

Fuzzy Linear Regression Approach for Power System State Estimation

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Abstract— the state estimation (SE) is the most important implemented function in the control center of electric power system. Without reliable and effective SE all monitoring and control functions of the control center are immaterial. The SE provides the data base by which all those functions can be performed. A hybrid estimator incorporating fuzzy logic concepts (if then rules) are introduced in the past. In this paper, a fast method for power system state estimation is introduced which is based on weighted least square (WLS) state estimation and fuzzy linear regression (FLR) model. The FLR model is based on inequality constrained linear programming (LP) optimization. In this approach, the WLS method will run for first iteration and then the FLR model will continue until the convergence is reached. The WLS-FLR approach provides not only the estimated states but also, an uncertainty range for those estimated states with less computational time. The effect of fuzziness degree and the measurements accuracy on the estimated states are discussed in this paper. The effectiveness of the proposed method is demonstrated by using 6-bus, and IEEE 30-bus systems.

Keywords— *State Estimation; Weighted Least Square (WLS); Linear programming; Fuzzy Linear Regression (FLR); Uncertainty.*

I. INTRODUCTION

The most important part of the system operation is to have an accurate picture of the system states. A simple SCADA (Supervisory Control and Data Acquisition) system is able to provide the system operators with measured information and the system operation conditions which can be filtered by state estimator to create a more accurate picture of the system status.

Using the state estimation, the effect of normal errors of measurements and the variance of the estimated states can be reduced by utilizing the redundancy available in the measurement system. The gross errors and invalid topological information together with the model parameter errors can also be detected and identified.

If the inaccuracy in the measurements is modeled by known probability distribution function, then the set of feasible estimates can be modeled by the same function. Unfortunately, it is difficult to characterize statistics of observation errors in practice. In such circumstances, it is desirable to provide not only just a single 'optimal' estimate of each state variable but also an uncertainty range within which the true value of the state variable be lay. The idea of an uncertainty range is recognizable in engineering practice, where the accuracy of a particular measurement is often described in percent e.g. $\pm 3\%$, rather than by quantifying the standard deviation or variance. The range is governed by the tolerance of the measuring instrument which is usually provided by the manufacturer.

The concepts of uncertainty in the general context of engineering analysis, estimation and optimization are introduced in [1]. These concepts have been extended and developed and applied in several areas, e.g. in water distribution networks [2]. The authors in [3] introduced bounds on measurements. A development of the approach [4] introduced the term set, bounded state estimation (SBSE) to increase the robustness of the estimation introduced bounds on the measurements. The concepts of robust control theory allowed the uncertainty in both the parameters and the measurements to be applied [2]. A developed method [2] of uncertainty analysis based on linear fractional transformations (LFT) is introduced. The ellipsoid-of-confidence bounds can be obtained by recasting the LFT problem into a semi definite programming problem (SDP).

Different methods have been introduced to estimate the uncertainty interval around the system state variables [5, 6]. The uncertainty in [5] is modeled through deterministic upper and lower bounds on measurement errors, which take into account known meter accuracies. In this method, WLS is used to estimate the expected value of the state variables, and then a LP formulation is utilized for estimating the tightest possible upper and lower bound on these estimates. The linear formulation was limited to modeling uncertainty only in the measurements which

was due to meters inaccuracies. In fact, other inaccuracies of the network mathematical model must be added to the uncertainty in the measurements. As an extension, authors in [6] have introduced another uncertainty analysis method in which the uncertainties, are expressed in both measurements and system parameters. This analysis assumed that the uncertainties be known and bounded. The problem is formulated as a constrained nonlinear optimization problem and is solved by sequential quadratic programming (SQP) technique.

The main drawback in those formulations was the computational upper and lower bounds arise from performing two LP or two SQP depending on the formulation used. For instance, minimizing a particular state variable over the all of the measurement inequality constraints provides the lower bound on that state variable. In a like manner, maximizing that state variable again over the same constraints provides the upper bound of that state. Consequently, the CPU execution time for both formulation is relatively high.

