

# Plastic Zones and Residual Stresses of Galvanized Fiber Reinforced LDPE Composite Laminates with a Hole

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**Abstract**—Fiber-reinforced composite materials and, in particular, polymer matrix composites have gained widespread applicability in high-performance air-craft and aerospace structures due to their high specific-stiffness and specific-strength (stiffness and strength per mass), thus, offering a considerable weight saving capability. The matrix, however, is the most significant limitations to broader and more efficient use of composite materials. It limits the overall performance of the composite system due to its inferior elastic/strength properties and large thermoelastic property mismatch with reinforcing fibers, as well as its high sensitivity to environmental conditions. In the present study, an elastic-plastic stress analysis in symmetric and antisymmetric Angle-ply laminated plastic-metal fiber composite plate is carried out by using the finite element method. The composite plate reinforced with (long galvanized wire fiber) is manufactured by using moulds under the action of 30 MPa pressure and heating up to 190°C. A laminated plate consists of four plastic matrix layers bonded symmetrically or antisymmetrically by applying pressure and heat. The first order shear deformation theory and nine-node Lagrangian finite element is used. In the numerical solution the transverse load is increased gradually.

**Keywords**— *Residual stresses; Thermoplastic composite; Finite element; Elastic-plastic solution; Thermoelastic property*

## Introduction

The high specific-designing composite structures, which are more efficient than metallic structures.

In addition, thermoplastic composites have the unique characteristic that they may be remelted, reprocessed, and reformed thereby offering a degree of post processing freedom unavailable in thermoset composites. This characteristic makes thermoplastic easily repairable as they can be remelted locally and resolidified (repair of transverse cracking and

delamination), and recyclable.

Jegley, [5] presented a manufacturing process of thermoforming and the results of a study of the effects of impact damage on compression loaded trapezoidal corrugation sandwich and semisandwich graphite-thermoplastic panels.

Representative experimental investigations on the forming of advanced thermoplastic composites can be found in references. (Jegley, Chen, Cantwell, Shi, Wang.) [5], [1], [6], [4], [2][3].

Elastic-plastic stress analysis was carried out in a laminated composite plate (Bahei-El-Din, Karakuzu). [8], [9]

Finite element method gives the excellent solution to the elastic-plastic stress analysis of laminated plate (Karakuzu, Sayman). [9], [7]

In the present study, an elastic-plastic stress analysis in symmetric and antisymmetric Angle-ply laminated plastic-metal fiber composite plate is carried out by using the finite element method. The composite plate reinforced with (long galvanized wire fiber) is manufactured by using moulds under the action of 30 MPa pressure and heating up to 190°C. A laminated plate consists of four plastic matrix layers bonded symmetrically or antisymmetrically by applying pressure and heat. The first order shear deformation theory and nine-node Lagrangian finite element is used. In the numerical solution the transverse load is increased gradually.

## I. MACROMECHANICAL PROPERTIES OF A LAMINATE

### 2.1. Mathematical Formulation

The laminated plate of constant thickness is composed of orthotropic layers bonded symmetrically or antisymmetrically about the middle surface of the plate. In the solution of this problem, the cartesian coordinates are used where the middle surface of the plate coincides with the x-y plane (**Figure 2.1**)

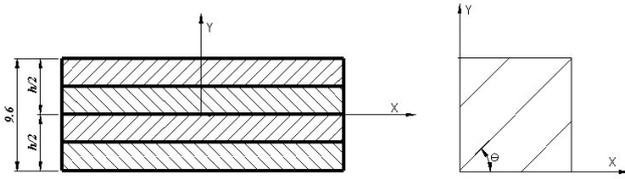


Figure 2.1

The stress-strain relations for an orthotropic layer can be written as;

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & Q_{16} \\ Q_{12} & Q_{22} & Q_{26} \\ Q_{16} & Q_{26} & Q_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} \quad (2.1)$$

$$\begin{Bmatrix} \tau_{yz} \\ \tau_{xz} \end{Bmatrix} = \begin{bmatrix} Q_{44} & Q_{45} \\ Q_{54} & Q_{55} \end{bmatrix} \begin{Bmatrix} \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} \quad (2.2)$$

where  $Q_{ij}$  are reduced stiffness and can be defined in terms of elastic constants  $D_{ij}$  and the orientation angle  $\theta$ . Here, we use the theory of plates with transverse shear deformations theory which uses the assumption that particles of the plate originally on a line that is normal to the undeformed middle surface remain on the straight line during deformations, but this line is not necessarily normal to the deformed middle surface. By using this assumption, the displacement components of a point of coordinates  $x, y, z$  for small deformations are (Jones, Tsai, Gibson), [17], [14], [18]

$$\begin{aligned} u(x, y, z) &= u_0(x, y) + z\psi_x(x, y) \\ v(x, y, z) &= v_0(x, y) - z + \psi_y(x, y) \\ w(x, y, z) &= w(x, y) \end{aligned} \quad (2.3)$$

where  $u_0, v_0$ , and  $w$  are displacements at any point of the middle surface and  $\psi_x, \psi_y$  are the rotations of normals to the  $y$  and  $x$  axes, respectively.

The bending strains vary linearly through the plate thickness, whereas shear strains are assumed to be constant through the thickness as (Lin) [13],

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{bmatrix} \frac{\partial u_0}{\partial x} \\ \frac{\partial v_0}{\partial y} \\ \frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x} \end{bmatrix} + z \begin{bmatrix} \frac{\partial \psi_x}{\partial x} \\ -\frac{\partial \psi_y}{\partial y} \\ \frac{\partial \psi_x}{\partial y} - \frac{\partial \psi_y}{\partial x} \end{bmatrix} \text{ or } \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{bmatrix} + z \begin{bmatrix} K_x \\ K_y \\ K_{xy} \end{bmatrix}$$

$$\begin{Bmatrix} \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} = \begin{bmatrix} \frac{\partial w}{\partial y} - \psi_y \\ \frac{\partial w}{\partial x} - \psi_x \end{bmatrix} \quad (2.4)$$

The total potential energy of a laminated plate under static loadings is given as:

$$\Pi = U_b + U_s + V \quad (2.5)$$

where  $U_b$  is the strain energy of bending,  $U_s$  is the strain energy of shear, and  $V$ , represents potential energy of external forces. They are as,

$$U_b = \frac{1}{2} \int_{-h/2}^{h/2} \int_A (\sigma_x \varepsilon_x + \sigma_y \varepsilon_y + \tau_{xy} \gamma_{xy}) dA dz$$

$$U_s = \frac{1}{2} \int_{-h/2}^{h/2} \int_A (\tau_{yz} \gamma_{yz} + \tau_{xz} \gamma_{xz}) dA dz$$

$$V = \frac{1}{2} \int_A w p dA - \int_{\partial R} (N_n^b u_n^0 + N_s^b u_s^0) ds \quad (2.6)$$

where  $dA = d_x \cdot d_y$ ,  $p$  is the transverse loading per unit area, and  $N_n^b$  and  $N_s^b$  are the in-plane loads applied on the boundary  $\partial R$ .

