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Application Of Queueing Theory In Optimization Of Service Process, A Case Study Of Gt Plaza Fast Food

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***Abstract*—Queues are common sights in fast food joints and other service based outlets these days. Queuing theory which is the mathematical study of waiting lines was applied for this study. This research seeks to determine the average time customers spend on the queue and the actual time of service delivery, as well as examining the impact of idle time and the cost associated with it. Primary data were collected from a reliable fast food outlet: GT Plaza fast food, Edo State; the data were collected based on the arrival and service pattern of customers. . The Chi-square test was used to ascertain whether the arrival and service pattern follows a Poisson and exponential distribution. The result obtained was in affirmative hence Markovian birth and death process model was used for analysis. The result revealed that service rate is 0.1521 and arrival rate is 0.2157, the probability that servers are idle is 0.2786 and the cost incurred from waiting is N938, 597. Based on the result of data analysis we recommend that the GT Plaza fast food should increase the number of servers to a minimum of three so as to help reduce the time customers spend on the queue and also reduce cost incurred from waiting and the management should educate their staff on the application of the right attitude to enhance operational efficiency and excellence.**

Keywords—queuing theory, utilization factor, arrival and service distribution times, eatery optimization.

1. INTRODUCTION

The study of waiting lines, (queuing theory), is one of the oldest and most widely used quantitative analysis techniques. Waiting lines are an everyday occurrence, affecting people shopping for groceries buying gasoline, making a bank deposit, or waiting on the telephone for the first available airline reservationists to answer. The word queue comes via French and the Latin Cauda meaning "tail". Customers waiting in line to receive services in any service system are inevitable and that is why queue management has been a major challenge.

Queuing theory is also known as the theory of overcrowding; it is the branch of operational research that explores the relationship between the demand on a service system and the delays suffered by the users of that system (Sharma, 2013)

The study of queues deals with quantifying the phenomenon of waiting in lines using representative measures of performance, such as average queue length, average waiting time in queue and average facility utilization (Taha, 2002)

1.1 STATEMENT OF THE PROBLEM

In most service based enterprise like he eatery, poor service pattern, queue discipline, wrong attitude of service personnel, [poor service facilities and delivery are common features. All this factors affect relationship with customers and the overall performance. This study seeks to investigate the expected waiting time of customers and the actual waiting time in the eatery, where the gap between the actual and expected waiting time can be analyzed to know how to improve on the efficiency and effectiveness of their business.

1.2 AIM AND OBJECTIVES OF THE STUDY

The aim of this study is optimized the amount or average time customers spend on a queue and actual time of service delivery by the application of queuing theory method

1.21. Objectives of this study are as follows

•To examine the impact of time wasting on the weak performance.

•To improve on the efficiency and effectiveness of their operations.

•To help managers improve customer's satisfaction through queue management.

1.3 SCOPE OF THE STUDY

The work will focus on the application of queuing theory in fast foods, a case study of GT Plaza fast food. The queue discipline used is first in first out (FIFO) and the arrival is strictly random. We also consider a Poisson distributed arrivals and exponentially distributed service times.

1.4 SIGNIFICANCE OF THE STUDY

Fast food managers and other service based businesses will benefit a lot from this study as it can be applied in their various firm's weather production or service based, thereby reducing the amount of time spent on queues leading to customer's satisfaction and enhanced overall efficiency and effectiveness.

1.5 LITERATURE REVIEW

Queuing theory has become one of the most important, valuable and arguably one of the most universally used tool by operation researchers. It has applications in diverse fields including telecommunications, traffic engineering, computing and design of factories, shops, offices, banks and hospitals.

A queuing model of a system is an abstract representation whose purpose is to isolate those factors that relate to the system's ability to meet service demands whose occurrences and durations are random. (J. Sztrik, 2010)

The study of queue deals with quantifying the phenomenon of waiting in lines using representative measures of performance, such as average queue length, average waiting time in queue and average facility utilization (H.A. Taha, 2002)

Some of the analysis that can be derived using queuing theory include the expected waiting time in the queue, the average waiting time in the system, the expected queue length, the expected number of customers served at one time, the probability of balking customers, as well as the probability of the system to be in certain states, such as empty or full (Patel, Rajeshkumar, Pragnesh and Makwana 2012).

