Parabola Model with the Application to the Single-Point Mooring System

Chunlei Sun, Yuheng Rong, Xin Wang, Mengjiao Tian
Faculty of Science
Jiangsu University, Jiangsu 212013
Zhenjiang, China
1841590798@qq.com

***Abstract*—Set in the 2016 CUMCA Problem A, from the two aspects of marine environmental load and system's parameters, the study describes the design of single-point mooring system. It is mainly divided into three parts, including the upper buoy, the chain and the lower anchor. Also, the chain line is simplified as a quadratic parabola. Furthermore, statics model where there are global mechanic analysis and local force analysis is set up.**

Keywords—Single-point mooring system; Parabola; Static equilibrium

I. INTRODUCTION

Since developed by the U.S. navy in the 1940s, mooring system has been widely used in ocean observation, engineering, aquaculture and other fields. A single-point mooring system, consisting of a cylindrical buoy, 4 cylindrical steel pipes, a cylindrical steel drum, a heavy ball, a chain and a special anchor [1]. The cylindrical buoy is 2 meters wide, 2 meters high and weighs 1000kg. Every pipe is 0.5 meters wide, 1 meters long and weighs 10kg. There is a underwater acoustic communication set in the steel drum that is 1 meters high and 0.3 meters wide. They weigh 100kg totally. The model and parameters of the chain are shown in TABLEI. It provides horizontal recovery force, under the action of external force in the environment, to keep the mooring floating body positioning. In fact, it has a series of advantages of simple structure, good direction and low cost, so it's one of the most common ways of positioning the field of military and civilian in China.

TABLE I. THE MODEL AND PARAMETERS OF THE CHAIN

Model	***The mass of the unit length(n(kg/m))***
I	3.2
II	7
III	12.5
IV	19.5
V	28.12

In the process of designing a mooring system, the following matters should be paid attention to. Firstly, the tilt angle of the tangent at the link of the anchor and chain does not exceed 16 degrees, otherwise the anchor will be dragged, causing the node to shift. Secondly, when the steel drum is vertical, the operation of the underwater acoustic communication set in it is the best. If the drum's tilt angle is more than 5 degrees, the equipment works poorly. To reduce the angle, a heavy ball will hang at the link of the drum and chain. It is ultimately required that we determine the model, length of the chain and the weight of the heavy ball to make the draft depth of the buoy and steel drum's tilt angle as small as possible.

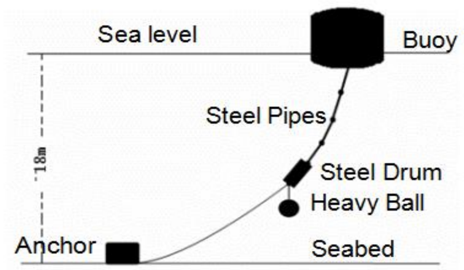

Fig. 1. *The single point mooring system*

The remainder of this paper is organized as follows. Section II focuses on static analysis of the mooring system. Section III shows system parameters at two wind speeds of 12m/s2 and 24m/s2. Section IV describes the change of the heavy ball's weight when wind speed increases. Section V designs the optimal solution to the system in a certain sea area and sensitivity analysis in this case. Conclusion and prospect are presented in Section VI.

II. Static Analysis [2]

A. The analysis of forces applied on the buoy

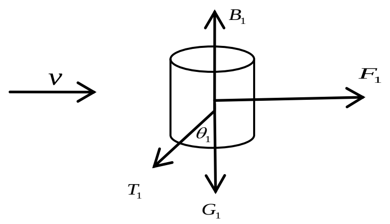

Fig. 2. *The analysis of forces applied on the buoy*

The buoyancy applied on the buoy is

$$B_1 = \rho g \pi (\frac{D_1}{2})^2 h = \frac{\pi}{4} D_1^2 \rho g h \qquad (1)$$

where ρ is the density of seawater, D_1 is the cylindrical buoy's diameter, h is its draft depth.