In-plane forces  $N_x, N_y$ , and  $N_{xy}$  and moments  $M_x, M_y$ , and  $M_{xy}$ , defined per unit length are given;

$$\begin{Bmatrix} N_x & M_x \\ N_y & M_y \\ N_{xy} & M_{xy} \end{Bmatrix} = \int_{-h/2}^{h/2} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_{xy} \end{Bmatrix} (1, z) dz$$

$$\begin{Bmatrix} Q_x \\ Q_y \end{Bmatrix} = \int_{-h/2}^{h/2} \begin{Bmatrix} \tau_{xz} \\ \tau_{yz} \end{Bmatrix} dz \quad (2.7)$$

Equilibrium requires that  $\pi$  is the stationary or  $\pi$  is a minimum that the second variation of  $\Pi$  is positive at the stationary point. It may be regarded as the principle of

$$\Pi = \Pi(u_0, v_0, w, \psi_x, \psi_y) \quad (2.8)$$

virtual displacement (Bathe). [12] By using the finite element method, the total energy is expressed as a function of displacement components as.

## 2.2. Finite Element Analysis

By using characteristic values found from the experiments mentioned in chapter three, we applied the finite element analysis by using computer to the laminates for variation of angles cross-ply laminate position (symmetric and antisymmetric), thickness and hole effect in the laminates and following results are found. The area of the laminates taken as  $150 \times 150 \text{ mm}^2$  (Figure 2.2.)

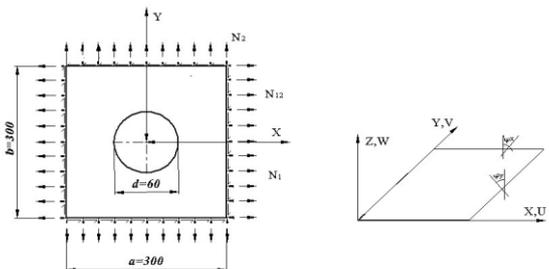


Figure 2.2. Loading laminated plate

The nine-node finite element is used in the present study. The quadratic interpolation functions for the nine-node Lagrangian finite element and mesh model.

In the elastic-plastic solution the tangential modular matrix is used instead of the elasticity matrix (Cristfield). The yield function  $f$  is as;

$$f = \sigma_e - \sigma_0 = 0 \quad (2.9)$$

where  $\sigma_e$  is the effective stress and  $\sigma_0$  is the yield stress: The tangential modular matrix is found as;

$$D_t = D \left( I - \frac{a a^T D}{a^T D a} \right) \quad (2.10)$$

Where  $D$ ,  $I$  and  $a$  are the elasticity matrix, unit matrix and flow vector, respectively. The flow vector is found by using the result Prandtl-Reuss equation (Mendelson and Chakrabarty) [15], [16]

$$a = \partial f - \partial \sigma \quad (2.11)$$

Once the nodal displacement are calculated, the strain components of each layer can be found and the stress components can be calculated and used to check the yield state of the material.

Since the calculated stresses do not generally coincide with the true stress in a nonlinear problem, the unbalanced nodal forces and the equivalent nodal forces must be calculated. The equivalent nodal point forces corresponding to the element stress at each iteration can be calculated as (Hu): [10]

$$\{R\}_{equivalent} = \int_{vol} [B]^T [\sigma] dA = \int_{vol} [B_b]^T [\sigma_b] dA + \int_{vol} [B_s]^T [\sigma_s] dA \quad (2.12)$$

When the equivalent nodal forces are known, the unbalanced nodal forces can be found by;

$$\{R\}_{unbalanced} = \{R\}_{applied} - \{R\}_{equivalent} \quad (2.13)$$

These unbalanced nodal forces are applied for obtaining increments in the solution and must satisfy the convergence tolerance in a nonlinear analysis. A widely used iteration procedure is the modified Newton Iteration Method as (Bathe). [12] This iterative schema can be derived from the Newton-Raphson Method Equations.

The difference between the plastic and elastic solution gives the residual stresses. The residual stresses may increase the possible failure of the laminated plates. In this solution, 169 nodes and 36 elements are used.

The stiffness matrix of the plate element is obtained by using the minimum potential energy method or the principle of virtual bending and shear stiffness matrices are:

Where

$$[K_b] = \int_A [B_b]^T [D_b] [B_b] dA$$

$$[K_s] = \int_A [B_s]^T [D_s] [B_s] dA$$

$$[D_b] = \begin{bmatrix} A_{ij} & B_{ij} \\ B_{ij} & D_{ij} \end{bmatrix} \quad [D_s] = \begin{bmatrix} k_1^2 A_{44} & 0 \\ 0 & k_2^2 A_{55} \end{bmatrix} \quad (2.14)$$

$$(A_{ij}, B_{ij}, D_{ij}) = \int_{-h/2}^{h/2} Q_{ij}(1, z, z^2) dz \quad (i, j=1, 2, 6)$$

$$(A_{44}, A_{55}) = \int_{-h/2}^{h/2} (Q_{44}, Q_{55}) dz \quad (2.15)$$

$[B_b]_{6 \times 45}$ ,  $[B_s]_{2 \times 45}$  and  $D_b$  and  $D_s$ , are the bending and shear parts of the material matrix, respectively.  $A_{55}$ , is neglected in comparison with  $A_{44}$  and  $A_{55}$  and shear correction factors for rectangular cross-sections are given as (Lin, Barbero), [13], [11]

$$k_1^2 = k_2^2 = 5/6.$$

Once the nodal displacements are calculated, the strain components of each layer can be found by partial derivation formulas and tangential modular matrix. The stress components can be calculated and used to check the yield state of the material.

## I. EXPERIMENTAL PROCEDURES

### 3.1. Introduction

In this study; low density polyethylene (LDPE) F2.12 was used as a thermoplastic matrix. The preparation of composite laminates consisted of two important steps. The first step was that, melting granule, raw material of thermoplastics, as a flat plate. The second was combining these plates with fibers as composite plates. These two steps were applied successfully in the laboratory.

Thermoplastic plates were obtained by melting the granule in the rectangular mould. The material temperature was increased up to 160°C without pressure in five minutes. Then, the materials were waited five minutes under 2.5 MPa pressure. And the last, the temperature was decreased to 30°C under 15 MPa pressure in three minutes. Consequently, polyethylene plate was obtained.

### 3.2 Production of Laminates

Approximately 30 grams of granulized plastic material was inserted into an iron mould of 160 mm size and 1 mm thickness. To avoid sticking, acetate was put between the surfaces. The compression pressure and time date were reported for the low density. PE were as before chapter. Then, a single laminate was produced by cooling down to 30°C degrees under 15 MPa pressure within 3 minutes.

### 3.3. Production of These Composite Materials

#### 3.3.1. Unidirectionally Galvanized Wire Fiber Reinforced Laminates

An iron square frame of 160 x 160 mm<sup>2</sup> area was driven by 0.5 mm diameter holes along its two facing

edges every other mm. Then, 0.5 mm - diameter galvanized fiber was passed through these holes producing a single directional fibered frame.

Then, this frame was sandwiched between two polyethylene laminates and inserted in a mould suitable to its thickness. Thereafter, it was put in a 2.5 MPa pressure under 160°C degrees for 5 minutes, by a press using acetate in between the faces to avoid sticking. Then, it was cooled down to 30°C degrees within 3 minutes under 15 MPa pressure. In this way, a matrix material was produced.

