \cdot e^{-2(1-\alpha)bt} - \frac{a}{b}, \quad \forall\, t \ge 0. \tag{8}$$

Hence, from (6), (7), and (8), we have

$$\begin{cases}
W(x(t)) \le \left[\left(\|x(0)\|^{2-2\alpha} + \dfrac{a}{b}\right) \cdot e^{-2(1-\alpha)bt} - \dfrac{a}{b}\right]^{1/(1-\alpha)}, \\
\qquad if \quad 0 \le t \le t_c, \\
W(x(t)) = 0, \quad if \quad t \ge t_c.
\end{cases} \tag{9}$$

Consquently, we conclude that

$$\begin{cases}
\|x(t)\| \le \left[\|x(0)\|^{2-2\alpha} + \dfrac{a}{b}\right]^{1/(2-2\alpha)} \cdot e^{-bt}, & if \quad 0 \le t \le t_c, \\
x(t) = 0, & if \quad t \ge t_c.
\end{cases}$$

in view of (1) and (9). This completes the proof. □

III. Numerical Simulations

Consider the uncertain generalized Rossler chaotic systems of (1) with

$b_1 = -b_2 = -b_4 = -1$, $b_3 = 0.2$, $-1 \le \Delta a_1 \le 1$, (10a)

$0 \le \Delta a_2 \le 1$, $-6 \le \Delta a_3 \le -5$, $x(0) = [4 \quad 2 \quad -2]^T$. (10b)

Clearly, one has $(\underline{a}_1, \bar{a}_1) = (-1,1)$, $(\underline{a}_2, \bar{a}_2) = (0,1)$, and $(\underline{a}_3, \bar{a}_3) = (-6,-5)$, in view of (A1). Our goal, in this example, is to design a feedback control such that the unsystem (1) realize the strong stabilization with the guaranteed exponential convergence rate $b = 1$. From (2), with $a = 50$, $p = 3$, $q = 1$, we deduce $\alpha = 0.6$,

$$u_1(t) = x_2(t) + x_3(t) - 2x_1(t) - 50x_1^{0.2}(t), \quad (11a)$$

$$u_2(t) = -x_1(t) - 2x_2(t) - 50x_2^{0.2}(t), \quad (11b)$$

$$u_3(t) = -0.2 - x_1(t)x_3(t) + 4x_3(t) - 50x_3^{0.2}(t). \quad (11c)$$

Consequently, by Theorem 1, we conclude that the uncertain systems (1) achieve strong stabilization with parameters of (10) and feedback control law of (11). Besides, the exponential convergence rate and the guaranteed critical time are given by $b = 1$ and $t_c = 0.086$, in view of (3).

The typical state trajectories of uncontrolled systems and controlled systems are depicted in Figure 1 and Figure 2, respectively. From the foregoing simulations results, it is seen that the uncertain systems of (1) with (10) achieve the strong stabilization under the control law of (11).

Conclusions

In this paper, the concept of strong stabilization has been introduced and the stabilization of uncertain generalized Rossler chaotic systems has been investigated. Based on the differential and integral inequalities approach, a novel control has been proposed to realize strong stabilization for the uncertain generalized Rossler chaotic systems with any pre-specified exponential convergence rate. The critical time can also been correctly estimated. Finally, numerical simulations have been given to demonstrate the feasibility and effectiveness of the obtained results.

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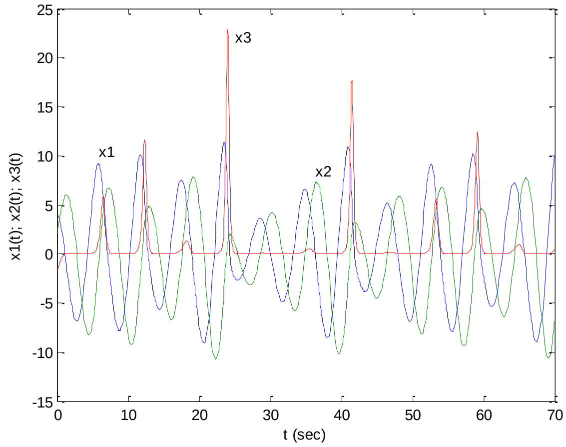

Figure 1: Typical state trajectories of the uncertain systems (1) with (10).

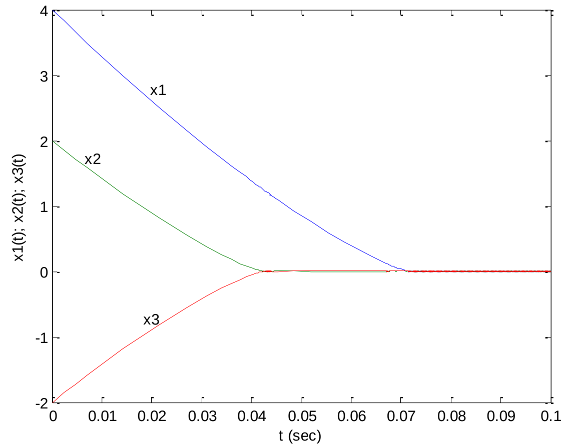

Figure 2: Typical state trajectories of the feedback-controlled system of (1) with (10) and (11).

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