Queuing models are used to represent the various types of queuing systems that arise in practice, the models helps in finding an appropriate balance between the cost of service and the amount of waiting (A. Nafees, 2007)

Queuing models provide the analyst with a powerful tool for designing and evaluating the performance of queuing systems (Bank, Carson, Nelson & Nicol, 2001).

Any system in which arrivals place demands upon a finite capacity resource maybe termed as queuing systems, if the arrival times of these demands are unpredictable, or if the size of these demands is unpredictable, then conflicts for the use of the resource will arise and queues of waiting customers will form and the length of these queue depend on two aspects of the flow pattern: First, they depend on the average rate. Secondly, they depend on the statistical fluctuations of this rate (L. Klenrock, 1975)

Ullah (2011) presented a comparison between Petri net (PN) and queuing network tools to determine the optimum values for flexible manufacturing system (FMS) measures of performance. A queuing theory was presented by Tsarouhas (2011) to calculate the total processing time for the processing time per pizza line at workstation in food production lines. McGuire (2010) proposed and tested a model which defines the psychological processes that mediate the relationship between perceived wait duration (PWD) and satisfaction. Caputo and Pelagagge (2011) reported that of the scarce literature existing on modeling material delivery to assembly lines, kitting has received the greater attention. However, most available models utilize queuing theory to analyze dynamic performances of kitting systems and kit-replenished assembly systems. Mahmoud and Lu (2011) reported that Markov chains and queuing theory are widely used analysis, optimization and decision-making tools in many areas of science and engineering. Real life systems could be modelled and analyzed for their steady-state and time-dependent behavior.

Diaz (2010) presented financial engineering derivative interest rate swap as well as new scheduling applications including inventory management and queuing models. Gudmundsson and Goldberg (2007) developed a model to study a commercially available industrial part feeder that uses an industrial robot arm and computer vision system. The problem of optimizing belt speeds and hence throughput of this feeder are addressed that avoid starvation, where no parts are visible to the camera and saturation, where too many parts prevent part pose detection or grasping. There have been a number of books focusing on the application of queuing theory on manufacturing systems, such as those by Papadopoulos *et al.* (2013), Smith and Tan (2013), Guy *et al.* (1997), Gershwin (1994), Yao *et al.* (1994), Buzacott and Shanthikumar (1993) and Narahari (1992).

Koo *et al.* (1995) proposed a manufacturing system modeling approach using computer spreadsheet software, in which a static capacity planning model and stochastic queuing model are integrated. Most stochastic performance measures such as throughput time or work in process as well as deterministic measures can be captured directly from the proposed model. Several special manufacturing features such as machine breakdown and batch production can be included in the model. The performance of the proposed model was evaluated by comparing its results with those obtained from other existing approaches. Their finding for this comparison stated that the maximum allowed a relative error was 10%. Sukhotua and Peters (2005) discussed a number of approaches in the facility design for modelling material flow u sing queuing networks. In these approaches, Poisson arrival or Markovian job routing assumptions were used. However, for many manufacturing environments, these assumptions lead to an inaccurate estimation of the material handling system's performance and thus lead to poor facility designs. The proposed modelling approach has showed to provide more accurate results than previous methods used in facility design based on numerical comparisons with results from discrete-event simulation.

Marcheta *et al.* (2012) presented an analytical model to estimate the performances (the transaction cycle time and waiting times) for product tote movement. The model is based on an open queuing network approach. The model effectiveness in performance estimation was validated through simulation.

The use of simulation in improving cycle time had been discussed by many researchers (Sivakumar and Chong 2001, Domaschke and Brown 1998, Wang *et al.* 1993, Toh *et al.* 1995). Based on their study on many manufacturing systems, they concluded that simulation can improve cycle time between 15% to 45%. The use of simulation is a powerful technique that helps decision maker to solve difficult problems in the design, control, or improvement of complex systems to reduce cost, improve quality or productivity, and shorten time-to-market.

