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Effect Of Duo Fermion Spin On The Specific Heat And Entropy Of A Mixture Of Helium Isotopes

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***Abstract*—Spin normally determines the character, rate of collision and subsequently the properties of ultracold systems. This study investigated the effect of double spin degeneracy on the thermodynamic properties of Helium-3 and Helium-4 isotopes specifically the specific heat and entropy of a grand canonical ensemble. The approach used was statistical in nature where permutation and exclusion was done on fermions and bosons. From thermodynamics, the expression of partition function which was used to derive other expressions of specific heat and entropy as temperature dependence. Specific heat and entropy were found to increase with temperature. The kink in specific heat at a temperature of 35K implied that there was a phase transition.**

Keywords—Thermodynamics, degeneracy, Entropy, Specific heat

INTRODUCTION

Experiments on ultracold trapped atomic gases have opened a window onto the phases of quantum matter [1]. Bose gas remains conceptually a good starting point to understand the remarkable behaviour of superfluid Helium. It was in this context that Bogoliubov analyzed the effect of weak interactions and Pitaevskii constructed the classical theory for inhomogenous Bose-Einstein condensates [2].

A gas of bosonic atoms in an optical or magnetic trap had been reversibly tuned between superfluid and insulating ground states by varying the strength of a periodic potential produced by standing optical waves. This transition was explained on the basis of the Bose-Hubbard model with on-site repulsive interactions and hopping between nearest neighboring sites of the lattice [3]. Further, theoretical studies of bosonic atoms with spin and/or pseudospin had also been undertaken. These studies have revealed a variety of interesting Mott phases and superfluid-insulating transitions in these systems. On the fermionic side, the experimental studies have mainly concentrated on the observation of paired superfluid states and the BCS-BEC crossover in such systems near a Feshbach resonance [4]. It has also been possible to generate mixtures of fermionic and bosonic atoms in a trap. Initially, the main focus of such experimental studies were to generate quantum degenerate Fermi gases, through sympathetic cooling with bosons. However, a host of theoretical studies followed, which established such Bose-Fermi mixtures to be interesting physical systems in their own right, exhibiting exciting Mott phases in the presence of an optical lattice [5]. In all of these works, the spin of the fermions in the mixture is taken to be frozen out due to the presence of the magnetic trap. Research works thereafter considered a Bose-Fermi mixture in an optical trap, where the spins of the fermions were dynamical degrees of freedom [6]. It was shown that the interaction between the bosons and the fermions in such a mixture enhanced the s-wave pairing instability of the fermions.

Thermodynamic properties of Helium-3 and Helium-4 isotopes has also been done where the partition function relations as a means of studying the thermodynamic properties of Helium-3 and Helium-4 isotopes was developed [7].

Similarly, thermodynamic properties of normal liquid helium-3 by using the lowest order constrained variational (LOCV) method was done where the free energy, pressure, entropy, chemical potential and liquid phase diagram as well as the helium-3 specific heat were evaluated and discussed [8].

In this work, we considered a Bose-Fermi mixture of Helium isotopes (Helium-4 and Helium-3) interacting weekly in pairs. It was assumed that the atoms were confined using an optical trap so that Fermions spins were not frozen out. However, the effect of the harmonic trap potential was ignored.

Theoretical framework

According to [9] , the partition function of Helium isotopes with duo spin degeneracy was derived as

$$Z(T)=\sum^{\infty}\left\{2\left[4+2\exp\left(\frac{\mu_b-\mu_f}{K_BT}\right)+\exp\left[2\left(\frac{-\mu_f+\varepsilon_j}{K_BT}\right)\right]\right]\right\}\frac{\exp\left[\frac{\left(\mu_bN_b+\mu_fN_f-\xi\right)}{K_BT}\right]}{\left[2+\exp\left[\frac{2\varepsilon_j-\mu_b-\mu_f}{K_BT}\right]\right]} \tag{1}$$

Where Z is the partition function, N_b denotes number of bosons, N_f number of fermions μ_b is the chemical potential of bosons, μ_f is the chemical potential of fermions, T is the temperature, K is the Boltzmann constant, ζ is the dual energy of helium isotopes