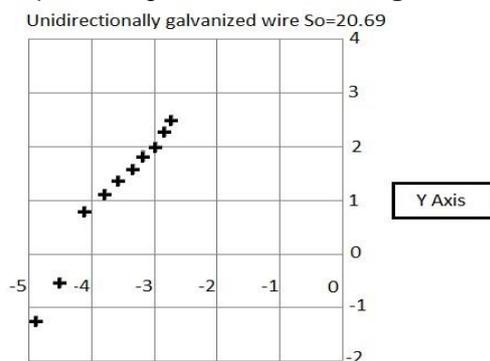
### 3.3.2. Finding $V_m$ and $V_f$

Before building composite material the weight of fiber and matrix materials were measure

**Table 3.1** Volume fractions

	Weight (gr)	Specific weight (gt/cm <sup>3</sup> )	Volume fraction
Fiber	13.764	7.8	0.046
PE(polyethylene)	32.680	0.9	0.953

After specimen preparation, the tensile tests were carried out. From these tests, both tensile strength and  $\sigma$ - $\epsilon$  diagrams were found. (k, n) plastic coefficients were calculated from this diagram by using elastic-plastic region as shown in **Figure 3.1**.



**Figure 3.1.** Diagram of tensile test.

From above diagram, following values were found.

**Table 3.2.** Tensile test of 1-direction.

Num. of tests	$A_0$ (mm <sup>2</sup> )	X(MPa)	K(MPa)	n
20	23.700	21.010	47.183	0.713

Similarly, loading process applied to 2-direction of specimen the results were in the following table:

**Table 3.3.** Tensile test of 2-direction.

Num. of tests	$A_0$ (mm <sup>2</sup> )	Y(MPa)
5	23.50	5.22

Then to find shear stress in 1-2 directions was investigated. The results are given in **Table 3.4**.

**Table 3.4.** Results of shear tests

Num. of tests	$A_0$ (mm <sup>2</sup> )	S(MPa)
5	10.75	5.85

To find  $E_1$ , and  $\nu_{12}$ , two strain gauges bounded on the specimen one of which is parallel to loading direction and the other perpendicular to loading direction. The test results are in **Table 3.5**.

**Table 3.5.**  $E_1$  and  $\nu_{12}$  test results.

P(kg)	10
$A_0$ (mm <sup>2</sup> )	33.75
$\sigma_1$ (MPa)	2.976
$\epsilon_1$	692 E-6
$\epsilon_2$	278 E-6
$E_1$ (MPa)	4300
$\nu_{12}$	0.4

Similarly, to find  $E_2$ , the loading operation was applied to 2-direction and  $E_2$ , was calculated then  $\nu_{12}$  was calculated the stiffness properties should satisfy the reciprocal relations The results are in **Table 3.6**.

**Table 3.6.**  $E_2$  and  $\nu_{12}$  test results

P(kg)	5
$A_0$ (mm <sup>2</sup> )	27.50
$\sigma_1$ (MPa)	18.18
$\epsilon_1$	1899 E-6
$E_1$ (MPa)	957
$\nu_{12}$	0.0898

Shear modulus is found by applying the loading to the x-direction.

**Table 3.7.** Shear modulus test results.

P(kg)	4
$A_0$ (mm <sup>2</sup> )	28.405
$\epsilon_x$	1838.72 E-6
$E_x$ (MPa)	765.86
$G_{12}$ (MPa)	241.48

### 3.4. Plates With a Hole

In this case elastic-plastic stress analysis is carried out in plates with a hole under the in plane loading greatest are found as shown in;

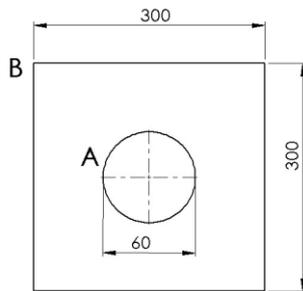


Figure 3.2.

## II. RESULTS AND DISCUSSIONS

### 4.1. Unidirectionally Galvanized Wire Reinforced Laminates

The mechanical properties of the laminated plate are found by using strain-gauges and Instron test machine.

Table 4.1.1. Mechanical properties and yield points of a layer

E <sub>1</sub>	E <sub>2</sub>	G <sub>12</sub>	ν <sub>12</sub>
4300 MPa	957 MPa	242 MPa	0.4

Table 4.1.2. Mechanical Properties

Axial Strength	X	21.01 MPa
Transverse Strength	Y	5.22 MPa
Shear Strength	S	5.85 MPa
Hardening Parameter	K	47.18 MPa
Strain Hardening Parameter	n	0.71

When the laminated plates composed of four layers are loaded transversely, yield points for transverse load are given in Table 4.1.3

Table 4.1.3. The yield points of the laminated plates loaded transversely. (S. Symmetric, A. Antisymmetric)

	(30°/-30°) <sub>2</sub> S and A	(45°/45°) <sub>2</sub> S and A	(60°/-60°) <sub>2</sub> S and A
Yield point MPa	S 0.0438 and A 0.0551	S 0.0438 and A 0.0550	S 0.0396 and A 0.0537

As seen from this table the yield point in the symmetric angle-ply laminated plate is higher than in the antisymmetric angle-ply laminated plate. The yield points in the antisymmetric angle-ply laminated plates are higher than the yield points in angle-ply symmetric laminated plates.

In symmetric angle -ply and angle-ply symmetric laminated plates the maximum stress components are at the upper and lower surfaces, and they are equal absolutely. Therefore, in symmetric laminated plates, stress components are given at the upper surface only at node 85, at the mid-point. But in antisymmetric laminated plates, stress components are given at the upper and lower surfaces, because they are different at these surfaces.

Table 4.1.4. Elastic-plastic, elastic and residual stress components in (30°/-30°)<sub>2</sub> angle-ply symmetric laminated plate for 100 iterations.

	Layer	σ <sub>x</sub> (MPa)	σ <sub>y</sub> (MPa)	T <sub>xy</sub> (MPa)	T <sub>yz</sub> (MPa)	T <sub>xz</sub> (MPa)
Elastic-Plastic Stresses	30°	11.296	6.579	3.821	-0.013	-0.033
	+30°	-11.296	-6.579	-3.821	-0.013	-0.033
Elastic Stresses	30°	11.320	6.606	3.821	0.012	-0.033
	+30°	-11.320	-6.606	-3.821	0.012	-0.033
Residual Stresses	30°	-0.023	-0.027	0.002	-0.001	-0.000
	+30°	0.023	0.027	-0.002	-0.001	-0.000

Table 4.1.5. Elastic-plastic, elastic and residual stress components in (30°/-30°)<sub>2</sub> angle-ply symmetric laminated plate for 150 iterations.

	Layer	σ <sub>x</sub> (MPa)	σ <sub>y</sub> (MPa)	T <sub>xy</sub> (MPa)	T <sub>yz</sub> (MPa)	T <sub>xz</sub> (MPa)
Elastic-Plastic Stresses	30°	11.863	6.651	4.210	-0.023	-0.038
	+30°	-11.863	-6.651	-4.210	-0.023	-0.038
Elastic Stresses	30°	12.372	7.220	4.174	0.013	-0.036
	+30°	-12.372	-7.220	-4.174	-0.013	-0.036
Residual Stresses	30°	-0.508	-0.569	0.036	-0.010	-0.002
	+30°	0.508	0.569	-0.036	-0.010	-0.002

Table 4.1.6. Elastic-plastic, elastic and residual stress components in (30°/-30°)<sub>2</sub> cross-ply symmetric laminated plate for 200 iterations.