The application of fuzzy logic in power system state estimation was initiated by Shabani, Prasad, and Smolleck in 1996 [7]. The proposed method is based on fuzzy if then rules for improving the WLS estimator. Fuzzy logic control adaptively adjusts the weighting factor in the proposed estimator [7]. For modeling the uncertainty in D.C state estimation, a fuzzy linear state estimation based on Tanaka's fuzzy linear regression model was proposed [8] and developed [9,10]. The measuring system used for estimation process in [9,10] were generated using NR load flow program without adding noise. In fact, random errors were added to the generated data to simulate typical measurement errors. The uncertainty in [10] is assumed to be present in the measurements only. The uncertain measurements are expressed as fuzzy number with triangular or trapezoidal membership functions. In the trapezoidal membership measurements, a WLS is solved for the inner breakpoints, and then the outer breakpoints are calculated using fuzzy arithmetic and LP.

The major disadvantage of [9,10] was the linear optimization problem will be solved in every iteration of NR method to compute the incremental change of state variables. Consequentially, for real large-scale power systems this method introduces a significant amount of computation and CPU time.

The drawback mentioned above will be improved using WLS state estimation with fuzzy linear regression model (WLS-FLR approach). In this work, the measuring system was generated numerically using NR power flow with random variations 1% around the base case to simulate typical measurement errors. The state estimation results obtained by WLS-FLR approach are also, compared with the WLS state estimation method using some performance indices i.e. average absolute error, maximum absolute error, and root mean square error.

II. TANAKA'S FUZZY LINEAR REGRESSION OVERVIEW

Fuzzy linear regression was introduced by Tanaka et al. [11] in 1982. The general form of Tanaka's formulation is given by:

$$Y_{\sim} = f(x, A) = A_0 + A_1x_1 + A_2x_2 + \dots + A_nx_n = Ax \quad (1)$$

where, Y_{\sim} is the output (dependent fuzzy variable), $\{x_1, x_2, \dots, x_n\}$ is a non fuzzy set of crisp independent parameters and $\{A_0, A_1, \dots, A_n\}$ is a fuzzy set of symmetric members, unknowns, needs to be estimated. Each fuzzy element in that set may be represented by a symmetrical triangular membership function, shown in Fig. 1, defined by a middle and a spread values, p_i and c_i , respectively. The middle is known as the model value and the spread denotes the fuzziness of that model value.

The membership function μ_{A_i} for each of the coefficients A is expressed as:

$$\mu_{A_i}(a_i) = \begin{cases} 1 - \frac{|p_i - a_i|}{c_i}, & p_i - c_i \leq a_i \leq p_i + c_i \\ 0 & \text{otherwise} \end{cases} \quad (2)$$

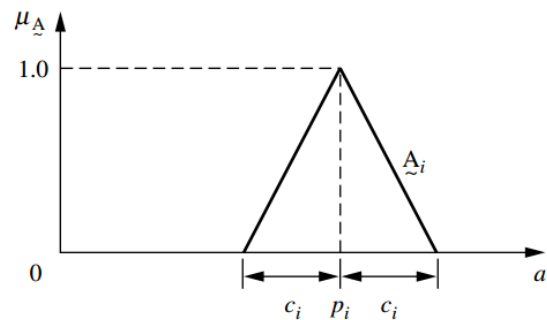


Fig. 1. Membership function for the fuzzy coefficient A_i

Therefore, since the fuzzy function A is a function of two parameters, p and c , (1) may be rewritten as:

$$Y_{\sim} = (p_0, c_0) + (p_1, c_1)x_1 + (p_2, c_2)x_2 + \dots + (p_n, c_n)x_n = A_{\sim}^* x_i \quad (3)$$

The membership function for the output fuzzy parameter, Y_{\sim} is given by

$$\mu_{Y_{\sim}}(y) = \begin{cases} \max(\min[\mu_{A_i}(a_i)]), & \{a | y = f(x, a)\} \neq \emptyset \\ 0, & \text{otherwise} \end{cases} \quad (4)$$

Now, by substituting (2) into (4), the membership function for the output fuzzy parameter, Y_{\sim} is given by

$$\mu_{Y_{\sim}}(y_i) = \begin{cases} 1 - \frac{|y - \sum_{i=1}^n p_i x_i|}{\sum_{i=1}^n c_i |x_i|} & x_i \neq 0 \\ 1, & x_i = 0, y = 0 \\ 0, & x_i = 0, y \neq 0 \end{cases} \quad (5)$$

The membership function for the fuzzy output is illustrated in Fig. 2. From regression point of view, the

foregoing equations may be applied to m samples where the output can be either non-fuzzy, (certain or exact), in which no assumption of ambiguity is associated with the output or fuzzy (uncertain), where uncertainty in the output is involved due to human judgment or meters inaccuracy [12].