From sea wind load F=0.625×Sv^2(N), where S is the projection area of the plane of the wind speed, v is wind speed, and water flow F=374×Sv^2(N), where S is the projection area of the plane of the water speed, v is flow rate, we can get that the marine environmental load on it is

$$F_1 = 0.625 \cdot D_1 (l_1 - h) v_1^2 + 374 \cdot D_1 h v_2^2 \qquad (2)$$

where v_1 is wind speed, v_2 is flow rate, l_1 is the cylindrical buoy's height.

The buoy remains in a state of rest, so its static equilibrium equations are

$$B_1 = G_1 + T_1 \cos\theta_1 \qquad (3)$$

$$F_1 = T_1 \sin\theta_1 \qquad (4)$$

Where G_1 is its gravity, T_1 is the tension at the top of the first pipe, θ_1 is the tilt angle of the buoy.

B. The analysis of forces applied on pipes

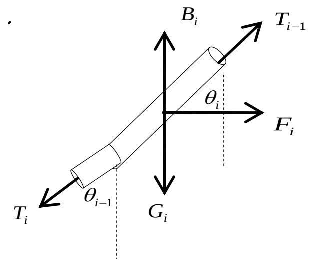

Fig.3. *The analysis of forces that applied on pipe*

The buoyancy applied on Section $i-1$ $(i = 2,3,4,5)$ of steel pipes is

$$B_i = \rho g \pi (\frac{D_i}{2})^2 l_i = \frac{\pi}{4} D_i^2 \rho g l_i \qquad (5)$$

Where D_i is its diameter, l_i is its length.

The marine environmental load on it is

$$F_i = 374 \cdot D_i l_i v_2^2 \cos\theta_i \qquad (6)$$

Section $i-1$ of pipes remains in a state of rest, so its static equilibrium equations are

$$F_i + T_{i-1} \sin\theta_{i-1} = T_i \sin\theta_i \qquad (7)$$

$$T_{i-1} \cos\theta_{i-1} + B_i = T_i \cos\theta_i + G_i \qquad (8)$$

Where G_i is its gravity, T_i is the tension at the bottom of Section $i-1$ of the steel pipe, θ_i is the tilt angle of the steel pipe.

C. The analysis of forces applied on the steel drum and the heavy ball

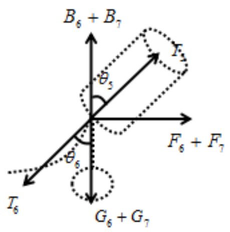

Fig. 3. *The analysis of forces applied on the drum and ball*

The buoyancy applied on them is

$$B_6 + B_7 = \rho g \pi (\frac{D_6}{2})^2 l_6 + \rho g \cdot \frac{m_7}{\rho_0} \qquad (9)$$

The marine environmental load on them is

$$F_6 + F_7 = 374 v_2^2 \cdot [D_6 l_6 \cos\theta_6 + \pi (\frac{3m_7}{4\pi\rho_0})^{\frac{4}{3}}] \qquad (10)$$

Their static equilibrium equations are

$$F_6 + F_7 + T_5 \sin\theta_5 = T_6 \sin\theta_6 \qquad (11)$$

$$T_5\cos\theta_5 + B_6 + B_7 = T_6\cos\theta_6 + G_6 + G_7 \quad (12)$$

where G_6, G_7 are their gravity, B_6, B_7 are the buoyancy acted on them, T_6 is the tension at the top of the chain, θ_5 is the drum's tilt angle, θ_6 is the tilt angle of the chain line's tangent at the junction with the drum.

D. The analysis of forces applied on the anchor

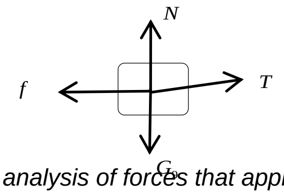

Fig. 4. *The analysis of forces that applied on the anchor*

Since the tilt angle of the chain line's tangent at the link with the anchor is very small and even none, it is assumed that the tension that the anchor take, has no component force in the vertical direction. As the anchor's volume is very small, its buoyancy is negligible. The static equilibrium equations of the anchor are approximately

$$f = T \quad (13)$$

$$N = G_9 \quad (14)$$

Where f is the static friction force exerted on it, N is the seabed touchdown, G_9 is the gravity of the anchor.