	Layer	σ <sub>x</sub> (MPa)	σ <sub>y</sub> (MPa)	T <sub>xy</sub> (MPa)	T <sub>yz</sub> (MPa)	T <sub>xz</sub> (MPa)
Elastic-Plastic Stresses	30°	12.432	6.711	4.612	-0.022	-0.039
	+30°	-12.432	-6.711	-4.612	-0.022	-0.039
Elastic Stresses	30°	13.424	7.834	4.529	-0.014	-0.039
	+30°	-13.424	-7.834	-4.529	-0.014	-0.039
Residual Stresses	30°	-0.991	-1.123	0.083	-0.008	0.001
	+30°	0.991	1.123	-0.083	-0.008	0.001

As seen from these tables, when we increase the iteration numbers the residual stresses become greater.

Table 4.1.7. Elastic-plastic, elastic and residual stress components in (30°/-30°)<sub>2</sub> angle-ply antisymmetric laminated plate at node 85 for 100 iterations.

	Layer	σ <sub>x</sub> (MPa)	σ <sub>y</sub> (MPa)	T <sub>xy</sub> (MPa)	T <sub>yz</sub> (MPa)	T <sub>xz</sub> (MPa)
Elastic-Plastic Stresses	30°	10.563	5.405	3.532	-0.019	-0.039
	+30°	-10.609	-5.426	3.574	-0.017	-0.043
Elastic Stresses	30°	11.455	6.442	3.410	-0.016	-0.040
	+30°	-11.487	-6.452	3.449	-0.016	-0.040
Residual Stresses	30°	-0.892	-1.037	0.122	-0.003	0.001
	+30°	0.878	1.026	-0.126	-0.001	0.004

**Table 4.1.8.** Elastic-plastic, elastic and residual stress components in  $(30^\circ/-30^\circ)_2$  angle-ply antisymmetric laminated plate at node 85 for 150 iterations.

	Layer	$\sigma_x$ (MPa)	$\sigma_y$ (MPa)	$T_{xy}$ (MPa)	$T_{yz}$ (MPa)	$T_{xz}$ (MPa)
Elastic-Plastic Stresses	$30^\circ$	12.424	6.617	4.493	-0.020	-0.045
	$+30^\circ$	-12.473	-6.622	4.541	-0.019	-0.052
Elastic Stresses	$30^\circ$	13.704	8.119	4.279	-0.016	-0.046
	$+30^\circ$	-13.743	-8.158	4.320	-0.016	-0.046
Residual Stresses	$30^\circ$	-1.279	-1.542	0.215	-0.004	0.001
	$+30^\circ$	1.269	1.536	0.221	-0.003	-0.006

As seen from these tables, residual stress components in symmetric laminated plates are less than the stresses in antisymmetric laminated plates.

**Table 4.1.9.** Elastic-plastic, elastic and residual stress components in  $(30^\circ/-30^\circ)_2$  angle-ply antisymmetric laminated plate at node 85 for 200 iterations

	Layer	$\sigma_x$ (MPa)	$\sigma_y$ (MPa)	$T_{xy}$ (MPa)	$T_{yz}$ (MPa)	$T_{xz}$ (MPa)
Elastic-Plastic Stresses	$30^\circ$	12.981	6.660	4.888	-0.029	-0.052
	$+30^\circ$	-13.037	-6.665	4.945	-0.024	-0.062
Elastic Stresses	$30^\circ$	14.682	8.742	4.584	-0.017	-0.049
	$+30^\circ$	-14.724	-8.741	4.629	-0.017	-0.049
Residual Stresses	$30^\circ$	-1.702	-2.082	0.304	-0.011	-0.003
	$+30^\circ$	1.688	2.076	0.316	-0.007	-0.013

When we increase the orientation angles in angle-ply laminated plates, the residual stress components become smaller.

**Table 4.1.10.** Elastic-plastic, elastic and residual stress components in  $(45^\circ/-45^\circ)_2$  angle-ply symmetric laminated plate for 100 iterations

	Layer	$\sigma_x$ MPa	$\sigma_y$ MPa	$T_{xy}$ MPa	$T_{yz}$ MPa	$T_{xz}$ MPa
Elastic-Plastic Stresses	$45^\circ$	6,965	6,965	3.343	-0.020	-0.020
	$+45^\circ$	-6,965	-6,965	-3.343	-0.020	-0.020
Elastic Stresses	$45^\circ$	6,960	6.660	3.343	0.020	-0.020
	$+45^\circ$	-6,960	-6.660	-3.343	-0.020	-0.020
Residual Stresses	$45^\circ$	0.005	0.005	-0.001	-0.000	-0.000
	$+45^\circ$	-0.005	-0.005	-0.001	-0.000	-0.000

**Table 4.1.11.** Elastic-plastic, elastic and residual stress components in  $(45^\circ/-45^\circ)_2$  angle-ply symmetric laminated plate for 150 iterations

	Layer	$\sigma_x$ (MPa)	$\sigma_y$ (MPa)	$T_{xy}$ (MPa)	$T_{yz}$ (MPa)	$T_{xz}$ (MPa)
Elastic-Plastic Stresses	$45^\circ$	7,485	7,485	3,725	-0.026	-0.026
	$+45^\circ$	-7,485	-7,485	-3,725	-0.026	-0.026
Elastic Stresses	$45^\circ$	7,661	7.661	3,680	-0.022	-0.022
	$+45^\circ$	-7,661	-7.661	-3,680	-0.022	-0.022
Residual Stresses	$45^\circ$	-0.176	-0.176	0.045	-0.004	-0.004
	$+45^\circ$	0.176	0.176	-0.045	-0.004	-0.004

**Table 4.1.12.** Elastic-plastic, elastic and residual stress components in  $(45^\circ/-45^\circ)_2$  cross-ply symmetric laminated plate for 200 iterations.

	Layer	$\sigma_x$ (MPa)	$\sigma_y$ (MPa)	$T_{xy}$ (MPa)	$T_{yz}$ (MPa)	$T_{xz}$ (MPa)
Elastic-Plastic Stresses	$45^\circ$	7,825	7,824	4.148	-0.028	-0.028
	$+45^\circ$	-7,825	-7,824	-4.148	-0.028	-0.028
Elastic Stresses	$45^\circ$	8,363	8,363	4.017	-0.024	-0.024
	$+45^\circ$	-8,363	-8,363	-4.017	-0.024	-0.024
Residual Stresses	$45^\circ$	-0.538	-0,539	0.131	-0.004	-0.004
	$+45^\circ$	0,538	0,539	-0.131	-0.004	-0.004

As seen from these tables, when we increase the iteration numbers the residual stresses become greater.

**Table 4.1.13.** Elastic-plastic, elastic and residual stress components in  $(45^\circ/-45^\circ)_2$  angle-ply antisymmetric laminated plate at node 85 for 100 iterations.

	Layer	$\sigma_x$ (MPa)	$\sigma_y$ (MPa)	$T_{xy}$ (MPa)	$T_{yz}$ (MPa)	$T_{xz}$ (MPa)
Elastic-Plastic Stresses	$45^\circ$	7,635	7,633	3.908	-0.028	-0.028
	$+45^\circ$	-7,665	-7,662	3,952	-0.030	-0.030
Elastic Stresses	$45^\circ$	8,285	8,285	3.693	-0.026	-0.026
	$+45^\circ$	-8,309	-8,308	3.734	-0.026	-0.026
Residual Stresses	$45^\circ$	-0,650	-0,652	0.215	-0.002	-0.002
	$+45^\circ$	0.645	1.647	0,218	-0.004	-0.004

**Table 4.1.14.** Elastic-plastic, elastic and residual stress components in  $(45^\circ/-45^\circ)_2$  angle-ply antisymmetric laminated plate at node 85 for 150 iterations.