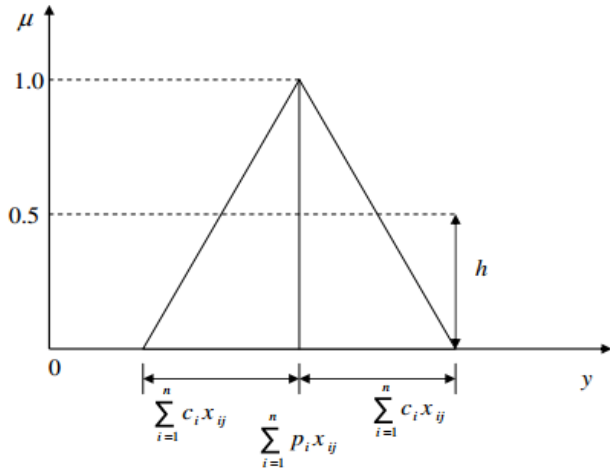


Fig. 2: Fuzzy output function

Here, the fuzzy out will be considered as follow, when human judgment or imprecise measurements are involved in determining the output, the output may certainly be fuzzy. The output in such situations is best represented by a fuzzy number as $Y_j = (y_j, e_j)$, where y_j is the middle value and e_j represents the uncertainty in measurement j , as seen in Fig. 3.

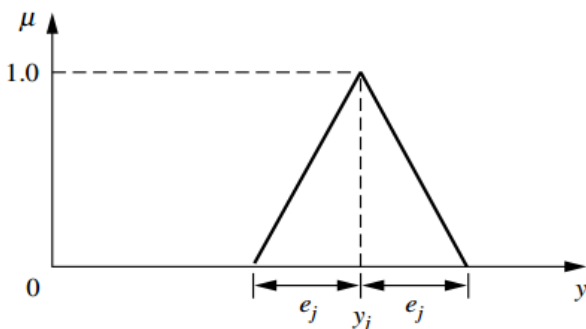


Fig. 3. An example of fuzzy output

The membership function for the observed fuzzy output is given as:

$$\mu_{Y_j}(y) = 1 - \frac{|y_j - y|}{e_j} \quad (6)$$

An estimate of (6) can be obtained from (5) as:

$$\mu_{Y_j^*}(y) = 1 - \frac{|y_j - \sum_{i=1}^n p_i x_{ij}|}{\sum_{i=1}^n c_i |x_{ij}|} \quad \text{for } j = 1, \dots, m \quad (7)$$

In summary, the main objective of fuzzy linear regression is to determine the fuzzy parameters A_j^* that minimize the sum of spread as in

$$\min \left\{ \sum_{j=1}^m \sum_{i=1}^n c_i x_{ij} \right\} \quad (8)$$

Subject to:

$$\sum_{i=1}^n p_i x_{ij} - (1-h) \sum_{i=1}^n c_i x_{ij} + (1-h)e_j \leq y_j \quad (9)$$

$$\sum_{i=1}^n p_i x_{ij} + (1-h) \sum_{i=1}^n c_i x_{ij} - (1-h)e_j \geq y_j \quad (10)$$

Note that the term $(1-h)e_j$ is due to the introduction of fuzziness (uncertainty) in the measurements. As mentioned, the (9) represents the y_j when it lies in the interval to the left of the middle value with the uncertainty with respect to it added to that interval. In like manner, (10) represents the y_j when it lies in the interval to the right of the middle value with the uncertainty with respect to it added to that interval. The proof and detailed derivation for both cases of Tanaka's model may be found in [11,13].

III. FUZZY LINEAR REGRESSION APPROACH

The main purpose of SE in electric power system is to find the estimate \hat{x} of the true state x which best fits the measurements z related to x through the nonlinear model [14]:

$$z = h(x) + e \quad (11)$$

- where, z is the m -dimensional measurement vector;
- x is the n -dimensional ($n < m$) state vector of voltage magnitudes and angles;
- $h(x)$ is the nonlinear vector function relating measurements to states;
- e is the m - dimensional measurement error vector;
- n is the number of state variables;
- m is the number of measurements.