However, the technology is still underutilized due to several reasons: (1) simulation modeling is a time-consuming and knowledge-intense process that requires knowledge not only about simulation but also application and implementation tools; (2) most simulation models developed with current technology are customized “rigid” ones that cannot be reused or easily adapted to other similar problems; and (3) transforming related knowledge and information from application domain to simulation is a unstructured or ill-defined process dependent on the skill and experience of individual modelers (Zhou *et al*. 2010)

Muhammad Marsudi and Hani Shafeek (2014) carried out queuing analysis to examine multi-stage production line performance to facilitate more realistic resource planning. It is one of a few such studies to improve the performance of multiproduct multi-stage production lines. This work aims to help managers in improving the efficiency, effectiveness and selecting the most suitable policy for assembly systems. The paper adopts an analytical approach based on real life data from an international battery company producing battery covers for camera model EC-196. The battery production line consists of six independent workstations namely injection molding, first color spray, second color spray, ultra-violet (UV) station, assembly station and a packing workstation. The relevant data for each workstation was collected and the chi-squared goodness test was applied to determine the arriving and leaving distributions data of processing parts. Queuing analysis reported in this work provides a basis for estimating and analyzing production systems by measures such as utilization, percentage of idle workstation, number of batches in system, number of batches in queue, expected time spent in queue, and expected time spent in system. The comparison between results and standard data in the company showed an accuracy of 93.80%.

Dong (2014) considered a two-stage production lot sizing problem which used an inventory system with random demand arrivals. To solve the problem, they proposed a numerical approach for the problem. Baek and Moon (2014) considered a lost sales production-inventory system in an uncertain environment. They also used queuing theory to present a stochastic model for the system. Also, some production-inventory papers in the literature assumed that the lead time is negligible or it can be ignored in practice when it is short in contrast to other time factors (Karimi-Nasab and Seyedhoseini 2013). Most of such research papers did not consider lost sales. Many of them, however, considered back orders as a rational managerial policy.

Chang and Lu (2011) considered a serial production system controlled by the base-stock policy. They presented a phase-type approximation for a controlled base-stock serial production system. They also proposed a cost model to determine the optimal base-stock level.

Jewkes and Alfa (2009) considered a production system in which a supplier produces semi-finished items on a make-to-stock basis for a manufacturer that customizes the

Items on a make-to-order basis. The manufacturer attempts to determine the optimal point of differentiation and its optimal semi-finished goods buffer size. They used matrix geometric methods to evaluate performance through various measures for this system.

Some researchers worked on multi-item inventory systems.

Shavandi et al. (2012) proposed a new constrained multi-item pricing and inventory model. They covered three categories of perishable products in their model. Taleizadeh et al. (2012) dealt with a multi-product inventory control problem in which periods between two replenishments were assumed to be independent random variables.

Sapna Isotupa (2006) considered lost sales of (s, Q) inventory system with two customer groups and illustrated the Markov processes. Boute et al. (2007), in a two-echelon supply chain, show that by including the impact of the order decision on lead times, the order pattern can be smoothed to a considerable extent without increasing stock levels. Karimi-Nasab and

Konstantaras (2013) considered special sales offer for a single item from the supplier under stochastic replenishment intervals. Jain and Raghavan (2009) studied batch ordering in multi-echelon supply chains and used queuing theory to capture the behavior of the manufacturing supply chain network. Babai et al. (2011) investigated stochastic demand and lead time and analyzed a single item inventory system through queuing theory. Bahri and Tarokh (2012) assumed that the delivery lead time is stochastic and follows an exponential distribution. Also, the shortage during the lead time is permitted and completely back ordered for the buyer.

Seyedhoseini et al. (2014) considered Poisson demand for customer in a cross docking problem. They employed queuing theory to provide a stochastic model. Salameh et al (2014) combined the separate works on substitution and joint replenishment and introduced a solution procedure for solving the joint replenishment model with substitution for two products within the framework of the classical economic order quantity model.

Karimi-Nasab and Sabri- Laghaie (2014) extended classical economic production quantity model to the case of stochastically generated poor quality items, while an imperfect screening scheme was devised to recognize such items from healthy ones.

Krommyda et al. (2015) studied an inventory control problem in which demand was satisfied using two mutually substitutable products. Their aim was to determine the order quantity for each product that maximizes the joint profit function.

Seyedhoseini et al. (2015) applied queuing theory to propose a mathematical model for inventory systems with substitute flexibility. (Rashid et al. 2015) also considered a location-inventory model. To prepare a stochastic inventory model, they used bi-level Markov process. Considering the effectiveness of queuing theory in inventory problems, we have also used queuing theory to develop a stochastic stock control model.