	Layer	$\sigma_x$ MPa	$\sigma_y$ MPa	$T_{xy}$ MPa	$T_{yz}$ MPa	$T_{xz}$ MPa
Elastic-Plastic Stresses	45°	7,960	7,958	4.318	-0.029	-0.029
	+45°	-7,992	-7,988	4,366	-0.031	-0.031
Elastic Stresses	45°	8,935	8,936	3,983	-0.028	-0.028
	+45°	-8,962	-8,961	4.027	-0.028	-0.028
Residual Stresses	45°	-0,975	-0,978	0.335	-0.001	-0.001
	+45°	0,970	0,972	0338	-0.003	-0.003

As seen from these tables, residual stress components in symmetric laminated plates are less than the stresses in antisymmetric laminated plates

**Table 4.1.15.** Elastic-plastic, elastic and residual stress components in  $(45^\circ/-45^\circ)_2$  angle-ply antisymmetric laminated plate at node 85 for 200 iterations.

	Layer	$\sigma_x$ MPa	$\sigma_y$ MPa	$T_{xy}$ MPa	$T_{yz}$ MPa	$T_{xz}$ MPa
Elastic-Plastic Stresses	45°	8,298	8,295	4.775	-0.034	-0.034
	+45°	-8,336	-8,332	4.813	-0.036	-0.036
Elastic Stresses	45°	9,586	9,586	4.273	-0.030	-0.030
	+45°	-9,614	-9,613	4.320	-0.030	-0.030
Residual Stresses	45°	-1,287	-1,291	0,483	-0.004	-0.004
	+45°	1,278	1,281	0,492	-0.006	-0.006

	Layer	$\sigma_x$ MPa	$\sigma_y$ MPa	$T_{xy}$ MPa	$T_{yz}$ MPa	$T_{xz}$ MPa
Elastic-Plastic Stresses	60°	6,610	12,402	4.472	-0.044	-0.02
	+60°	-6,625	-12,452	4,520	-0.050	-0.02
Elastic Stresses	60°	8,148	13,679	4,255	-0.045	-0.016
	+60°	-8,159	-13,719	4.296	-0.045	-0.016
Residual Stresses	60°	-1,538	-1,277	0.217	-0.001	-0.03
	+60°	1,534	1,268	0,224	-0.005	-0.01

When we increase the orientation angles in angle-ply laminated plates, the residual stress components become smaller. Residual stress components in angle-ply laminated plates are greater than in angle-ply laminated plates because in angle-ply laminated plates mechanical properties are more different than angle-ply laminated plates.

**Table 4.1.16.** Elastic-plastic, elastic and residual stress components in  $(60^\circ/-60^\circ)_2$  angle-ply symmetric laminated plate for 100 iterations.

	Layer	$\sigma_x$ MPa	$\sigma_y$ MPa	$T_{xy}$ MPa	$T_{yz}$ MPa	$T_{xz}$ MPa
Elastic-Plastic Stresses	60°	6,579	11,296	3.821	-0.033	-0.013
	+60°	-6,579	-11,296	-3.821	-0.033	-0.013
Elastic Stresses	60°	6,606	11,320	3.819	0.033	-0.012
	+60°	-6,606	-11,320	-3.819	-0.033	-0.012
Residual Stresses	60°	-0.027	-0.023	-0.002	0.000	-0.001
	+60°	0.027	0.023	-0.002	0.000	-0.001

**Table 4.1.17.** Elastic-plastic, elastic and residual stress components in  $(60^\circ/-60^\circ)_2$  angle-ply symmetric laminated plate for 150 iterations.

	Layer	$\sigma_x$ MPa	$\sigma_y$ MPa	$T_{xy}$ MPa	$T_{yz}$ MPa	$T_{xz}$ MPa
Elastic-Plastic Stresses	60°	41,110	41,047	-0,063	-0.176	-0.152
	+60°	-41,110	-41,047	0,063	-0.176	-0.192
Elastic Stresses	60°	43,897	43,829	-0,065	-0.150	-0.110
	+60°	-43,897	-43,829	0,065	-0.150	-0.110
Residual Stresses	60°	-2,787	-2,782	0.002	-0.027	-0.082
	+60°	2,787	2,782	-0.002	0.027	-0.082

**Table 4.1.18.** Elastic-plastic, elastic and residual stress components in  $(60^\circ/-60^\circ)_2$  angle-ply symmetric laminated plate for 200 iterations.

**Table 4.1.19.** Elastic-plastic, elastic and residual stress components in  $(60^\circ/-60^\circ)_2$  angle-ply antisymmetric laminated plate at node 85 for 100 iterations.

	Layer	$\sigma_x$ (MPa)	$\sigma_y$ (MPa)	$T_{xy}$ (MPa)	$T_{yz}$ (MPa)	$T_{xz}$ (MPa)
Elastic-Plastic Stresses	60°	6,712	12,431	4.612	-0.039	-0.022
	+60°	-6,712	-12,431	-4.612	-0.039	-0.022
Elastic Stresses	60°	7,834	13,424	4.529	-0.039	-0.014
	+60°	-7,834	-13,424	-4.529	-0.039	-0.014
Residual Stresses	60°	-1,121	-0,993	0.083	-0.000	-0.008
	+60°	1,121	0,993	-0.083	-0.000	-0.008

As seen from these tables, when we increase the iteration numbers the residual stresses become greater.

**Table 4.1.20.** Elastic-plastic, elastic and residual stress components in  $(60^\circ/-60^\circ)_2$  angle-ply antisymmetric laminated plate at node 85 for 150 iterations.

	Layer	$\sigma_x$ (MPa)	$\sigma_y$ (MPa)	$T_{xy}$ (MPa)	$T_{yz}$ (MPa)	$T_{xz}$ (MPa)
Elastic-Plastic Stresses	60°	6,554	11,864	4,099	-0.042	-0.020
	+60°	-6,554	-11,864	4,099	-0.047	-0.018
Elastic Stresses	60°	7,567	12,703	3,951	-0.041	-0.015
	+60°	-7,567	-12,703	3,989	-0.041	-0.015
Residual Stresses	60°	-1,014	-0,840	0.148	-0.001	-0.006
	+60°	1,014	0,840	0.153	-0.006	-0.003

As seen from these tables, residual stress components in symmetric laminated plates are less than the stresses in antisymmetric laminated plates.

	Layer	$\sigma_x$ (MPa)	$\sigma_y$ (MPa)	$T_{xy}$ (MPa)	$T_{yz}$ (MPa)	$T_{xz}$ (MPa)
Elastic-Plastic Stresses	60°	6,654	12,961	4.868	-0.051	-0.028
	+60°	-6,670	-13,017	4.924	-0.060	-0.023
Elastic Stresses	60°	8,729	14,655	4.558	-0.048	-0.017
	+60°	-8,741	-14,698	4.602	-0.048	-0.017
Residual Stresses	60°	-2,075	-1,694	0,310	-0.003	-0.011
	+60°	2,071	1,681	0.322	-0.012	-0.006

**Table 4.1.21.** Elastic-plastic, elastic and residual stress components in  $(60^\circ/-60^\circ)_2$  angle-ply antisymmetric laminated plate at node 85 for 200 iterations

When we increase the orientation angles in angle-ply laminated plates, the residual stress components become smaller. Residual stress components in angle-ply laminated plates are greater than in angle-ply laminated plates because in angle-ply laminated plates mechanical properties are more different than angle-ply laminated plates.