The following are the conditions for determining the specific heat (C_v) , entropy (S) and lambda transition temperature (T_λ) for a grand canonical ensemble

$$C_V=\left[\frac{\partial}{\partial T}\left(K_BT^2\frac{\partial}{\partial T}\ln Z\right)\right]_V \tag{2}$$

$$S=K_BT\left(\frac{\partial \ln Z}{\partial T}\right)+K_B\ln Z \tag{3}$$

$$\left(\frac{\partial C_V}{\partial T}\right)_{T=T_\lambda}=0 \tag{4}$$

Based on equations (2), (3) and (4), the specific heat (C_v), entropy (S) and the lambda transition temperature (T_λ) was found to be

$$C_v(T)=2K_BT\frac{\partial}{\partial T}\ln\left\{2\left[4+2\exp\left(\frac{\mu_b-\mu_f}{K_BT}\right)+\exp\left[2\left(\frac{-\mu_f+\varepsilon_j}{K_BT}\right)\right]\right]\frac{\exp\left(\frac{\mu_bN_b+\mu_fN_f-\xi}{K_BT}\right)}{\left[2+\exp\left(\frac{2\varepsilon_j-\mu_b-\mu_f}{K_BT}\right)\right]}\right\}+$$
$$K_BT^2\frac{\partial^2}{\partial T^2}\ln\left\{2\left[4+2\exp\left(\frac{\mu_b-\mu_f}{K_BT}\right)+\exp\left[2\left(\frac{-\mu_f+\varepsilon_j}{K_BT}\right)\right]\right]\frac{\exp\left(\frac{\mu_bN_b+\mu_fN_f-\xi}{K_BT}\right)}{\left[2+\exp\left(\frac{2\varepsilon_j-\mu_b-\mu_f}{K_BT}\right)\right]}\right\} \tag{5}$$

$$S(T)=K_BT\frac{\partial}{\partial T}\ln\left\{2\left[4+2\exp\left(\frac{\mu_b-\mu_f}{K_BT}\right)+\exp\left[2\left(\frac{-\mu_f+\varepsilon_j}{K_BT}\right)\right]\right]\frac{\exp\left(\frac{\mu_bN_b+\mu_fN_f-\xi}{K_BT}\right)}{\left[2+\exp\left(\frac{2\varepsilon_j-\mu_b-\mu_f}{K_BT}\right)\right]}\right\}+$$

$$K_B\ln\left\{2\left[4+2\exp\left(\frac{\mu_b-\mu_f}{K_BT}\right)+\exp\left[2\left(\frac{-\mu_f+\varepsilon_j}{K_BT}\right)\right]\right]\frac{\exp\left(\frac{\mu_bN_b+\mu_fN_f-\xi}{K_BT}\right)}{\left[2+\exp\left(\frac{2\varepsilon_j-\mu_b-\mu_f}{K_BT}\right)\right]}\right\} \tag{6}$$

$$T_\lambda=\frac{4\gamma}{15\alpha} \tag{7}$$

RESULTS AND DISCUSSION

Specific Heat

Values of specific heat are computed using Equation (5) leading to figure 3 which shows the variation of specific heat with temperature in the range 5K to 80K.