E. The shape of the chain line [3]-[6]

Catenary and quadratic parabola model are two ways of static analysis of the chain line. Since the catenary equation is transcendental and the tilt angle of the line's tangent at the link with the anchor is not equal to 0, the solutions are very complex and difficult to calculate. When the angle is small, quadratic parabola can be used instead of catenary. In this paper, quadratic parabolic is used to simulate the shape of the chain line.

a) When the chain line is tangent to the seafloor, we pick the origin *O* to be the point where the anchor is, put the seafloor line as the *x* axis and vertical line as the *y* axis, and set up the plane rectangular coordinates system *xoy*.

It assumed that *w* is the self-gravity per unit length of the chain, *F* (*F*=*f*) is horizontal force of its tension and x_0 is the length of its part on the seafloor. When the anchor line is tangent to the seabed, based on the geometric relationship between the curve and force, the parabolic curve equation is

$$y = \begin{cases} \frac{w}{2f}(x - x_0)^2, x > x_0 \\ 0, 0 \le x \le x_0 \end{cases} \quad (15)$$

The length of its part on the seafloor is

$$x_0 \approx l_7 - \sqrt{\frac{2fh_1}{w} + h_1^2} \quad (16)$$

where h_1 is its vertical projection's height, l_7 is its length.

b) When the chain line and the seabed have a certain angle, we pick the origin to be the point *O*, put the tangent as the *x'* axis and the perpendicular line of the *x'* axis as the *y'* axis to set up System *x'oy'*. Because the angle that named α between *ox* and *ox'* is very small, it can be assumed that the *y* axis and *y'* axis are approximately coincident and *f* is approximately parallel to the *x* axis.

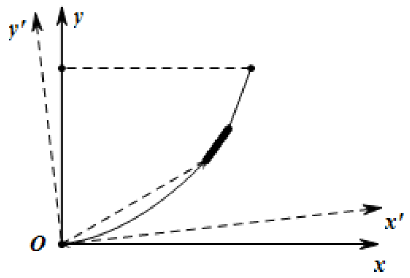

Fig. 5. *The model of the system*

In *x'oy'*, the equation is $y' = \frac{w}{2f}x'^2$,

so in System *xoy*, the equation turns into

$$y = \frac{w}{2f}x^2 + x\tan\alpha \quad (17)$$

On the basis of $L_1\tan\alpha + \frac{w}{2f}L_1^2 = L_1\tan\beta$

and $\tan\beta = \frac{h_1}{L_1}$, we can get

$$\alpha = \arctan\left(\frac{h_1}{L_1} - \frac{wL_1}{2f}\right) \quad (18)$$

The horizontal projection length of the chain line is

$$L_1 \approx \sqrt{l_7^2 - h_1^2} \quad (19)$$

F. *The analysis of forces of the whole system*

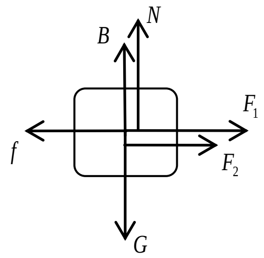

Fig. 6. *The analysis of forces of the whole system*

The whole system static equilibrium equations are

$$\begin{cases} f=\sum_{i=1}^{7}F_i \\ G=B+N=\sum_{i=1}^{9}G_i \\ B=\sum_{i=1}^{9}B_i \end{cases} \quad (20)$$

For the whole system, we can see that

$$H=h+\sum_{i=2}^{6}l_i\cos\theta_i+h_1 \quad (21)$$

where *G* is gravity of the whole system, *B* is its buoyancy, *H* is the depth of the sea.

III. System Parameters At Two Wind Speeds

From (19) and (20), we establish nonlinear multivariable equation systems. The buoy' s draft depth is

$$h=(\sum_{i=1}^{9}G_i-\sum_{i=2}^{9}B_i)\ /(\frac{\pi}{4}\rho_0 gD_1^2) \quad (22)$$

If we select the chain in model II with length of 22.05m, the heavy ball with weight of 1200kg and the density of seawater is 1025 kg/m3, the density of steel is 7850 kg/m3, we can calculate the self-gravity per unit length of the chain is $w=59.59N/m$. If seawater is still and sea is 18 meters deep, when $v_1=12m/s$ and $v_2=12m/s$, the chain line's equation is (15). In this case, the solutions can be solved by Newton-iterative method.