**Table 4.1.22.** The residual stresses in the angle-ply  $(30^\circ/-30^\circ)_2$ ,  $(45^\circ/-45^\circ)_2$  and  $(60^\circ/-60^\circ)_2$  symmetric laminated plates at node 85 for 100, 150 and 200 iterations

Iteration Numbers	Iteration Number	$\sigma_x$ MPa	$\sigma_y$ MPa	$T_x$ MPa	$T_{yz}$ MPa	$T_{xz}$ MPa
100	30°	-0,023	-0,027	0,002	-0.001	-0.001
	45°	0,005	0,005	-0,001	-0.000	-0.000
	60°	-0,027	-0,023	0,002	-0.000	-0.001
150	30°	-0,508	-0,57	0,036	-0.010	-0.002
	45°	-0,176	-0,176	0,045	-0.004	-0.004
	60°	-0,569	-0,57	0,036	-0.002	-0.010
200	30°	-0,991	-1.123	0,083	-0.008	-0.001
	45°	-0,538	-0,539	0.131	-0.004	-0.004
	60°	-1.121	-0,99	0,083	-0.000	-0.008

**Table 4.1.23.** The residual stresses in the angle-ply

$(30^\circ/-30^\circ)_2$ ,  $(45^\circ/-45^\circ)_2$  and  $(60^\circ/-60^\circ)_2$  antisymmetric laminated plates at node 85 for 100, 150 and 200 iterations.

Stresses	Iteration Number	$\sigma_x$ MPa	$\sigma_y$ MPa	$T_x$ MPa	$T_{yz}$ MPa	$T_{xz}$ MPa
Elastic-Plastic Stresses	100	7,635	7,633	3.908	-0.028	-0.028
	150	7,96	7,958	4.318	-0.029	-0.029
	200	8,29	8,295	4.755	-0.034	-0.034
Elastic Stresses	100	8,285	8,285	3,693	-0.026	-0.026
	150	8,935	8,936	3,983	-0.028	-0.028
	200	9,536	9,586	4,273	-0.030	-0.030
Residual Stresses	100	-0,65	-0,652	0.215	0.002	0.002
	150	-0,97	-0,978	0.335	-0.001	-0.001
	200	-1.29	-1,291	0.483	-0.006	-0.004

As seen from these Tables, residual stresses in  $(30^\circ/-30^\circ)_2$  and  $(60^\circ/-60^\circ)_2$  angle-ply laminated plates are higher than in  $(45^\circ/-45^\circ)_2$  angle-ply laminated plate for the same iterations.

**Table 4.1.24.** Elastic-plastic, elastic and residual stress components for symmetric angle-ply laminated plate  $(30^\circ/-30^\circ)_2$  at node 85 for 100, 150 and 200 iterations

Stresses	Iteration Number	$\sigma_x$ MPa	$\sigma_y$ MPa	$T_x$ MPa	$T_{yz}$ MPa	$T_{xz}$ MPa
Elastic-Plastic Stresses	100	11.296	6.579	3.821	-0.013	-0.033
	150	11.863	6.651	4.210	-0.023	-0.038
	200	12.432	6.711	4.612	-0.022	-0.039
Elastic Stresses	100	11.32	5.606	3.819	-0.012	-0.033
	150	12.372	7.220	4.174	-0.013	-0.036
	200	13.424	7.834	4.529	-0.014	-0.039
Residual Stresses	100	-0.023	-0.027	0.002	-0.001	-0.000
	150	-0.508	-0.569	0.036	-0.010	-0.002
	200	-0.991	-1.123	0.083	-0.008	-0.001

**Table 4.1.25.** Elastic-plastic, elastic and residual stress components for antisymmetric angle-ply laminated plate  $(30^\circ/-30^\circ)_2$  at node 85 for 100, 150 and 200 iterations.

Iteration Numbers	Iteration Number	$\sigma_x$ MPa	$\sigma_y$ MPa	$T_x$ MPa	$T_{yz}$ MPa	$T_{xz}$ MPa
100	30°	-0,892	-1,04	0,122	-0.003	-0.001
	45°	-0,650	-0,66	0,215	-0.002	-0.002
	60°	-1,014	-0,84	0,148	-0.001	-0.006
150	30°	-1,279	-1,54	0,215	-0.004	-0.001
	45°	-0,975	-0,98	0,335	-0.001	-0.001
	60°	-1,538	-1,28	0,217	-0.001	-0.004
200	30°	-1,702	-2,08	0,304	-0.011	-0.003
	45°	-1,287	-1,29	0.483	-0.004	-0.004
	60°	-2,075	-1,69	0,310	-0.003	-0.011

**Table 4.1.26.** Elastic-plastic, elastic and residual stress components for antisymmetric angle-ply laminated plate  $(30^\circ/-30^\circ)_2$  at node 85 for 100, 150 and 200 iterations.

Stresses	Iteration Number	$\sigma_x$ MPa	$\sigma_y$ MPa	$T_x$ MPa	$T_{yz}$ MPa	$T_{xz}$ MPa
Elastic-Plastic Stresses	100	10.563	5.045	3.532	-0.019	-0.039
	150	12.424	6.617	4.493	-0.020	-0.045
	200	12.981	6.660	4.888	-0.029	-0.052
Elastic Stresses	100	11.455	6.642	3.410	-0.016	-0.040
	150	13.704	8.159	4.174	-0.013	-0.036
	200	14.682	8.742	4.584	-0.017	-0.049
Residual Stresses	100	-0.892	-1.037	0.112	-0.003	-0.001
	150	-1.279	-1.54	0.215	-0.004	-0.001
	200	-1.702	-2.08	0.304	-0.011	-0.003

**Table 4.1.27.** Elastic-plastic, elastic and residual stress components for symmetric angle-ply laminated plate  $(45^\circ/-45^\circ)_2$  at node 85 for 100, 150 and 200 iterations.

Stresses	Iteration Number	$\sigma_x$ MPa	$\sigma_y$ MPa	$T_x$ MPa	$T_{yz}$ MPa	$T_{xz}$ MPa
Elastic-Plastic Stresses	100	6,965	6.965	3,343	-0.020	-0.020
	150	7,485	7,485	3,725	-0.026	-0.026
	200	7,825	7,824	4,148	-0.028	-0.028
Elastic Stresses	100	6,960	6,960	3,343	-0.020	-0.020
	150	7,661	7,661	3,680	-0.022	-0.022
	200	8,363	8,363	4,017	-0.024	-0.024
Residual Stresses	100	0.005	0.005	-0.001	0.000	0.000
	150	-0.176	-0.176	0.045	-0.004	-0.004
	200	-0.538	-0,534	0.131	-0.004	-0.004

**Table 4.1.28.** Elastic-plastic, elastic and residual stress components for antisymmetric angle-ply laminated plate  $(45^\circ/-45^\circ)_2$  at node 85 for 100, 150 and 200 iterations.

Stresses	Iteration Number	$\sigma_x$ MPa	$\sigma_y$ MPa	$T_x$ MPa	$T_{yz}$ MPa	$T_{xz}$ MPa
Elastic-Plastic Stresses	100	7,635	7,633	3,908	-0.028	-0.028
	150	7,960	7,958	4,318	-0.029	-0.029
	200	8,298	8,295	4,755	-0.034	-0.034
Elastic Stresses	100	8,285	8,285	3,693	-0.026	-0.026
	150	8,935	8,936	3,983	-0.028	-0.028
	200	9,536	9,586	4,273	-0.030	-0.030
Residual Stresses	100	-0,65	-0,652	0,215	0,002	0,002
	150	-0,97	-0,978	0,335	-0,001	-0,001
	200	-1,29	-1,291	0,483	-0,006	-0,004

**Table 4.1.29.** Elastic-plastic, elastic and residual stress components for symmetric angle-ply laminated plate  $(60^\circ/-60^\circ)_2$  at node 85 for 100, 150 and 200 iterations.