The estimate of the unknown state vector x is designated by \hat{x} and is finding out by the linearization (11) around some operating point x^0 . This is based on taking the first-order Taylor series expansion and ignoring the higher order terms as follows,

$$\Delta z = H(x^0) \Delta x + e \quad (12)$$

where, $\Delta z = z - h(x^0)$;

$H = \partial h(x^0) / \partial x$ is the Jacobian matrix ($m \times n$);

$$\Delta x = \hat{x} - x^0$$

The estimates are usually solved by the Newton-Raphson (NR) method which computes the state corrections Δx at each iteration until appropriate convergence is attained. The linearized power system in (12) for the H^{th} measurement can be rewritten as:

$$\Delta z_j = \Delta x_1 H_{j1} + \Delta x_2 H_{j2} + \dots + \Delta x_n H_{jn} \quad (13)$$

The change in the system state variables Δx can be defined as a fuzzy membership function having a middle and a spread values, p_i and c_i , respectively. Then, (13) can be expressed as:

$$\Delta z_j = (p_1, c_1)H_{j1} + (p_2, c_2)H_{j2} + \dots + (p_n, c_n)H_{jn} \quad (14)$$

where, p_i is the middle value, which represents the value of the change in the system state variables Δx_i , at the current iteration of the linearized model. The spread c_i on the other hand, which is symmetric, corresponds to the incremental confidence interval of that state variable. Therefore, Δx can be defined:

$$\Delta x = [(p_1, c_1) (p_2, c_2) \dots (p_n, c_n)] \quad (15)$$

Fuzzy linear regression model is modified as an alternative method in order to be used for modeling the uncertainty in power system state estimation. The optimal state estimate vector \hat{x} will be determined by minimizing the sum of the spread of all state variables. The change in state variables, subject to a number of constraint representing measurements can be expressed as:

$$\min \left(\sum_{j=1}^m \sum_{i=1}^n c_i H_{ij} \right) \quad (16)$$

Subject to:

$$\sum_{i=1}^n p_i H_{ij} - (1-h) \sum_{i=1}^n c_i H_{ij} + (1-h)e_j \leq \Delta z_j \quad (17)$$

$$\sum_{i=1}^n p_i H_{ij} + (1-h) \sum_{i=1}^n c_i H_{ij} - (1-h)e_j \geq \Delta z_j \quad (18)$$

Note that h is the degree of the fuzziness and is specified by the decision maker. In the context of power system state estimation e_j may represent the transducer tolerance which is usually provided by the manufacturer of the meter itself, i.e. (± 3). This model is inequality linear programming optimization model and will be solved by the function linprog incorporated in the MATLAB optimization Toolbox™ [15]. In the NR approach, the state variable is updated by

$$\hat{x}_{k+1} = \hat{x}_k + \Delta x_k \quad (19)$$

where, the incremental change in state variable Δx_k , is computed by fuzzy linear model above, (16-18), and it can be expressed

$$\Delta x = [p_1, p_2, \dots, p_n]^T \quad (20)$$

where, p_i correspond to the middle value of the incremental change of the system state variables at the iterations k . Since the optimal spreads represent a quantified measure of how uncertain we are about their respective middles i.e. state variables, and then the interval of confidence due to uncertainty can be constructed by adding or subtracting the spreads to or

from their respective middles. For instance, the lower and upper bound of the incremental changes at iteration k can be calculated as:

$$\Delta x_k^- = \Delta x_k - [c_1, c_2, \dots, c_n]^T \quad (21)$$

$$\Delta x_k^+ = \Delta x_k + [c_1, c_2, \dots, c_n]^T \quad (22)$$

Finally, the lower and upper bound of the interval of all state variables at iteration k can be computed:

$$\hat{x}_{k+1}^- = \hat{x}_k + \Delta x_k^- \quad (23)$$

$$\hat{x}_{k+1}^+ = \hat{x}_k + \Delta x_k^+ \quad (24)$$

It is important to mention that the problem of power system state estimation consists of determining the best estimate solution of the state variables of the power system which best fits the redundant set of measurements z . The FLR model formulation above provides the set of estimates (middle values) along with an upper and lower bound for that estimated middle values without needing to any other additional estimator with high computational time.

Since the weighted least square is faster than linear programming optimization, the WLS will run for first iteration as in [16] and then the FLR model (16-20) will continue until the stopping criterion is reached. The tolerance of 0.0000001 was taken as convergence criteria. After the WLS-FLR approach was converged, the upper and lower bounds of the estimated states can be computed using, (21-24). Note that in this study it is found that the WLS state estimation seems to have no effect on the computation of the estimated middle value of the estimated states when applied for first iteration. Consequentially, the CPU time required for FLR method may be improved especially in case of large-scale power systems.