2. METHODOLOGY

The chapter entails empirical research including procedures and methods adopted in this research

2.1 RESEARCH DESIGN

The main aim of this research is to show how the management of GT plaza fast food will go about the reduction of the waiting time of customers. This piece of work will also check if increasing the number of server will reduce the waiting time as well as putting the profit in consideration. Hence, the objectives of this project will be achieved by analyzing the real life observed data, then constructing a new model of system and using statistical analytical tools like Poisson, exponential and chi-square distribution to study pattern and reaction to change in the system.

2.2 AREA OF STUDY

This research focuses on the waiting area of the hall of one GT plaza fast food outlets in Edo State Nigeria

2.3 FORMULATION OF MODEL FOR THE SYSTEM

To formulate a model for this system, the following assumptions were considered:

- The arrival of customers into the system is discrete form Poisson distribution with arrival rate λ
- The queuing discipline is first come first serve.
- We considered two-service channel i.e. (M│M│2)
- The service channel can only render service of finite rate exponentially distribution with service rate, μ.
- The calling population (i.e. the number of customers calling for service is finite.
- The waiting area for the customers in the system is N, which is either limited or unlimited. Hence, the model can be formulated appropriately by using a system for the investigation system. Kendall's notation is introduced; (V│W│X│Y│Z)

V- Which is the arrival distribution or pattern is Poisson as indicated earlier.

W- The service time distribution is distribution is exponential as indicated earlier.

X- The number of available server in the system is two from the assumption above.

Z- This represents the queue discipline which is first come first served (FIFO).

Hence, with the above assumptions and approach the formulated model is (M│M│2│N│FCFS) by Kendall's notation.

2.4.1 ALGORITIM FOR SOLVING THE PROBLEM

There is no unique model for single channel; the particular approach one adopted depends on the following factors.

- The purpose of the solution exercise.
- The information available about the system.
- The tools for the work on the system.

For this investigation, analytical method of solution was adopted

By

1. Considering the system characteristics.

2. Collection of data based on.

- Arrival distribution.
- Service time distribution.

3. Estimating the parameter λ and μ from the data.

4. Testing of the data distributions for statistical conformity to the assumed theoretical probability distribution using the chi-square tests for goodness for fit.

5. Analytical solution to the system model using the values of the parameters λ, μ and N (system capacity) calculated from the data.

3. METHOD OF DATA ANALYSIS

First we use the chi-square distribution to study the pattern and reactions to change of the system i.e. identification of customers, server and queue characteristics that are apparent in the system. After which we analyze the data collected from the fast foods based on

- The time arrival of each customer.

- The time service commerce for each customer and leaves the system using the M│M│S model since the system has to do with multiple servers and a single queue.

3.1 CHI SQUARE DISTRIBUTION

This is a unique statistical test designed to investigate the agreement of a set of observed frequency and expected frequency on the assumption of a theoretical model for the phenomenon being studied. The test is used for investigating dependency or independency of two attributes of classification.

A measure for the discrepancy existing between observed and expected frequency is supplied by the chi-squared (x2) statistic given by

$$X_c^2 = \sum \frac{(Fi-Ei)2}{Ei}$$

Where F_i= observed frequency of the observation

e_i= expected frequency of observation

3.2 M│M│S MODEL

In this model, it shows that

If 1μ is the mean service time for one server to handle one customer, then the mean rate of service completion when there are customers in the system is

$$\lambda_n = \lambda,\ \mu_n = \begin{cases} n\mu & \text{if } n = 0,1,\ldots,c \text{ i.e } n \le c \\ c\mu & \text{if } n = c+1, c+2 \text{ i.e } n \ge c \end{cases} \quad \ldots\ldots\ldots\ldots (3.5.1)$$

The probability of zero customers in the system (Po) and the probability of n customer in the system (P_O) are given by:

$$P_o = \left\{ \frac{\left(\frac{\lambda}{\mu}\right)^c}{C!\left|1-\frac{\lambda/\mu}{c}\right|} + \frac{(\lambda/\mu)^1}{1!} + \frac{(\lambda/\mu)^2}{2!} + \cdots + \frac{(\lambda/\mu)^{c-1}}{(c-1)!} \right\}^{-1} \ldots\ldots(3.5.2)$$

The capacity utilization in this system is $\frac{\lambda}{c\mu}$

We can use the above equation of $\frac{\lambda}{c\mu} < 1$

If $\frac{\lambda}{c\mu}$ >1, then the waiting line grows larger and larger i.e. becomes infinite if the process runs long enough.