1) When $v_1=12m/s$,tilt angles ($\theta_i/°$) of steel pipes and the steel drum from top to bottom are 1.1103, 1.1176, 1.1249, 1.1323, 1.1398. The length of its part on the seafloor (x_0) is 9.2744 meters long.

2) When $v_1=24m/s$,tilt angles ($\theta_i/°$) of steel pipes and the steel drum from top to bottom are 4.4330, 4.4617, 4.4908, 4.5203, 4.5501. The length of its part on the seafloor (x_0) is 0.3802 meters long.

IV. The Change Of The Heavy Ball's Weight

Under the assumption of Section III, wind speed increases to 36m/s. In this case, the line's equation is (17) and the other equations are the same to the front ones. The tilt angle of the steel drum is 8.506 degrees and the tilt angle of the tangent at the link of the chain and anchor is 19.931 degrees. In this case, the anchor will be dragged and acoustic communication sets work poorly. It is necessary that the heavy ball's weight should be increased to adjust the system.

As the weight of the ball gradually increases, the angles are all decreasing and the tilt angle of the drum firstly decreases to 5 degrees. When the weight of the ball continues to increase to 2200kg, the tangent's tilt angle is just 15.9034 degrees and the drum's tilt angle is 4.9841 degrees. In this condition, the system works properly.

V. System Design And Sensitivity Analysis

A. *System Design* [7] [8]

Due to some factors such as tidal, sea environment including water depth, flow speed and wind speed, is changing. Based on the analysis of forces of the system, we establish a multi-objective optimization model

$$\begin{cases} \min[\theta_5,h,R] \\ 0°\le\theta_5\le5° \\ 0°\le\alpha\le16° \end{cases} \quad (23)$$

where the model, length of the chain and the ball's weight are decision variables. We make data discrete among the range of variables and enumerate all the combination plans to seek for the optimal solution.

When water depth changes between 16 meters and 20 meters, sea water velocity can reach 1.5m/s and wind speed can reach 36m/s, the better solution is that the chain's model is V, its length is 20.03m and the ball weighs 3900kg.

B. *Sensitivity Analysis* [9]

Under the optimal solution, sensitivity analysis is conducted to assess the influence of sea water velocity, wind speed and water depth on the system parameters[8].

a) When $v_1 = 36m/s$, $v_2 = 1.5m/s$,under different water depths, the parameters change as follows.

TABLE II. PARAMETERS UNDER DIFFERENT WATER DEPTHS

Water depth (H/m)	*Title angles* ($\theta_i/°$)	*Partial length* (x_0/m)	*Draft depth* (h/m)
16	4.62/ 4.68/ 4.75/ 4.82 /4.89	2.59	1.61
17	4.62/ 4.68/ 4.75/ 4.82/ 4.89	1.24	1.62
18	4.62/ 4.68/ 4.75/ 4.82/4.89	0	1.62
19	4.62/ 4.68/ 4.75/ 4.82/ 4.89	0	1.62
20	4.62/ 4.68/ 4.75/ 4.82/ 4.89	0	1.62

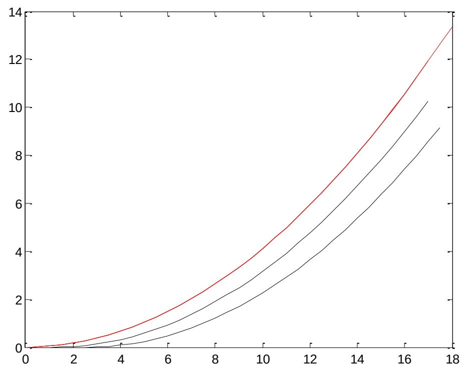

Fig. 7. *The shapes of chain under different water depths*

From TABLE II and Fig. 7, it is clear that water depth mainly affects the tilt angle of the tangent. The deeper the water, the bigger the tilt angle and the smaller the area instead. But it has little effect on the tilt angles of pipes and the drum.

b) When $v_1 = 36m/s$, $H = 18m$, under different water velocities, the parameters change as follows.