IV. SIMULATION RESULTS AND DISCUSSION

This section presents typical results obtained by applying the WLS-FLR approach to the 6-bus, and IEEE 30-bus test systems. The programs for WLS-FLR method was coded in MATLAB M-files and run on a TOSHIBA Pentium IV machine. In this work, the measuring system was generated numerically using NR power flow with random variations of 1% around the base case. To simulate parametric uncertainty, elements of the admittance matrix have been perturbed by adding uniformly distributed random values to the nominal values of those elements over an interval [-1%, 1%].

Simulated test data for 6-bus sample power system is given in [14] shown in Fig. 4, which has 19 number of measurements. The redundancy level of the measurement is 1.3. The uncertainty in measurements is assumed to be of $\pm 3\%$ of nominal values, while the

uncertainties due system parameters are bounded by $\pm 1\%$.

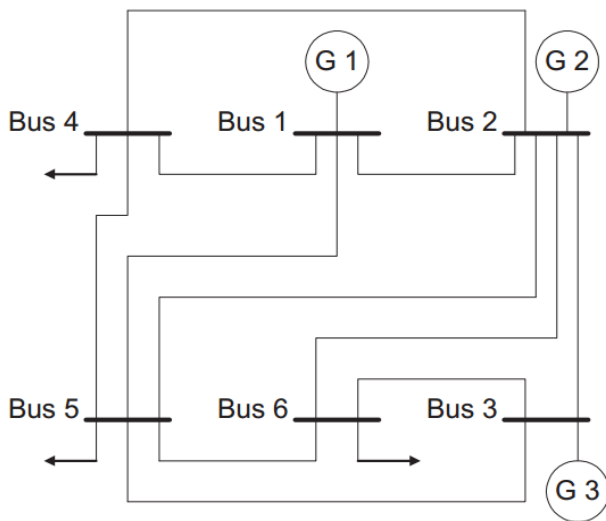


Fig. 4. 6-bus test system

Table I presents typical results obtained by the WLS state estimation and WLS-FLR method when applied to the 6-bus network. As for the uncertainty interval, the estimated upper and lower bound are shown in Table 1. It may be noticed that the estimated center points appear to be fuzzy due to the inaccuracies in the measurements and system parameters as expected. The information given in table I are also presented in Fig. 5 and Fig. 6 for voltage magnitudes and phase angles respectively.

It should be noted that the estimated center points (middle value) lie exactly in the middle of the confidence interval. This particular outcome is expected since a symmetric spread was adopted by FLR to model the uncertainties. The estimated center points of the interval obtained by the WLS-FLR method are also, identical to those obtained by WLS algorithm. It is obviously that the estimated center points of interval obtained by WLS-FLR are close to the actual values. Note that in this particular test case, it is found that the degree of fuzziness h seems to have no

significant effect on the computation of spreads. This is due to the fact that FLR technique estimates the incremental changes of state variables in the linearized domain which are relatively very small. The WLS-FLR approach converged in 4 iterations with execution time 0.0321s, see Table III.