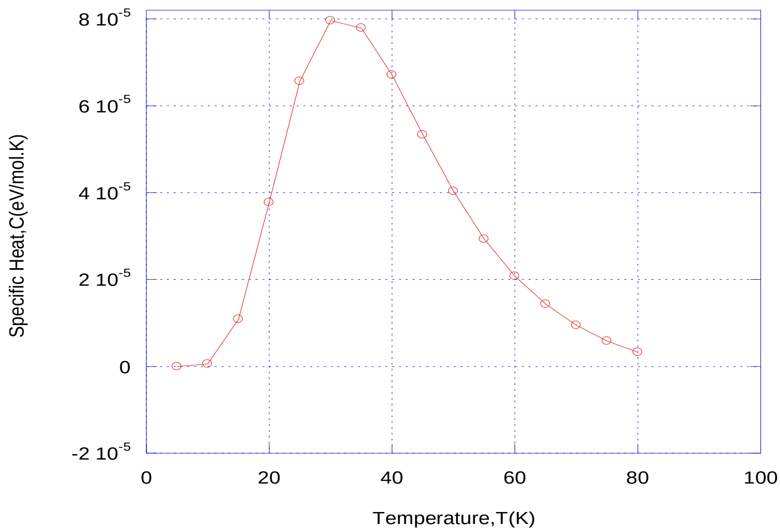

Figure 1: *Variation of Specific Heat with Temperature in the range 5K to 80K*

The specific heat is zero at very low temperatures below 5K. In this temperature range, particles are all occupying the lower quantum states. As the temperature is increased, fermions shift quickly to the higher states. The specific heat rises exponentially to a peak value of 8.0 x 10^{-3} *eV/Mol.K* at a temperature of 35K and reduces gradually to very low values. At this temperature, a phase-like transition appears to take place in a manner that is characteristic of a second-order phase transition. This type of Gaussian shaped curve relating specific heat to temperature has been observed by other scientists while investigating relationship between specific heat and temperature for varied materials under varied conditions [10]. This findings concurs with [11]. In this study, a bulk mixture of helium-3 and helium-4 is considered in which there is free movement of helium-3 atoms into liquid helium-4. Helium-3 atoms are seen as impurities in liquid helium-4 and weak interactions develop between the helium-3 and helium-4 atoms which, consequently, shift the transition temperature of liquid helium-4 to a higher value.

Entropy

Using equation (6), the variation of Entropy with temperature is found to be a curve with a gently decreasing slope nearly saturating at 80K.

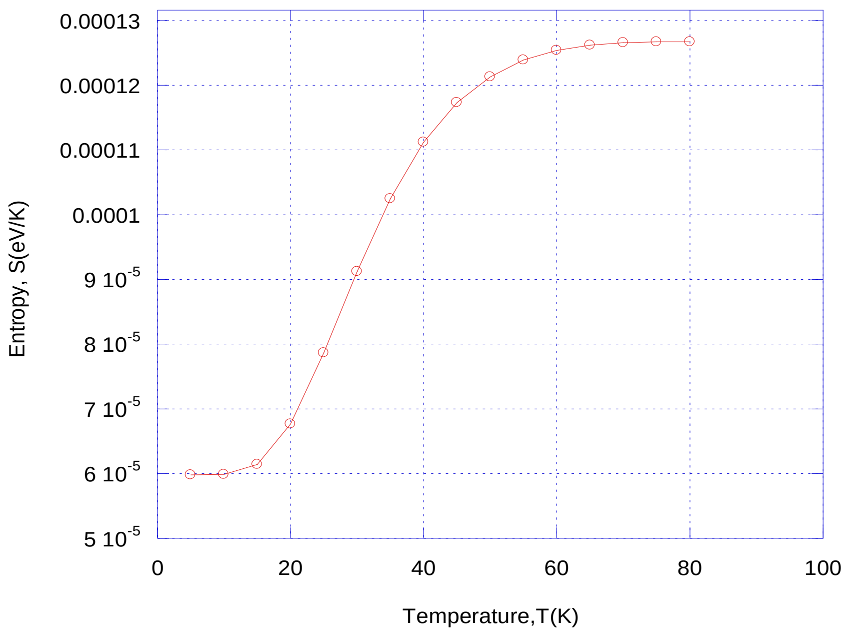

Figure 2: *Variation of Entropy with Temperature in the range 5K to 80K*

Entropy is a measure of disorder in a system. On the graph shown above, entropy is seen increasing instantaneously with increase in temperature. This shows excitation of the particles in the system. The particles in the system do not spin at low temperatures but they gain momentum as the temperature increases. This results tally with that of [12, 13].

CONCLUSIONS

Different research works on the study of thermodynamic properties of Helium isotopes have looked at two models: the independent particle model in isolated system and the ensemble model consisting of large collection of system. In these model, particles are considered to be interacting weakly in pairs.

At low temperatures (below 5 K), the specific heat and entropy are diminishingly small, this is majorly because of the microscopic occupation of the ground state. This concurs with what [14] observed for disordered bosons. The entropy which is determined statistically, naturally, increases with temperature.

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