When C = 1 (there is one service facility), equations (3.5.2) reduces to

$$P_n = \left(\frac{\lambda}{\mu}\right)^n P_0 \quad \ldots\ldots\ldots\ldots\ldots\ldots(3.5.3)$$

From equations (3.5.2), we have

$$P_n = P_0 \frac{\left(\frac{\lambda}{\mu}\right)^n}{n!} \text{ if } n \le C$$

But n can only take on values of 0 or 1 if $n \le C = 1$. Thus

$$P_n = P_0 \left(\frac{\lambda}{\mu}\right)^n \ldots\ldots\ldots\ldots(3.5.4)$$

If C = 1, equation 3.5.2 also reduces to equation 3.5.4

With C service facilities, the average number of customers in the queue is

$$N_q = \frac{(\lambda/\mu)^{c+1} P_0}{C.C!\left[1-\frac{\lambda/\mu}{C}\right]^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots(3.5.5)$$

The average number in the system (waiting plus service) is

$$N_s = N_q + \frac{\lambda}{\mu} \quad \ldots\ldots\ldots\ldots\ldots\ldots(3.5.6)$$

The expected waiting time in the queue for an arrival is

$$T_q = \frac{N_q}{\lambda} \quad \ldots\ldots\ldots\ldots\ldots\ldots (3.5.7)$$

The expected total time spent in the system (waiting plus service) is

$$T = \frac{N_s}{\lambda} \quad \ldots\ldots\ldots\ldots\ldots\ldots (3.5.8)$$

3.3 METHOD OF DATA COLLECTION

The data was collected primarily by direct observation at the fast food joint. Thus, the researcher recorded the following events as it happened in the system using a wrist watch

1. The time of arrival of each customer.

2. The time service commences for each customer in the system.

3. The time the customer leaves the system.

These events were observed at the withdrawal station of the fast food joint. A form was designed for this exercise and the above required information was recorded in the form.

One week of three working days was spent to collect relevant data.

The data under study was collected from GT plaza fast food and χ 2 (chi square) test of goodness of fit to test the hypothesis that

H_0 = arrival distribution is Poisson

H_1 = arrival distribution is not Poisson

Let n = number of arrivals in 10 mins

F_n = frequency of number of arrivals

P_n = probability of number of arrivals.

TABLE 3.1

N	F	Nf_n	P_n	e_n
0	6	0	0.1178	8.4816
1	14	14	0.2519	18.1368
2	26	52	0.2694	19.3968
3	18	54	0.1921	13.8312
4	6	24	0.1027	7.3944
5	2	10	0.0439	3.1608
TOTAL	**72**	**154**		

Source: arrival pattern in GT Plaza fast food

$$P_n = \frac{e^{-\lambda}\lambda^n}{n!} \text{ where } \lambda = \frac{\sum_n f_n}{\sum f_n} = \frac{154}{72} = 2.1389$$

$$P_n = \frac{e^{-2.1389} 2.1389^n}{n!}$$

$e_n = \sum f_n (P_n) = 72P_n$

Since χ2 test is used when approximation is good. Experience and theoretical investigations indicates that the approximation is usually satisfactory, provided that ei ≥ 5. If the expected frequency of a cell does not exceed five, this cell should be combined with one or more other cells until the above condition is satisfied.

Hence table 3.1 now yields

Table 3.2

N	0	1	2	3	4 or 5
f_n	6	14	26	18	8
e_n	8.4816	18.1368	19.3968	13.8312	10.5552

$\chi^2 \text{ cal} = \sum \frac{(Fn-En)2}{En} = 5.7927$

To find the χ^2 we check the distribution table. Degree of freedom = number of observed value – number of estimated parameter -1 = n-1 -1. Number of observed value from table 4.2 is 5 and the number of estimated parameter is 1. Hence the degree of freedom = 5-1 -1 =3 Taking =5%, from the χ^2 distribution table, 5 critical value of χ^2 for 3 DF is

$\chi^2 \text{ tab} = \chi^2_{3,\,0.95} = 7.8147$

Hence the null hypothesis is accepted and thus there is no reason on the basis of this test for doubting that queuing model can be applied to this data. This also implies that the arrival pattern follows a Poisson distribution.