Stresses	Iteration Number	$\sigma_x$ MPa	$\sigma_y$ MPa	$T_x$ MPa	$T_{yz}$ MPa	$T_{xz}$ MPa
Elastic-Plastic Stresses	100	6,579	11,296	3,821	-0.033	-0.013
	150	6,651	11,863	4,211	-0.038	-0.023
	200	6,712	12,431	4,612	-0.039	-0.022
Elastic Stresses	100	6,606	11,320	3,819	-0.033	-0.012
	150	7,220	12,372	4,174	-0.036	-0.013
	200	7,834	13,424	4,529	-0.039	-0.014
Residual Stresses	100	0.027	-0.023	0.002	0.000	-0.001
	150	-0.569	-0.569	0.036	-0.002	-0.010
	200	1,121	0,993	0,083	-0,000	-0,008

**Table 4.1.30.** Elastic-plastic, elastic and residual stress components for antisymmetric angle-ply laminated plate  $(60^\circ/-60^\circ)_2$  at node 85 for 100, 150 and 200 iterations.

Stresses	Iteration Number	$\sigma_x$ MPa	$\sigma_y$ MPa	$T_x$ MPa	$T_{yz}$ MPa	$T_{xz}$ MPa
Elastic-Plastic Stresses	100	6,554	11,864	4,099	-0.042	-0.030
	150	6,610	12,402	4,472	-0.044	-0.020
	200	6,654	12,961	4,868	-0.051	-0.028
Elastic Stresses	100	7,567	12,703	3,951	-0.041	-0.015
	150	8,148	13,679	4,255	-0.045	-0.016
	200	8,729	14,655	4,558	-0.048	-0.017
Residual Stresses	100	-1,014	-0,840	-0,148	-0,001	0,006
	150	-1,538	-1,277	0,217	-0,001	-0,004
	200	-2,075	-1,694	0,310	-0,003	-0,011

As seen from these Tables, residual stresses in antisymmetric angle-ply  $(30^\circ/-30^\circ)_2$ ,  $(45^\circ/-45^\circ)_2$ ,  $(60^\circ/-60^\circ)_2$  higher than in symmetric  $(30^\circ/-30^\circ)_2$ ,  $(45^\circ/-45^\circ)_2$ ,  $(60^\circ/-60^\circ)_2$  angle-ply laminated plates for the same iterations. Because in antisymmetric laminated case whose mechanical properties and layer orientations are more different from in symmetric case. The differency in mechanical properties, layer orientations and in the structure of the plate produces the higher residual stresses.

**Table 4.1.31.** Elastic-plastic stress components at upper and lower surfaces of the symmetric laminated plates for 100 iterations.

	Laminated Plates	$\sigma_x$ MPa	$\sigma_y$ MPa	$\tau_{xy}$ MPa	$\tau_{yz}$ MPa	$\tau_{xz}$ MPa
Upper surface	$(30^\circ/-30^\circ)_2$	11,296	6,579	3,821	-0,013	-0,033
	$(45^\circ/-45^\circ)_2$	6,965	6,965	3,343	-0,02	-0,02
	$(60^\circ/-60^\circ)_2$	6,58	11,3	3,82	-0,03	-0,03
Lower surface	$(30^\circ/-30^\circ)_2$	-11,29	-6,579	-3,82	-0,013	-0,033
	$(45^\circ/-45^\circ)_2$	-6,965	-6,965	-3,34	-0,02	-0,02
	$(60^\circ/-60^\circ)_2$	-6,58	-11,3	-3,82	-0,03	-0,03

**Table 4.1.32.** Elastic-plastic stress components at upper and lower surfaces of the antisymmetric laminated plates for 100 iterations.

	Laminated Plates	$\sigma_x$ MPa	$\sigma_y$ MPa	$\tau_{xy}$ MPa	$\tau_{yz}$ MPa	$\tau_{xz}$ MPa
Upper surface	(30°/-30°) <sub>2</sub>	10,563	5,405	3,532	-0,019	-0,039
	(45°/-45°) <sub>2</sub>	7,635	7,633	3,908	-0,028	-0,028
	(60°/-60°) <sub>2</sub>	6,554	11,864	4,099	-0,042	-0,030
Lower surface	(30°/-30°) <sub>2</sub>	-10,609	-5,426	3,574	-0,017	-0,043
	(45°/-45°) <sub>2</sub>	-7,665	-7,662	3,952	-0,080	-0,030
	(60°/-60°) <sub>2</sub>	-6,568	-11,911	4,142	-0,047	-0,018

**Table 4.1.33.** Elastic-plastic stress components at upper and lower surfaces of the symmetric laminated plates for 150 iterations.

	Laminated Plates	$\sigma_x$ MPa	$\sigma_y$ MPa	$\tau_{xy}$ MPa	$\tau_{yz}$ MPa	$\tau_{xz}$ MPa
Upper surface	(30°/-30°) <sub>2</sub>	11,863	6,651	4,210	-0,023	-0,038
	(45°/-45°) <sub>2</sub>	7,485	7,485	3,725	-0,026	-0,026
	(60°/-60°) <sub>2</sub>	6,651	11,863	4,211	-0,038	-0,023
Lower surface	(30°/-30°) <sub>2</sub>	-11,863	-6,651	-4,210	-0,023	-0,038
	(45°/-45°) <sub>2</sub>	-7,485	-7,485	-3,725	-0,026	-0,026
	(60°/-60°) <sub>2</sub>	-6,651	-11,863	-4,211	-0,038	-0,023

**Table 4.1.34.** Elastic-plastic stress components at upper and lower surfaces of the antisymmetric laminated plates for 150 iterations.

	Laminated Plates	$\sigma_x$ MPa	$\sigma_y$ MPa	$\tau_{xy}$ MPa	$\tau_{yz}$ MPa	$\tau_{xz}$ MPa
Upper surface	(30°/-30°) <sub>2</sub>	12,424	6,617	4,493	-0,020	-0,045
	(45°/-45°) <sub>2</sub>	7,960	7,958	4,318	-0,029	-0,029
	(60°/-60°) <sub>2</sub>	6,610	12,402	4,472	-0,044	-0,020
Lower surface	(30°/-30°) <sub>2</sub>	-12,473	-6,622	4,493	-0,019	-0,052
	(45°/-45°) <sub>2</sub>	-7,992	-7,988	4,318	-0,031	-0,031
	(60°/-60°) <sub>2</sub>	-6,625	-12,452	-4,472	-0,050	-0,017

**Table 4.1.35.** Elastic-plastic stress components at upper and lower surfaces of the symmetric laminated plates for 200 iterations.

	Laminated Plates	$\sigma_x$ MPa	$\sigma_y$ MPa	$\tau_{xy}$ MPa	$\tau_{yz}$ MPa	$\tau_{xz}$ MPa
Upper surface	(30°/-30°) <sub>2</sub>	12,432	6,711	4,612	-0,022	-0,039
	(45°/-45°) <sub>2</sub>	7,825	7,824	4,148	-0,028	-0,028
	(60°/-60°) <sub>2</sub>	6,712	12,431	4,612	-0,039	-0,022
Lower surface	(30°/-30°) <sub>2</sub>	-12,432	-6,711	-4,612	-0,022	-0,039
	(45°/-45°) <sub>2</sub>	-7,825	-7,824	-4,148	-0,028	-0,028
	(60°/-60°) <sub>2</sub>	-6,712	-12,431	-4,612	-0,039	-0,022

**Table 4.1.36.** Elastic-plastic stress components at upper and lower surfaces of the antisymmetric laminated plates for 200 iterations.