TABLE III. PARAMETERS UNDER DIFFERENT WATER VELOCITIES

Water velocity ($v_2/m/s$)	*Title angles* ($\theta_i/°$)	*Partial length* (x_0/m)	*Draft depth* (h/m)
0.3	1.00/ 1.00/ 1.00/ 1.01/ 1.02	8.76	1.58
0.6	1.45/ 1.46/ 1.48/ 1.49/1.50	7.17	1.59
0.9	2.21/ 2.23/ 2.26/ 2.28/2.31	5.00	1.60
1.2	3.26/ 3.31/ 3.35/ 3.39/3.44	2.52	1.61
1.5	4.62/ 4.68/ 4.75/ 4.82/4.89	0.00	1.62

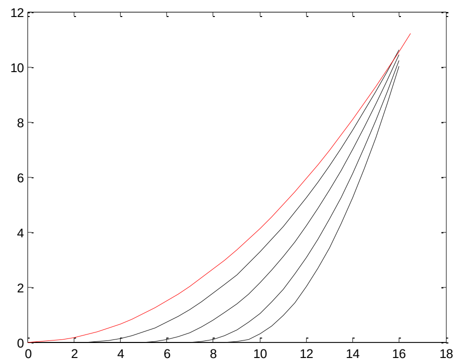

Fig. 8. *The shapes of chain under different water velocities*

From TABLE III and Fig. 8, we can find that water velocity has a great impact on the shape of the chain. And it is positively correlated with the tilt angles of pipes and the drums, but negatively correlated with the tilt angle of the tangent.

c) When $v_2 = 1.5m/s$, $H = 18m$, under different wind speeds, the parameters change as follows.

TABLEIV. PARAMETERS UNDER DIFFERENT WIND SPEEDS

wind speed ($v_1/m/s$)	*Title angles* ($\theta_i/°$)	*Partial length* (x_0/m)	*Draft depth* (h/m)
12	3.87/ 3.93/ 4.00/ 4.06/ 4.13	1.27	1.62
18	3.98/ 4.05/ 4.12/ 4.18/4.25	1.04	1.62
24	4.15/ 4.21/ 4.28/ 4.35/ 4.41	0.72	1.62
30	4.36/4.43/ 4.49/ 4.56/ 4.63	0.32	1.62
36	4.62/ 4.68/4.75/ 4.82/4.89	0.00	1.62

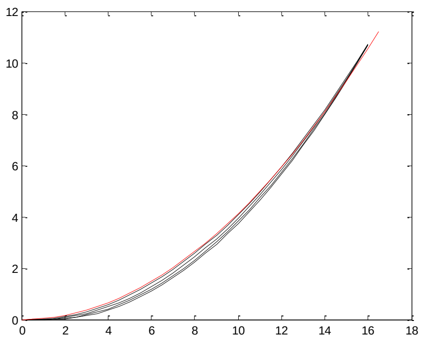

Fig. 9. *The shapes of chain under different wind speeds*

Comparing TABLE III and TABLE IV, we can see that the influence of wind speed on the tilt angles of pipes and the drums is similar to water velocity's. However, it has less influence on the chain's shape.

VI. CONCLUSIONS AND PROSPECT

Based on the 2016 CUMCA Problem A, the Single-Point mooring system is studied by setting up statics model and parabola Model in this paper. Within the allowable range of errors, the complex calculation is replaced with the approximation to simplify the model and the Newton-iterative method is used to seek exact solutions to the equations. After a series of studies and calculations, it is clear that water velocity, wind speed and water depth are three main variables that affect the system parameters. Water velocity and wind speed mainly affect tilt angles of pipes and the drums. Water depth mainly affects the tilt angle of the tangent and swimming area of the buoy. Besides, they all has a great effect on the chain's shape.

The statics and parabola model of the system needs to be improved. In theory, the dynamic analysis and the non-linearity of the mooring line will be the focus of research in the future.

ACKNOWLEDGE

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