Appendix 1 is the table showing the time of arrival, the time service begins, the time service ends and the service time. We use χ2 (chi-square) test of goodness of fit to test the hypothesis that H0 = service time distribution is exponential H1= service time distribution is not exponential Let n= service time in minutes F_n= Frequency of the service times P_n = probability of service time

TABLE 3.3

T	F_n	P_n	e_n
0≤ T<6	95	0.5985	92.169
6≤T<12	47	0.2403	37.0062
12≤T<18	7	0.0965	14.861
18≤T<24	5	0.0387	5.9598

$e_n = \sum f_n (P_n) = 154P_n$

$P_n = \mu.e^{\mu\text{-}T}$

Where $\mu = \frac{\text{system capacity}}{\text{time taken to be served}}$

$= \frac{154}{1012} = 0.1521$

$P(a \le T<b) = \int_a^b \mu.e^{\mu\text{-}T}\, dT$

$= -e^{-\mu b} + e^{-\mu a}$

$= e^{-\mu a} - e^{-\mu b}$

$P(0\le T<6) = e^{-0.1521(0)} - e^{-0.1521(6)} = 0.5985$ $P(6\le T<12) = e^{-0.1521(6)} - e^{-0.1521(12)} = 0.2384$ $P(12\le T<18) = e^{-0.1521(12)} - e^{-0.1521(18)} = 0.0965$ $P(18\le T<24) = e^{-0.1521(18)} - e^{-0.1521(24)} = 0.0387$

As stated earlier χ^2 is used only when the approximation is good. Approximation is usually satisfactory provided that the ei≥5. Hence, table 4.3 above now yields

TABLE 3.4

T	0≤T<6	6≤T<12	12≤T<18	18≤T<30
f_n	95	47	7	5
e_n	92.169	37.0062	14.861	5.9598

$\chi^2\text{cal} = \sum \frac{(Fi-Ei)2}{Ei} = 5.8983$

To find the χ^2tab, we check the chi-square distribution table. Degree of freedom = 4-1-1=2 Taken α = 5%, from the χ^2 distribution table, 5% critical value of χ^2 for 2 DF is $\chi^2\text{tab} = \chi^2_{2,\,0.95} = 5.99$ Therefore the null hypothesis is accepted and this implies that the service pattern follows an exponential distribution.

2.8 PARAMETERS ESTIMATION

From Appendix 1 (i.e. the data collected at the eatery) The system capacity, N =154 customers Inter-arrival time for 154 customers, T =714 minutes Time taken by 154 customers to be served, S=1012 minutes Now the

Arrival rate, $\lambda = \frac{N}{T} = \frac{154}{714} = 0.2157$

Service rate, $\mu = \frac{N}{S} = \frac{154}{1012} = 0.1521$

Traffic intensity $(\ell) = \frac{\lambda}{\mu} = \frac{0.2157}{0.1521} = 1.4181$

This implies that μ = 0.1521

λ = 0.2157 C = 2

$\frac{\lambda}{\mu}$ = 1.4181

We now calculate the probability values a) Probability that the servers are idle is equ (3.5.2)

$$P_0 = \frac{1}{\frac{\left(\frac{\lambda}{\mu}\right)^c}{c!\left[1-\frac{\lambda/\mu}{c}\right]}+1+\frac{(\lambda/\mu)^1}{1!}+\cdots+\frac{(\lambda/\mu)^{c-1}}{(c-1)}}$$

$$P_o = \frac{1}{\frac{1.4181}{2!\left[1-\frac{1.4181}{2}\right]}+1+\frac{1.4181}{1}}$$

$$= \frac{1}{1.1702+2.4181} = \frac{1}{3.5883} = 0.2786$$

b) Probability of n customers in the system is (from equ. (3.5.3)

$$P_n = \frac{\left(\frac{\lambda}{\mu}\right)^n}{c!\,c^{n-c}} = \frac{(1.4181)^n 0.2786}{2!\times 2^{n-2}} = \frac{(1.4181)^n 0.2786}{2^{n-1}}$$