	Laminated Plates	$\sigma_x$ MPa	$\sigma_y$ MPa	$\tau_{xy}$ MPa	$\tau_{yz}$ MPa	$\tau_{xz}$ MPa
Upper surface	(30°/-30°) <sub>2</sub>	12,981	6,660	4,888	-0,029	-0,052
	(45°/-45°) <sub>2</sub>	8,298	8,295	4,755	-0,034	-0,034
	(60°/-60°) <sub>2</sub>	6,654	12,961	4,868	-0,051	-0,028
Lower surface	(30°/-30°) <sub>2</sub>	-13,037	-6,665	4,965	-0,024	-0,062
	(45°/-45°) <sub>2</sub>	-8,336	-8,323	4,813	-0,036	-0,036
	(60°/-60°) <sub>2</sub>	-6,670	-13,017	4,924	-0,060	-0,023

**Table 4.1.37.** Residual stress components at upper and lower surfaces of the symmetric laminated plates for 100 iterations

	Laminated Plates	$\sigma_x$ MPa	$\sigma_y$ MPa	$\tau_{xy}$ MPa	$\tau_{yz}$ MPa	$\tau_{xz}$ MPa
Upper surface	(30°/-30°) <sub>2</sub>	-0,023	-0,027	0,002	-0,001	-0,000
	(45°/-45°) <sub>2</sub>	-0,005	-0,005	-0,001	-0,000	-0,000
	(60°/-60°) <sub>2</sub>	-0,027	-0,023	0,002	-0,000	-0,001
Lower surface	(30°/-30°) <sub>2</sub>	1,412	0,027	-0,002	-0,001	-0,001
	(45°/-45°) <sub>2</sub>	0,023	-0,005	0,001	-0,000	-0,000
	(60°/-60°) <sub>2</sub>	0,027	0,023	-0,002	-0,000	-0,001

The absolute value of the stress components in the symmetric laminated plates at the upper and lower surfaces are the same for  $\sigma_x$ ,  $\sigma_y$  and  $\tau_{xy}$  stress components, it is different at the lower and upper surfaces in the antisymmetric laminated plates.

**Table 4.1.38.** Residual stress components at upper and lower surfaces of the antisymmetric laminated plates for 100 iterations.

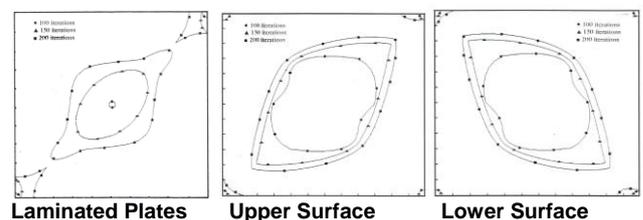
	Laminated Plates	$\sigma_x$ MPa	$\sigma_y$ MPa	$\tau_{xy}$ MPa	$\tau_{yz}$ MPa	$\tau_{xz}$ MPa
Upper surface	(30°/-30°) <sub>2</sub>	-0,991	-1,123	0,085	-0,008	0,001
	(45°/-45°) <sub>2</sub>	-0,538	-0,539	0,131	-0,004	-0,004
	(60°/-60°) <sub>2</sub>	-1,121	-0,993	0,083	0,000	-0,008
Lower surface	(30°/-30°) <sub>2</sub>	0,991	1,123	-0,085	-0,008	-0,001
	(45°/-45°) <sub>2</sub>	0,538	0,539	-0,131	-0,004	-0,004
	(60°/-60°) <sub>2</sub>	1,121	0,993	-0,083	0,000	-0,008

**Table 4.1.39.** Residual stress components at upper and lower surfaces of the symmetric laminated plates for 200 iterations.

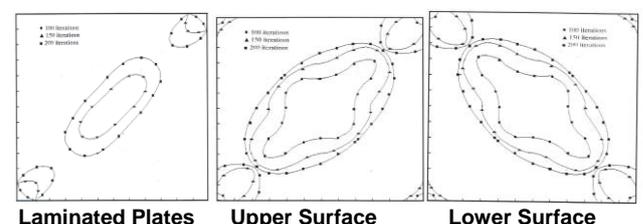
	Laminated Plates	$\sigma_x$ MPa	$\sigma_y$ MPa	$\tau_{xy}$ MPa	$\tau_{yz}$ MPa	$\tau_{xz}$ MPa
Upper surface	(30°/-30°) <sub>2</sub>	-1,702	-2,082	-0,304	-0,011	-0,003
	(45°/-45°) <sub>2</sub>	-1,287	-1,291	0,483	-0,004	-0,004
	(60°/-60°) <sub>2</sub>	-2,075	-1,694	0,310	-0,003	-0,011
Lower surface	(30°/-30°) <sub>2</sub>	1,688	2,076	0,316	-0,007	-0,013
	(45°/-45°) <sub>2</sub>	1,278	1,281	0,492	-0,006	-0,006
	(60°/-60°) <sub>2</sub>	2,071	1,681	0,322	-0,012	-0,006

## I. FIGURES

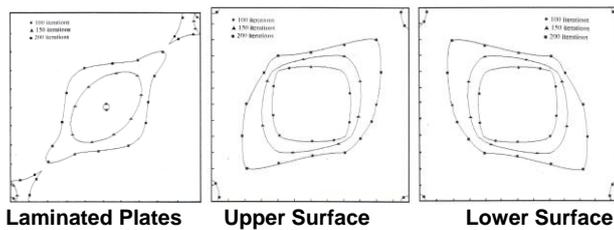
**Figure 5.1.** The expansion of plastic zone in (30°/-30°)<sub>2</sub> symmetric laminated plates;



**Figure 5.2.** The expansion of plastic zone in symmetric, (45°/-45°), laminated plates.



**Figure 5.3.** The expansion of plastic zone in symmetric,  $(60^\circ/60^\circ)_2$  laminated plates.



It is seen that the expansion of plastic Zone is the same at the upper and lower surfaces of the symmetric angle-ply laminated plates. It spreads in different directions at the upper and lower surfaces of the antisymmetric angle-ply laminated plates. Also, plastic zone spreads at the corners of the plates because of the shear stresses in angle-ply laminated plates.

## II. CONCLUSIONS

Elastic-plastic stress analysis has been carried out by using the first order shear deformation theory in thermoplastic composite laminated plates under transverse loading. The expansion of plastic zone and residual stresses are obtained in symmetric and antisymmetric angle-ply laminated plates. The following results of elastic-plastic stress analysis are found.

Unidirectionally Galvanized Wire Reinforced Composite Laminated Plates:

a) The yield points in the antisymmetric angle-ply laminated plates are higher than the yield points in angle-ply symmetric laminated plates.

b) When we increase the orientation angles in angle-ply laminated plates, the residual stress components become smaller.

c) The residual stresses in,  $(30^\circ/30^\circ)_2$  and  $(60^\circ/60^\circ)_2$ , angle-ply laminated plates are higher than in,  $(45^\circ/45^\circ)_2$ , angle-ply laminated plate for the same iterations.

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