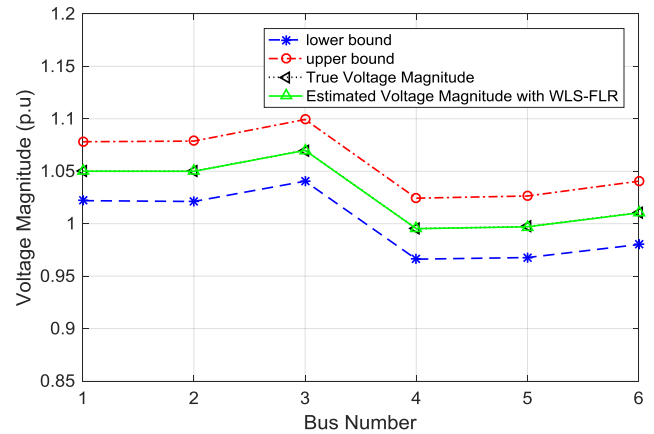


Fig. 5. Estimated voltage and uncertainty bounds for six-bus network

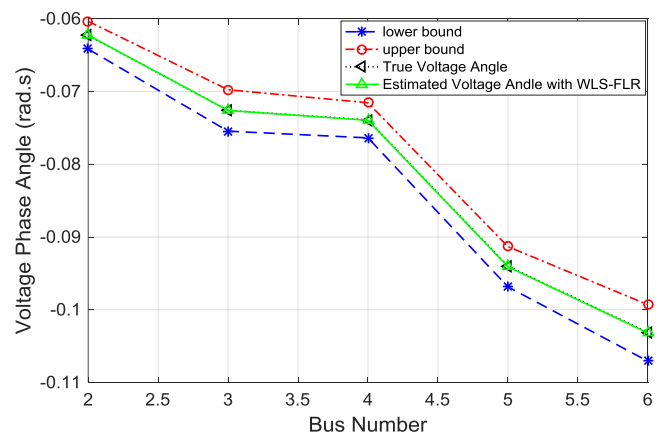


Fig. 6. Estimated voltage phase angle and uncertainty bounds for six-bus network

TABLE I. ESTIMATED STATE VARIABLES (VOLTAGE MAGNITUDE AND ANGLE) AND THEIR LOWER AND UPPER POUNDS FOR 6-BUS NETWORK WITH ($h=0.5$)

Bus No.	WLS		Lower bound		WLS-FLR (Middle)		Upper bound	
	Voltage	Angle	Voltage	Angle	Voltage	Angle	Voltage	Angle
1	1.0500	0	1.0220	0	1.0500	0	1.0781	0
2	1.0499	-0.0623	1.0212	-0.0642	1.0499	-0.0623	1.0787	-0.0603
3	1.0699	-0.0726	1.0405	-0.0755	1.0699	-0.0726	1.0993	-0.0698
4	0.9952	-0.0740	0.9662	-0.0764	0.9952	-0.0740	1.0243	-0.0715
5	0.9970	-0.0941	0.9677	-0.0968	0.9971	-0.0941	1.0264	-0.0913
6	1.0103	-0.1031	0.9801	-0.1071	1.0103	-0.1032	1.0405	-0.0993

Similarly, simulated test data for IEEE 30-bus power system [17] has 98 number of measurements and there are 59 state variables (voltage magnitudes and voltage angles). The uncertainty measurement is assumed to be of $\pm 5\%$ of nominal values. The degree of freedom is 39. The values are in p.u for voltage magnitude while the phase angle values are in radian.

Table II provides the estimated voltage magnitude and angles with their lower and upper bounds for IEEE 30-bus test system using WLS-FLR technique. In this table, columns 2 and 3 represent the estimated states using WLS while columns 4 to 8 depict the estimated states with their lower and upper bounds using WLS-FLR. The results obtained by the FLR model are identical to those obtained by WLS-FLR. The WLS-

FLR approach converged in 5 iterations with execution time 0.2840s, as shown in Table III.

Table III provides the computational time for the test cases used. The CPU execution time of the FLR method required for convergence is relatively higher than WLS and WLS-FLR methods. This slightly more CPU time of the fuzzy linear regression may be attributed to having to solve a constrained state estimation linear programming problem as mentioned earlier. It may be seen from Table III that the CPU execution time of the WLS-FLR method is lower than FLR model. This outcome is expected since WLS run for first iteration which eliminates the time required to solve LP problem for one iteration.

TABLE II. ESTIMATED STATE VARIABLES (VOLTAGE MAGNITUDE AND ANGLE) AND THEIR LOWER AND UPPER BOUNDS FOR 30-BUS NETWORK WITH $h=0.5$.