Now we calculate the Queue measure

a) The expected number in the waiting line, from equ (3.5.6) we have that

$$N_q = \frac{\left(\frac{\lambda}{\mu}\right)^{c+1}}{CC!\left[1-\frac{\lambda/\mu}{C}\right]^2} P_0$$

$$= \frac{(1.4181)^3(0.2786)}{2\times 2!\left[1-\frac{1.4181}{2}\right]^2} = \frac{(2.8518)\times 0.2786}{0.3386} = 2.3465$$

b) The expected number in the system (waiting plus in service) is by equ.(3.5.7) $N_s = N_q + \frac{\lambda}{\mu}$ = 2.3465 + 1.4181 = 3.7646

c) The expected waiting time in the queue is by equation (3.5.8)

$T_q = \frac{Nq}{\lambda} = \frac{2.3465}{0.2157} = 10.8785$

If we assume an 8 hr workday, there will be an average of 51 customers arriving per day, the expected total lost time of customers waiting will be T_L = λ × 8 × T_q

= 0.2157 × 8 × 10.8785 = 18.7719

Assuming the cost associated with this time lost is ₦50, the average cost per day from waiting is 18.7719 × ₦50 = ₦938.597 Now, we want to find out if increasing the number of servers can help to reduce the amount of time spent on queue and the hence minimize the cost incurred by waiting. Hence we compare solutions:

Let λ = 0.2157

μ =0.1521

$\frac{\lambda}{\mu}$ = 1.4181 which is same in the data above but let C = 3 Now we have

$$P_o = \frac{1}{\frac{(1.4181)^3}{3!\times\left[1-\frac{1.4181}{3}\right]}+1+\frac{(1.4181)^1}{1!}+\frac{(1.4181)^2}{2!}}$$

$$= \frac{1}{0.9014+1+1.4181+1.0055} = \frac{1}{4.325} = 0.2312$$

The expected number in the waiting line is

$$N_q = \frac{(1.4181)^4}{3\times 3!\left(1-{}^{1.4181}/_{3}\right)^2}(0.2312)$$

$= \frac{0.9350}{5.0048} = 0.1868$

Expected number in the system is

$N_s = N_q + \frac{\lambda}{\mu}$

= 0.1868 + 1.4181 = 1.6049 Then expected waiting time in the queue is

$T_q = \frac{Nq}{\lambda} = \frac{0.1868}{0.2157} = 0.8660$

The expected total time lost waiting on one day

T_L = λ× 8 hours × T_q

= 0.2157 × 8 × 0.8660 = 1.4944 hours

The average lost per day from waiting is = 1.4944 × ₦50 = ₦74.72

2.9 DISCUSSION

Considering the analytical solution, the capacity of the system under study is 154 customers and the arrival rate is 0.2157 while the service rate is 0.1521. This shows that the arrival rate of the system is greater than the service rate, this imply that customers have to queue up, though the queue will not be long. Probability that the servers are idle is 0.29 which shows that the servers will be 29% idle and 71% busy.

The expected number in the waiting line is 2.3465. The expected number in the system is 3.7646. The expected waiting time in the queue is 10.8785 and the expected total time lost waiting in one day is 18.7719 hours.

From the foregoing Queue measures, the average cost per day for waiting is N938.597 and from the calculation of the comparing solutions, the average cost per day from waiting is N74.72. There had been a saving in the expected cost of N938.597 - N74.72 = N863.877

3. CONCLUSION

This research offers a practical model for eatery service optimization. It is based on customers' requirements for shorter waiting time and needs for making best use of resources. Developing an optimal waiting scheme can effectively improve customer service efficiency, which can enhance customers' satisfaction and loyalty. Considering the time we are in now (jet age) coupled with the fast development of economy, people cherish time more and more. Competition for fast food service is fiercer. The need to solve the conflict between customers and the servers deserves more considerations. Eatery or restaurant managers should always take the management as priority for development and profit. They can explore a set of management tools that are right for certain fast food operations by setting up relevant work rules for different positions.

Secondly, set up strict performance assessment system for employees. Constitute leveled goal responsibility system for every department. Responsibilities are for everyone.

From the model formulated, we can conclude that adding one more server will help reduce the time customers spend on queue and as well help to reduce the cost incurred from waiting. Hence the objective of this research is achieved.

The advantage of using this single system with multiple servers is that a slow server does not affect the movement of the queue i.e if a server is slow it does not affect the movement of the queue because next customer can go to the next available server instead of waiting for the slow server.

3.1 RECOMMENDATION

We recommend that, the management of the fast foods firm should increase the number of the servers to at least three so as to reduce the time customers has to wait, to receive services, thereby minimizing the cost incurred from waiting.

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