Bus No.	WLS		Lower bound		WLS-FLR (Middle)		Upper bound	
	Voltage	Angle	Voltage	Angle	Voltage	Angle	Voltage	Angle
1	1.0569	0	1.0284	0	1.0657	0	1.1030	0
2	1.0402	-0.0973	1.0097	-0.1008	1.0490	-0.0956	1.0882	-0.0904
3	1.0202	-0.1271	0.9866	-0.1342	1.0292	-0.1248	1.0719	-0.1155
4	1.0062	-0.1678	0.9711	-0.1774	1.0171	-0.1651	1.0630	-0.1529
5	1.0059	-0.2522	0.9695	-0.2654	1.0149	-0.2477	1.0603	-0.2301
6	1.0113	-0.2019	0.9734	-0.2122	1.0191	-0.1980	1.0647	-0.1839
7	1.0026	-0.2325	0.9615	-0.2458	1.0105	-0.2281	1.0595	-0.2103
8	1.0091	-0.2155	0.9696	-0.2273	1.0169	-0.2114	1.0642	-0.1955
9	1.0314	-0.2583	0.9891	-0.2755	1.0363	-0.2521	1.0834	-0.2286
10	1.0264	-0.2864	0.9939	-0.3077	1.0345	-0.2795	1.0750	-0.2514
11	1.0662	-0.2584	1.0276	-0.2821	1.0684	-0.2527	1.1091	-0.2233
12	1.0405	-0.2726	1.0170	-0.2956	1.0510	-0.2699	1.0850	-0.2441
13	1.0540	-0.2715	1.0296	-0.2961	1.0631	-0.2698	1.0965	-0.2435
14	1.0233	-0.2887	1.0019	-0.3197	1.0335	-0.2867	1.0651	-0.2536
15	1.0181	-0.2909	0.9946	-0.3180	1.0289	-0.2870	1.0631	-0.2561
16	1.0272	-0.2838	1.0012	-0.3091	1.0364	-0.2805	1.0715	-0.2519
17	1.0227	-0.2894	0.9913	-0.3190	1.0308	-0.2854	1.0704	-0.2518
18	1.0070	-0.3027	0.9759	-0.3300	1.0198	-0.2953	1.0638	-0.2606
19	1.0055	-0.3055	0.9723	-0.3314	1.0201	-0.2982	1.0678	-0.2650
20	1.0104	-0.3014	0.9768	-0.3272	1.0229	-0.2936	1.0691	-0.2599
21	1.0129	-0.2945	0.9811	-0.3146	1.0207	-0.2860	1.0604	-0.2574
22	1.0128	-0.2944	0.9825	-0.3155	1.0213	-0.2858	1.0602	-0.2560
23	1.0051	-0.2994	0.9781	-0.3328	1.0150	-0.2952	1.0520	-0.2576
24	0.9998	-0.3027	0.9773	-0.3307	1.0118	-0.2948	1.0464	-0.2589
25	0.9974	-0.2887	0.9985	-0.2974	1.0117	-0.2860	1.0249	-0.2746
26	0.9742	-0.2936	0.9833	-0.2760	0.9869	-0.2860	0.9904	-0.2960
27	1.0063	-0.2817	1.0089	-0.2845	1.0235	-0.2794	1.0380	-0.2742
28	1.0107	-0.2132	0.9705	-0.2254	1.0183	-0.2092	1.0660	-0.1931

29	0.9794	-0.3014	0.9979	-0.2806	1.0009	-0.3006	1.0039	-0.3206
30	0.9657	-0.3117	0.9702	-0.3149	0.9902	-0.3075	1.0102	-0.3000

TABLE III. CPU TIME WITH WLS, FLR and WLS-FLR METHODS.

Test System	WLS		FLR		FLR-WLS	
	CPU Time (s)	Iterations	CPU Time (s)	Iterations	CPU Time (s)	Iterations
6-bus	0.0072	5	0.0551	4	0.0321	4
IEEE 30-bus	0.0271	5	0.3495	5	0.2840	5

The accuracy of the WLS and WLS-FLR approaches are also, compared using some performance indices i.e. the average absolute (e_{av}) errors, maximum absolute (e_{max}) errors, and root mean square (e_{rms}) errors as illustrated in Table 4. It may be observed from Table IV that the estimated middle value obtained by WLS-FLR approach are more accurate rather than WLS state estimation.

Table IV. SUMMARY OF STATE ESTIMATION ERRORS FOR WLS AND WLS-FLR APPROACHES FOR IEEE 30-BUS SYSTEM

	WLS		WLS-FLR	
	Voltage	Angle	Voltage	Angle
e_{av}	0.0149	0.0050	0.0081	0.0025
e_{max}	0.0288	0.0126	0.0147	0.0149
e_{rms}	0.0173	0.0059	0.0086	0.0039

V. CONCLUSION

In this paper, WLS-FLR method has been illustrated through the application on 6-bus and IEEE 30-bus test systems. Fuzzy regression and linear programming models were employed to estimate the state variables and their respective lower and upper bounds. The advantage of this method is that errors in the measurements are expressed as fuzzy numbers with a triangular membership function that has middle and spread value reflected on the estimated states. Results obtained show that the computational performance of the WLS state estimator is improved by this approach.

In order to evaluate the WLS-FLR approach, a comparison between the WLS state estimation, FLR model, and WLS-FLR approach are performed based on the convergence and the time assessment for test systems used. Simulation results indicate that the WLS-FLR approach is more suitable for modeling the uncertainty in power system state estimation with both less computational time and high accuracy.

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